

How to allocate portfolio weight based on the FF3F model

Jingyuan Gao^{1,*†}, Qinyu Yang^{2,†}

¹School of Finance, Renmin University, Beijing, China

²Fairmont Preparatory Academy, Anaheim, United States

*Corresponding author: 2021200076@ruc.edu.cn

†These authors contributed equally

Abstract. Selecting the correct stocks and funds is essential for portfolio management and allocation. In order to reduce the risk of the portfolio, it is better to diversify the assets and select the stocks within more than one sector. Another step would be setting the portfolio with the firefighter case or without it. The firefighter case illustrates that we suppose there is a firefighter who has 25 years of service as a firefighter who will be retired in Florida. The Florida government gives the retired firefighter two choices. He takes the lump-sum payout and invests his nest egg and the lump sum in supporting him throughout his retirement. Another one is that he takes the pension benefit and invests his nest egg in supplementing the pension benefit. In this case, we compared those two and finally selected the one in which he maintained the pension and made the pension weight precisely equal to 71%. The Sharpe ratio for this scenario is relatively higher than the another. Moreover, choosing the correct financial model. Comparing the pros and cons between the CAPM model and the Fama French Three-Factor model, we found that the Fama French Three-Factor model is more related to reality and includes the market risk, making the portfolios more accurate and reliable.

Keywords: Fama French three-factor model, Capital asset pricing model, portfolio management.

1. Introduction

In investment, mutual funds and stocks are two main parts of the capital market. A mutual fund company is a company that manages funds on behalf of small investors to invest their savings in profitable businesses and stock portfolios. Mutual fund facilitates small investors to get the benefits of capital investment through skillful and professional management [1]. Stock is a security that represents the ownership of a fraction of a corporation. Also, there are different sections of the industry in stocks. For instance, the most common sections can be the value section or the technology section since they usually provide a higher return which is exactly what investors want to see. A value stock is a stock with a price that appears low relative to the company's financial performance. The higher returns for value stocks are compensation for higher risk [2]. Due to the high risk, instead of using the value stock, we chose a raw material stock-U.S. Steel; online shopping stock-eBay; mutual funds of an investment bank-Goldman Sachs; a technology and industrial mutual funds-Brandywine Global; a technology and energy sources mutual funds-Adams Diversified Equity Fund. Overall, this portfolio involves a broad selection among the investment sectors. The correlation between those stocks or mutual funds is pretty small, which is ideal for us. The benefits of stock-bond diversification increase when the stock market is uncertain [3].

The ideal portfolio is more likely to have the best combination between the risk and the expected return, the highest Sharpe ratio. Sharpe ratio is calculated by using the risk-free return divided by the risk-free standard deviation of the portfolio. The proposed algorithm for the Sharpe ratio maximization introduces a feasible and straightforward way of portfolio optimization [4]. By comparing an optimal portfolio that minimizes the Value-at-Risk and at the same time coincides with the market portfolio on the Sharpe ratio portfolio, a confidence interval for the suggested risk measure of the Sharpe ratio portfolio is constructed and applied to actual data [5].

The CAPM model and the FF3F are two relatively good models that can be used as references for research. It was demonstrated that such conditional models perform far better than unconditional specifications and about as well as the Fama-French three-factor model on portfolios sorted by size

and book-to-market characteristics. According to Lettau, the conditional consumption CAPM can account for the difference in returns between low-book-to-market and high-book-to-market portfolios and exhibits little evidence of residual size or book-to-market effects [6]. However, the CAPM model has shortcomings. It is a single factor model, which is too idealization. There is a big gap between the model and the actual situation. For instance, evidence has mounted that the CAPM is inconsistent with numerous empirical regularities of cross-sectional asset pricing data [6]. The current study finds the three-factor model provides significantly improved explanatory power over the CAPM, and evidence that the BM factor plays a role in asset pricing [7]. The model suggests that the prices of certain portfolios that are related to the Fama-French HML and SMB hedge portfolio returns will carry information about investment opportunities [8-10]. That means the FF3F model adds the size risk and value risk factors to the market risk factor in CAPM, making it closer to reality.

2. Firm description

2.1 The basic information of the companies

The US steel is in the iron and steel industry. The company has over 30 bulk carriers responsible for transportation in the country, and dozens of subsidiaries. A number of subsidiaries and associated companies are located outside of the United States, including Canada, the United Kingdom, Italy, Germany, Spain, France, Japan, India, Nigeria, and Brazil. A wide variety of products are manufactured by the company, including mining of iron ore, coal, and dolomite, the manufacture of steel pipes, steel plates, rails, chemicals, and oil drilling and oil production equipment. The company is also involved in construction, shipbuilding, and other activities, such as the manufacture of tanks and heavy artillery.

eBay is an online auction and shopping site where buyers and sellers from around the world exchange goods. With an international web trading platform, eBay is one of the most popular online trading platforms in the world, catering to both individual and corporate users. Globally, eBay operates 26 sites including the main site in the United States, as well as sites in Austria, Australia, Belgium, Brazil, Canada, China, France, Germany, Hong Kong, India, Ireland, Italy, South Korea, Malaysia, Mexico, the Netherlands, New Zealand, the Philippines, Poland, Singapore, Spain, Sweden, Switzerland, Taiwan, the United Kingdom, and Argentina.

Goldman Sachs is an international investment bank, one of the world's oldest and largest investment banks, headquartered in New York, and has been named to the Fortune U.S. 500 list for many consecutive years. Goldman Sachs Large Cap Core Fund investor class (GSPTX) constitutes a relatively significant share of the Consumer cyclical and healthcare

Brandywine Global focuses on Industrials and technology. Adams Diversified Equity Fund focuses on information technology and the energy industry (holdings include Apple, Google, Adams natural resources fund, Pepsi...)

2.2 Stock return

Here are the line charts of the historical stock return (this month's closing price/ Last month's closing price) data. We picked the monthly data for 5 years from Yahoo.finance.com. (from 2017/5/1 to 2022/3/1)

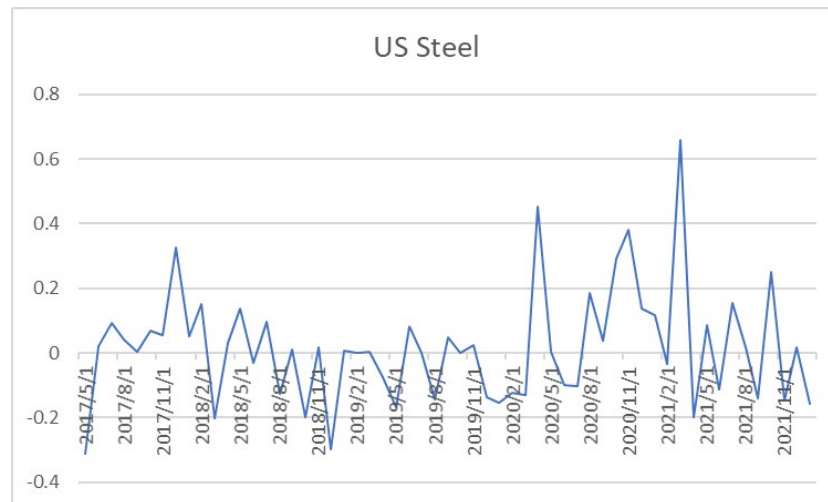


Figure 1. US Steel's return

For more than half the time the return is positive. At the highest point, the return is even above 0.6.

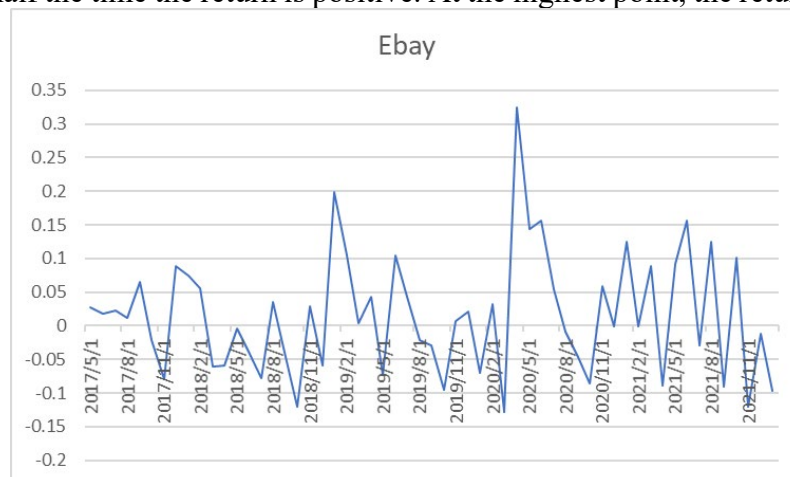


Figure 2. eBay's return

Most of the time the return is positive. In recent 2 years, the stock's return has been positive for almost all periods of time.

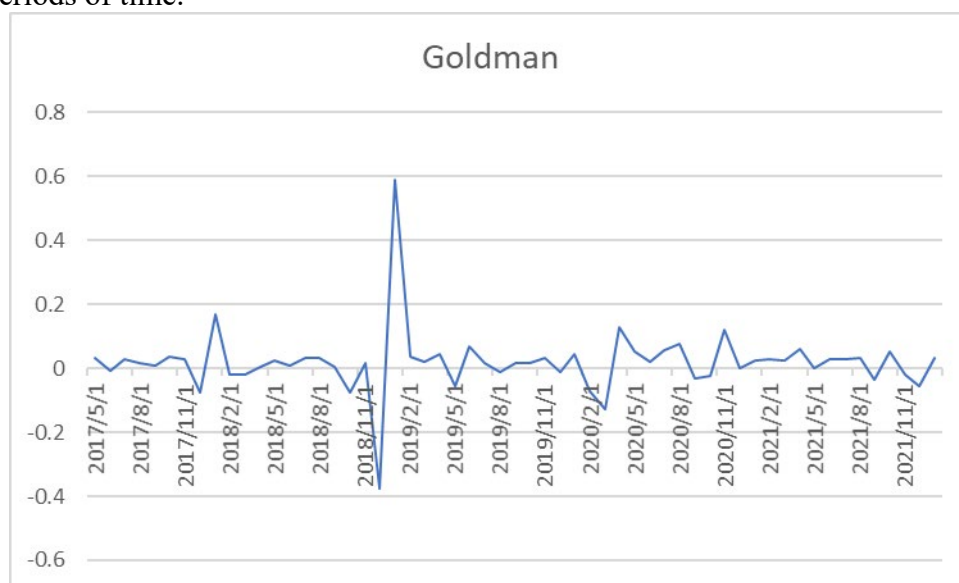


Figure 3. GSPTX's return

The return on this stock is steadily positive for nearly 5 years. In May 2019, the return was even closed to 0.6.

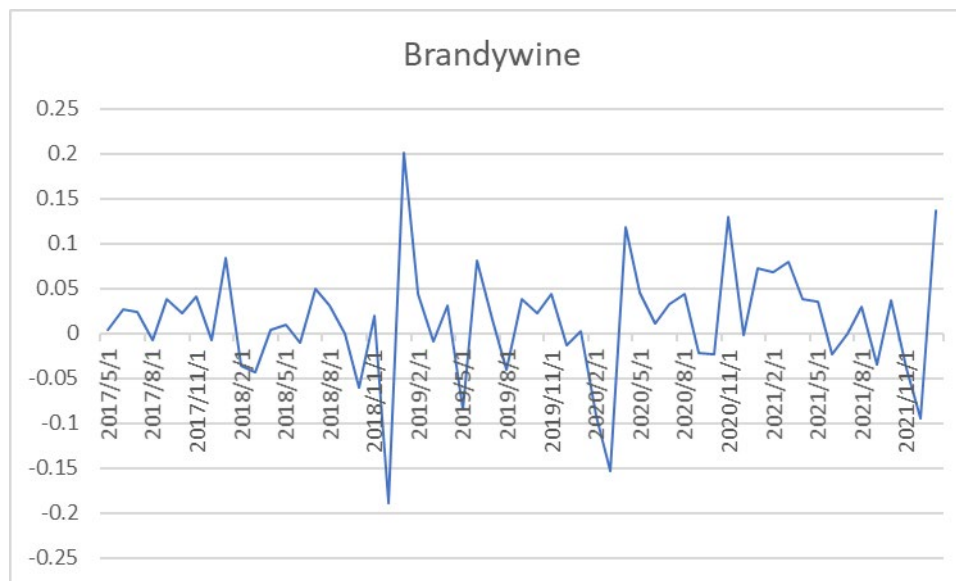


Figure 4. Brandywine's return

Most of the time the return is positive. But the return is small. Even the maximum value is only about 0.2.

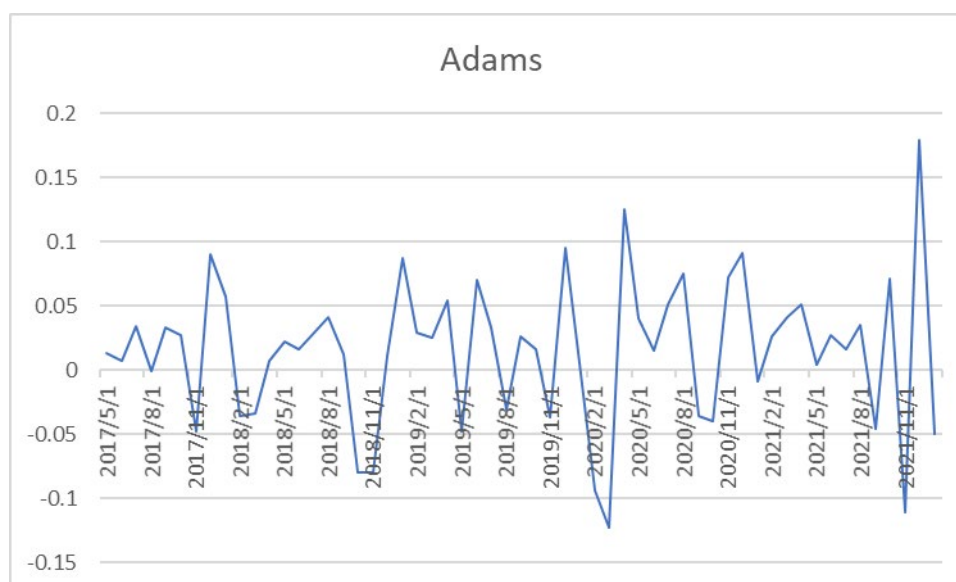


Figure 3. ADX's return

Most of the time the return is positive. The return is small. Even the maximum value is only about 0.15.

3. Methods

3.1 The CAPM model and the FF3F model

There are two ways of calculating the optimal weights for the portfolios. Firstly, we chose the CAPM model to do it. CAPM is known as the capital asset pricing model, which determines the expected return by pricing risky securities. In other words, it helps you to generate the expected return for assets and is close to actual society. Another way is the FF3F Model (Fama French Three-Factor Model), an asset pricing model developed in 1992 that expands on the capital asset pricing model (CAPM) by adding size risk and value risk factors to the market risk factor in CAPM. [1] After calculating the portfolio weights by using these two models, we find out that there's a slight difference between these values. We chose the FF3F eventually because it includes more risk factors such as SMB and HML. Also, it's more related to reality due to historical data within the calculation. In

addition, CAPM has some over-idealization, and it's a single factor model which influences the accuracy of the portfolio in some cases.

3.2 The formulas

These are the formulas throughout the whole process, listed following:

(1) The return equals the return rate, which measures the increase or decrease of the return for the following month.

(2) The variance illustrates the risk rate for a stock or portfolio; the higher variance corresponds to the higher risk.

(3) The covariance rate between two stocks emphasizes how these stocks are related. The higher the covariance links, the higher relations which demonstrate that when one stock increases or decreases, another stock will be more likely to go with the same trend. But the negative covariance rate will show an opposite tendency between two stocks, for example, if one stock increase, another stock will decrease relatively.

(4) The firefighter case we used sets the risk-free rate to 7.75% annually and with a weight of 71% in our portfolio. We can either choose to get the cash-out or put it inside of the firefighter case, which has the constant return back until you die.

(5) The Sharpe ratio measures the optimal combination between the risk and the expected return. In most cases, we want to maximize the Sharpe ratio in order to achieve our goal.

(6) The Fama French Three-Factor Model demonstrates the variable factors in reality, that matter and may bring effects to your portfolio.

$$r_t = (P_t + D) / (P_{t-1}) - 1 \quad (1)$$

$$\text{Var}(r) = 1/T * ((r_1 - r)^2 + (r_2 - r)^2 + \dots + (r_T - r)^2) \quad (2)$$

$$\text{Cov}(X, Y) = \sum E((X - \mu) E(Y - v)) / n - 1 \quad (3)$$

$$1.45M = 100K / (1+r) + 103K / (1+r)^2 \quad (4)$$

$$\text{Sharpe Ratio} = (R_p - R_f) / \sigma_p \quad (5)$$

$$E(r_i - r_f) = \beta_i M E(r_m - r_f) + \beta_i \text{SMB} E(\text{SMB}) + \beta_i \text{HML} E(\text{HML}) \quad (6)$$

4. Results analysis

4.1 The correlation between each stock

Table1. correlation

	US Steel	eBay	Goldman	Brandywine	Adams
US Steel	1				
eBay	0.37415583	1			
Goldman	0.23033401	0.46991285	1		
Brandywine	0.03178948	-0.0498422	-0.0451134	1	
Adams	0.26105465	0.15115826	-0.0323645	-0.0417324	1

According to table 1, the correlation between one equity and another is moderate, which is less than 0.5. So there is little correlation between the different stocks and mutual funds, which makes sure that each stock can hedge the risk of the other stocks effectively.

4.2 Final results

4.2.1 Cash-out case

Table 2. Final results (Cash-out case)

	US Steel	eBay	Goldman	Brandywine	Adams
expected return	0.0164747	0.00836954	0.01047682	0.00919937	0.00703904
weight	0.03715466	-0.0193674	-0.1215855	0.69420295	0.40959533
Portfolio expected return	0.00844557				
variance	0.00265025				
Standard deviation	0.05148063				
Sharpe ratio	0.14714045				

According to table 2, the expected return of US Steel, eBay, Goldman, Brandywine, and Adams is respectively 0.0165, 0.0084, 0.0105, 0.0092, and 0.0070. The portfolio expected return equals 0.00845 while the standard deviation equals 0.0515. We got the highest Sharpe Ratio in all possible situations, which is 0.147, by optimizing the Sharpe Ratio with Excel's solver. This is the best way to allocate the portfolio weight between each equity in the theory.

In our portfolio, US Steel took up 3.715%, eBay -1.937%, GSPTX -12.159%, Brandywine 69.42%, ADX40.96%.

The weight of eBay and Goldman was minus because we wanted to short them to make the Sharpe Ratio higher.

4.2.2 With pension

Table 3. Final results (with pension case)

	US Steel	eBay	Goldman	Brandywine	Adams	pension
expected return	0.0164747	0.00836954	0.01047682	0.00919937	0.00703904	0.0062
weight	-0.0148976	0.00097659	-0.0600423	0.20994051	0.15402276	0.71
Portfolio expected return	0.00655118					
variance	0.00018787					
Standard deviation	0.01370644					
Sharpe ratio	0.41443948					

According to table 3, the expected return of US Steel, eBay, Goldman, Brandywine, Adams, and the Florida pension is respectively 0.0165, 0.0084, 0.0105, 0.0092, 0.0070, and 0.062. We got the highest Sharpe Ratio with the same process. The Sharpe Ratio was 0.414, which was more than twice as much as in the cash-out case, for which we believed that the firefighter should keep the pension rather than cash out.

In this case, US Steel takes up -1.49%, eBay 0.098%, GSPTX -6.159%, Brandywine 20.99%, ADX 15.40%, and the pension 71%, which is fixed in our assumption. We wanted to short US Steel and GSPTX to make the Sharpe Ratio higher.

5. Conclusion

We have built two portfolios based on two hypothetical scenarios using the FF3F model, which is more related to reality compared to the CAPM model. Initially, we picked 5 equities according to their performance and the feedback from other people. Then we ran a correlation chart in Excel to check if they can hedge each other's risk effectively. After ensuring our choices are not too correlative, we used FF3F Model (Fama French Three-Factor Model) to calculate their expected return, variance, and some other essential data that we needed. Eventually, we allocated the weight between each equity to make the highest Sharpe Ratio in our portfolio with Excel's solver.

In the cash-out case, US Steel takes up 3.715%, eBay -1.937%, GSPTX -12.159%, Brandywine 69.42%, ADX 40.96%. The Sharpe Ratio is 0.147. While in another case, US Steel takes up -1.49%, eBay 0.098%, GSPTX -6.159%, Brandywine 20.99%, ADX 15.40%, the pension 71%. The Sharpe Ratio was 0.414, which was more than twice as much as in the cash-out case, for which we believed that the firefighter should keep the pension rather than cash out.

The FF3F model can be used as one of the bases when we are investing. It is also a great tool for beginners to understand how a portfolio is built and which factors should be mainly concerned. This model is closer to the real situation than the traditional CAPM model by adding size risk and value risk factors to the market risk factor in CAPM. However, there is still a certain gap between the FF3F model and the real situation. In the future, our research will be focused on figuring out which other important factors that might influence the construction of a portfolio and improve the model.

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