

Sharpe-Ratio Related Portfolio Selection

Zhenghao Dong

Goizueta Business School, Emory University, Atlanta, Georgia, 30322, United States

zhenghao.dong@emory.edu

Abstract. This article investigates the criteria that individual investors should consider within the Sharpe Ratio perspective. Based on risk, return, and correlation, this research used Excel program to find the optimal portfolio and efficient frontier which offer the highest expected return for a defined level of risk or the lowest risk for a given level of expected return. Among the historical real data of 16 assets, this study found that the optimal portfolio contains assets that has lowest correlation and highest individual risk/return ratio. With this finding, investors can pick combination of domestic and international stocks from different sectors that has low correlation while maintain highest individual risk/return ratio. Therefore, the individual investors can tailor their own risk tolerance to build personalized portfolio with highest Sharpe Ratio.

Keywords: Sharpe-Ratio; Assets Correlation; Portfolio Selection; Stock Market.

1. INTRODUCTION

1.1 Background

In the United States, the stock markets were divided into different sectors, each sector having different reward to variability characteristics. It includes sectors such as Aerospace & Defense, Agriculture & Forestry, Consumer & Business Services, Retail & Food Services, Financial Services, Pharmaceuticals & Healthcare. As an investor, it is crucial to select portfolio assets in the right sectors and right market trend time periods. The correlation between international stocks and US stocks shows a lower level of correlation compared to domestic stocks between sectors [1]. The correlation changes as the world financial market enters the bear market and bull market.

As an individual investor, researching and selecting stocks is no doubt the most important part. It is unlikely for an investor with limited funding to invest in the number of assets as the private equity firm would. Prioritizing the investment budget into a few funds or stocks are within the capability and feasibility of small investors.

This research makes comparisons among three assets into the selected total weight to demonstrate the effect of changing the assets and make analysis. This is three assets scenario reflects the reality of small investors. Although individual has different preference when selecting stocks, return and risks are the factors that are most important to all of them. Therefore, it is crucial to invest with the reward to variability mindset when making investment decision.

1.2 Related research

Bodnar and Zabolotsky explained that based on the research finding, a confidence interval for the suggested risk measure of the Sharpe Ratio portfolio is constructed and applied to the data. It provides the investor with a strategy about how to construct an optimal portfolio. Moreover, the Sharpe Ratio portfolio is consistent with Markowitz's optimization idea since it lies along the efficient frontier in the mean-variance space and provides a desirable alternative to the global minimum variance portfolio [2-4]. Huang et al. explained that different investors have different attitudes towards investing. One is controlling risk and maximizing return. Second is controlling return and minimizing downside risk. Experiments showed that the proposed methods yield successful results [5]. Wagner and Lau explained that as an investor, there are two risks. Diversifiable risk or systematic risk. In the long run, the riskier portfolio performed better, we can conclude that the higher return comes with more risk. By diversifying the assets in a portfolio, we can effectively reduce the diversifiable risk [6].

Vyklyuk et al. explained that the developed neural networks model is successful to predict the position of the investor who will be rewarded with extra risk premium on assets for the defined level of risk or a greater risk premium which corresponds to growth risk. This research predicts Sharpe ratio for the future. The results showed that such ratios provide useful information that allows the selection of portfolio with higher investment returns than the market index such as Equal Weighted index or Market Cap index [7]. Butler and Joaquin explained that the correlation between domestic stocks and international stocks increases when entering the bear market. However, the diversification effect is taking the international correlation as a buffer for a potential downside return. When we need international assets in the bear market, it doesn't serve its function as we expected due to the increase of correlation between assets [8].

1.3 Objective

The goal of this research is to generate the reward to variability ratio (Sharpe Ratio), which measures the return of an investment compared to its risk and find the efficient frontier – the set of optimal portfolios that offer the highest expected return for a defined level of risk or the lowest risk for a given level of expected return. This paper will analyze the relationship between the correlation of individual assets and the Sharpe Ratio of the portfolio. This research will reveal several methods to select individual stocks that can ultimately lead to maximized Sharpe Ratio within the feasible efficient frontier.

2. Method and data

2.1 Definition and Theory

The Sharpe Ratio (reward to variability ratio) is the equation used to measure the desirability of the portfolio [9, 10]. $(\text{Return} - \text{Risk-free}) / \text{Standard deviation}$ would give investors the Sharpe ratio of the portfolio.

$$S = \frac{E(r_p) - r_f}{\sigma_p} \quad (1)$$

Markowitz portfolio theory explained that plotting risk on x-axis and return on y-axis, investors are able to visualize the line that plotted all the feasible point with the given assets combinations. Given the same risk, rational investors will always choose the portfolio along the efficient frontier - above the global minimum-variance portfolio. Adjusting the weight of each assets will differentiate the portfolio along the efficient frontier.

In figure 1, point G refers to the Global minimum-variance portfolio, point P refers to the tangent portfolio, and Point F refers to the risk-free asset. Efficient frontier is the blue line above point G.

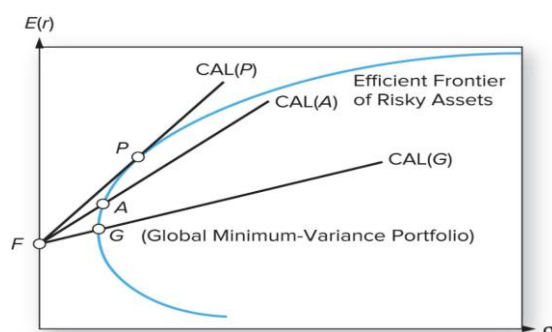


Fig. 1 Efficient frontier and Sharpe Ratio

2.2 Analysis

As shown in Table 1, this is the characteristics of optimal portfolio of 3 assets out of 16 assets that the research included. Based on normalized historical daily datasets among these 16 assets after

March 2020, the research selected the numerical value of the expected return, standard deviation, and correlation matrix, this serves as a foundation of the analyses below.

According to Table 2, the computation of Variance-Covariance matrix was computed with equation $Cor(AB) * Std(A) * Std(B)$.

To examine the relationship between risk and return on top of the variation of correlation between assets, this study used Excel to generate a graph that reflect the efficient frontier of the portfolio. The three input of the portfolio includes correlation between assets, weight of individual assets, the individual return, and standard deviation.

Table 1. Data features and correlation matrix

Asset	Annual data		Correlation Matrix		
	Expected Return	Standard Deviation	Asset 2	Asset 4	Asset 5
Asset (2)	8%	12%	1	0.10	-0.10
Asset (4)	13%	20%	0.10	1	0.15
Asset (5)	4%	6%	-0.10	0.15	1
Risk-free rate	2%				

Table 2. Variance-Covariance Matrix

	Asset A	Asset B	Asset C
Asset A	0.0144	0.0024	-0.0007
Asset B	0.0024	0.0400	0.0018
Asset C	-0.0007	0.0018	0.0036

To recall the Solver function of excel, the study set a constrain of weight total equals to 1. This is a true reflection of the reality since investors want to invest money into several different assets and all assets combined should equals to 100% of the money. Once successfully enter all the required input into the excel solver, the excel program is able to generate and visualize a Sharpe Ratio efficient frontier. In Table 3, it shows the optimal portfolio given the level of risks, the weight of assets changes in each expected return scenario. This is useful for investors to personalize their risk tolerance ability and pick the risk scenario that matches their level of favoured risk compared to the return. Table 4 and Table 5 are representation of global minimum variance portfolio and Optimal risky portfolio respectfully. Among the asset combination of Asset 2, Asset 3, and Asset 5, the Optimal Sharpe ratio is 0.77 according to Table 5.

Note that this is one of the computation out of hundreds that the research conducted based on all the combinations of 16 assets for finding the optimal portfolio. With different characteristics of the three assets that the Excel sheet began with, the Sharpe ratio result, weight, Optimal risky portfolio will all be different and specific.

Table 3. The weighted value and optimal results

Item	Condition 1	Condition 2	Condition 3	Condition 4	Condition 5
Min. Exp. Return (Constraint)	5.0%	6.0%	7.0%	8.0%	9.0%
w(A)	0.22	0.28	0.34	0.40	0.46
w(B)	0.02	0.10	0.18	0.27	0.35
w(C)	0.76	0.62	0.48	0.33	0.19
Sum (Weight Constraint)	1.00	1.00	1.00	1.00	1.00
Portfolio Variance (Target)	0.0026	0.0030	0.0042	0.0062	0.0090
Portfolio Standard Deviation	5.13%	5.48%	6.47%	7.87%	9.48%
Portfolio Expected Return	5.03%	6.00%	7.00%	8.00%	9.00%
Sharpe Ratio	0.59	0.73	0.77	0.76	0.74

Table 4. Global Minimum Variance Portfolio

Item	Value
w(A)	0.22
w(B)	0.02
w(C)	0.76
Sum	1.00
Portfolio Variance (Target)	0.0026
Portfolio Standard Deviation	5.13%
Portfolio Expected Return	5.03%
Sharpe Ratio	0.59

Table 5. Optimal Risky Portfolio

Item	Value
w(A)	0.35
w(B)	0.20
w(C)	0.45
Sum	1.00
Portfolio Variance	0.0045
Portfolio Standard Deviation	6.69%
Portfolio Expected Return	7.17%
Sharpe Ratio (Target)	0.77

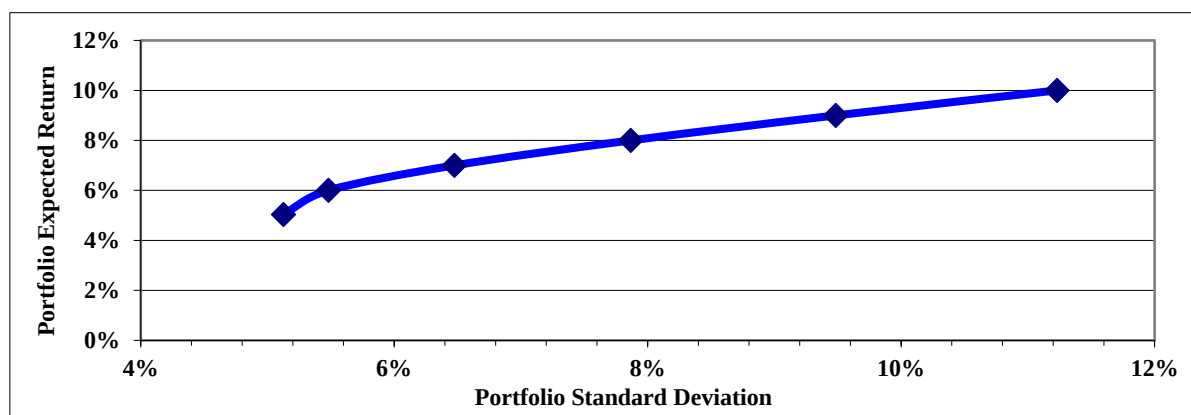


Fig. 2 Return vs. Risk

3. Results and Discussion

Taking the consideration of diversification effect, this study want to reduce the firm specific risk that led to volatility. Any added assets with the correlation of less than 1 will reduce the standard deviation. When introduce the concept of Sharpe Ratio, it generates the efficient frontier which allow the investor to visualize the portfolio combination that maximizes risk over return. Using method 2.1 and 2.2, investors can compute the efficient frontier for any possible combinations of three. Now this article focuses on the methods of selecting assets before we generate the efficient frontier to maximize the return. In other words, the characteristics that impact the investor's expected risk of return.

Utilizing the excel sheet, this study manually entered the numerical value of standard deviation, return, and correlation matrix for 16 combinations of assets ranging from cryptocurrency, ETFs, Mutual Funds, and stocks that performed well in specific sectors. Not surprisingly, the after several attempts, this research found out that controlling for the correlation between assets, the maximized Sharpe ratio was generated by the three assets with highest individual Sharpe Ratio. This is within

intuition, within the 16 assets, the top three assets with biggest individual Sharpe ratio will generate the biggest Sharpe Ratio overall when combined into a portfolio.

But this isn't enough, the correlation between sectors also plays an important role for the overall risk over return ratio. Holding assets return constant, the research now adjusting the numerical value entered into the correlation matrix of the Sharpe Ratio Excel Sheet to reveal the relationship between correlation and Sharpe Ratio. The study began with correlation of 1 between all three assets, and the maximized Sharpe Ratio portfolio is just a weight of 100% of 1 asset with highest individual Sharpe ratio and 0% among other two. In reality, this defeats the purpose of diversification effect and it is unlikely to have three assets with correlation of 1. To start from here, this research enter the correlation value corresponds to the 16 assets that initially computed, the result became clear after several trials. Holding return and standard deviation constant, the assets that generate the most Sharpe Ratio value is the combination between lower assets correlation. The lower the correlation is, the higher the Sharpe Ratio. This confirms the idea of diversification effect as well, since investors want to have assets with less similar pattern to avoid fluctuation that resulted a downside return.

Now the research is done with the Excel Calculation, the important question is, what does correlation even mean to the individual investors and how do investors find assets with lower correlation? The correlation table between US sectors are available on finance articles such as Investopedia and Yahoo Finance, with the historical value that is searchable everywhere, investors can refer to the correlation matrix among sectors to personalize the investment portfolio.

Clearly, the portfolio just contain US stocks will not be able to give us the lowest correlation even we diversified these through assets from different sectors. The correlation is lower across international assets [8]. But the bad news is that the correlation between international assets will increase when hitting the bear market. Investors are desperate to compute the Sharpe Ratio based on the factors of risks and return, but the correlation increase when we need it the most isn't an optimistic thing for the investor, but this still gives the portfolio with some advantage compared to domestic assets only portfolio.

4. Conclusion

The Sharpe Ratio helps the investors to understand the relationship between risk and return. It is crucial for risk averse investors to choose global minimum portfolio with lower expected return and profit driven investor investing in the tangent portfolio or higher return along the efficient frontier. This article illustrated the assets characteristics that generate the portfolio with optimal Sharpe Ratio. The among any given asset choices, the assets combination that has lowest correlation and the highest individual risk/return ratio will generate the portfolio with highest Sharpe Ratio. Specifically, investors can utilize the historical performance of stock and correlation between stocks, sectors, and country to further diversify the risk and optimize the investing portfolio with assets characteristics of highest Sharpe Ratio.

References

- [1] M. Landsberger, I. Meilijson, Demand for risky financial assets: a portfolio analysis. *Journal of Economic Theory*, 50 (1), 204-213, 1990.
- [2] W.H. Wagner, S.C. Lau, The effect of diversification on risk. *Financial Analysts Journal*, 27 (6), 48-53.
- [3] Z. Bai, H. Liu, W.K. Wong, Enhancement of the applicability of markowitz's portfolio optimization by utilizing random matrix theory. *Social Science Electronic Publishing*, 19 (4), 639-667, 2010.
- [4] N. Zhang, P. Chen, Z. Jin, S. Li, Markowitz's mean-variance optimization with investment and constrained reinsurance. *Journal of Industrial and Management Optimization*, 12 (1), 22-22, 2016.
- [5] K.K. Hung, C.C. Cheung, L. Xu, New sharpe-ratio-related methods for portfolio selection. In *Proceedings of the IEEE/IAFE/INFORMS 2000 Conference on Computational Intelligence for Financial Engineering (CIFEr)* (Cat. No. 00TH8520) (pp. 34-37). IEEE, 2000.

- [6] T. Bodnar, T. Zabolotskyy, How risky is the optimal portfolio which maximizes the Sharpe ratio? AStA Advances in Statistical Analysis, 101 (1), 1-28.
- [7] D. Vukovic, Y. Vykyuk, N. Matsiuk, M. Maiti, Neural network forecasting in prediction Sharpe ratio: Evidence from EU debt market. Physica A: Statistical Mechanics and its Applications, 542, 123331, 2020.
- [8] K. C. Butler, D.C. Joaquin, Are the gains from international portfolio diversification exaggerated? The influence of downside risk in bear markets. Journal of International Money and Finance, 21 (7), 981-1011, 2002.
- [9] O. Ledoit, M. Wolf, Robust performance hypothesis testing with the sharpe ratio. IEW - Working Papers, 2008.
- [10] L. M. Wolf, Robust performance hypothesis testing with the sharpe ratio. Journal of Empirical Finance, 2008.