

Analysis on Portfolio Selection under Markowitz Model and Index Model before and After Covid-19 Outbroke

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Abstract. The worldwide outbreak of COVID-19 in early 2020 has brought great challenges to global economic development. Against this background, how to choose a better portfolio to invest in becomes a problem. The purpose of the study is to find the effect of COVID-19 on different type of portfolios under one benchmark and four constraints by utilizing Markowitz model and Index model. This paper selects one stock index and six stocks from three different industries to form a portfolio. Portfolio which only consider risks and portfolio which consider both risks and returns under the Markowitz model and the index model with different constraints are presented respectively. The empirical results show that the portfolios that performed best before COVID-19 were difficult to choose. However, statistics after COVID-19 shares similarity. In general, the portfolio when the weight of SPX is zero shows the best performance in the Markowitz model, and the portfolio when the sum of the absolute values of all weights is less than or equal to two shows the best performance in the index model. This paper also contributes to provide a guideline to conduct investors to choose better investment portfolios by different models after COVID-19.

Keywords: Markowitz model, Index model, COVID-19.

1. Introduction

On March 9th, 2020, after the outbreak of the COVID-19, the S&P 500 index fell by more than 7%, and the US stock market had a trading curb again after more than 20 years. The outbreak of the COVID-19 has had a negative impact on social order, national health and economic development. The investment market is a microcosm of the economy, and the most reactive of these is the stock market. The sharp fluctuations in the stock market are the concrete embodiment of changes in economic fundamentals, changes in business conditions and fluctuations in investors' mentality. However, the uncertain scenario reinforces the importance of diversified risk allocation. Diversified portfolios tend to have a higher ability to combat risk. To study the impact of major public health events on investment portfolios, this paper selected two stocks in each of the three sectors and the S&P index as research objects, using mathematical analysis methods and using the Markowitz model and index model to observe the changes in portfolios before and after the COVID-19.

Since the 21st century, public emergencies have occurred in various countries around the world. By studying the impact of natural events and disasters in Australia on Australian stock market returns, it has been concluded that natural disasters have a subtle impact on returns [1]. In contrast, there was a study that revealed the returns of Taiwan's biotechnology industry were greatly affected by the spread of infectious diseases and conducted investors should adjust portfolio allocation accordingly during outbreaks of major infectious diseases [2]. It also has been found that the Ebola outbreak event effect was more pronounced for those stocks whose companies with operations in the West African countries (WAC) and the United States [3]. This is because psychological reactions such as psychological imbalance and loss of control might be caused by the uncertainty of infectious public health events [4].

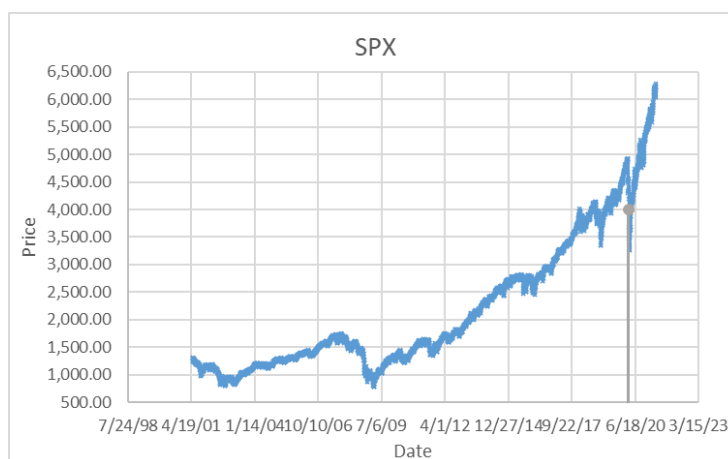
Since the outbreak and spread of COVID-19 in the world in early 2020, global stock markets have been violently shaken. Through intervention analysis, studies have shown that COVID-19 had a negative influence on most of the expected levels of returns of stock index [5]. With the outbreak and spread of the epidemic in Europe and the United States, COVID-19 has evolved from a major public health event to a financial crisis. More and more scholars have conducted in-depth and extensive research on the relationship between the epidemic and the stock market. It has been deduced that the

main reasons of the volatility in the U.S. stock market due to COVID-19 are government restrictions on commercial activities and social distancing requirements [6]. The general patterns of country-specific risks and systemic risks in the global financial markets were tried to be excogitated by some scholars [7]. A study based on Chinese stock returns also revealed that the impact of COVID-19 on different industries varies [8].

This paper uses the Markowitz model, which was first proposed by Harry Markowitz in 1952[9], and the index model, which was investigated by William F. Sharpe in 1964[10], to conduct an empirical study of the impact of COVID-19 on investment portfolios. It is based on a study which only carried out a theoretical analysis about the Markowitz model and the index model [11]. This paper aims to analyze the effect of COVID-19 on different portfolio types under varying constraints between the Markowitz model and the index model. Statistics gives the result that the greatest return of portfolio in the Markowitz model and the index model are under different constraint no matter before or after COVID-19. It has been shown that the highest return of minimum variance portfolio and maximum Sharpe ratio portfolio can be obtained in the Markowitz model when the weight of SPX is zero after COVID-19. When it comes to the index model, the portfolio has the greatest return when the sum of the absolute values of all weights is less than or equal to two after COVID-19. The research in this paper enriches the study of the economic consequences of infectious public health events which contributes to conduct investors to choose better investment portfolios varying different models after COVID-19.

2. Data

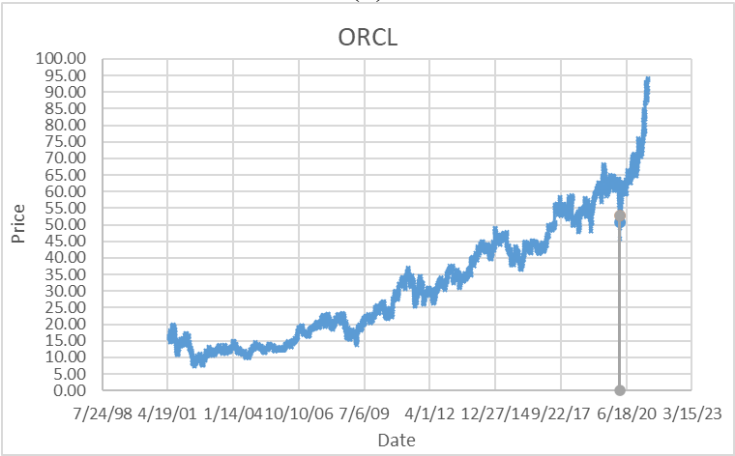
This paper investigates the portfolios performance covering three industries: technology, energy, consumer defensive. The figures of technology part were selected from Akamai Technologies Inc (AKAM), a company that provides Internet content protection and delivery solutions as well as business applications, and Oracle Corporation (ORCL), a company that provides products and services that solve the problems of enterprise information technology environments. The figures of energy part were chosen from Exxon Mobil Corporation (XOM) which is engaged in the exploration, production, trading, transportation and sales of crude oil and natural gas and Imperial Oil Limited (IMO) which operates in all phases of the oil industry in Canada, including exploration, production and sales of crude oil and natural gas. The consumer defensive part used the figures from the Coca-Cola Company (KO), a company that owns, licenses and sells non-alcoholic beverage brands, and PepsiCo, Inc (PEP), a company which is in possession of all aspects of the food and beverage. Data of stock price employed in this paper from May 11st, 2001 to May 12nd, 2021. March 9th, 2020 was selected as the before and after COVID-19 split point because on this day the U.S. stock market had a trading curb again after 23 years. In the two weeks since, the U.S. stocks have gone through three more circuit breakers. Figure 1 demonstrates the trend of these companies' stock prices.



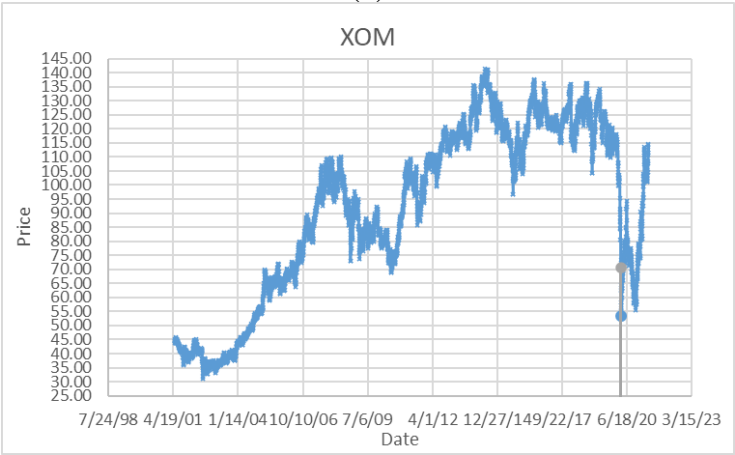
(a)



(b)



(c)



(d)

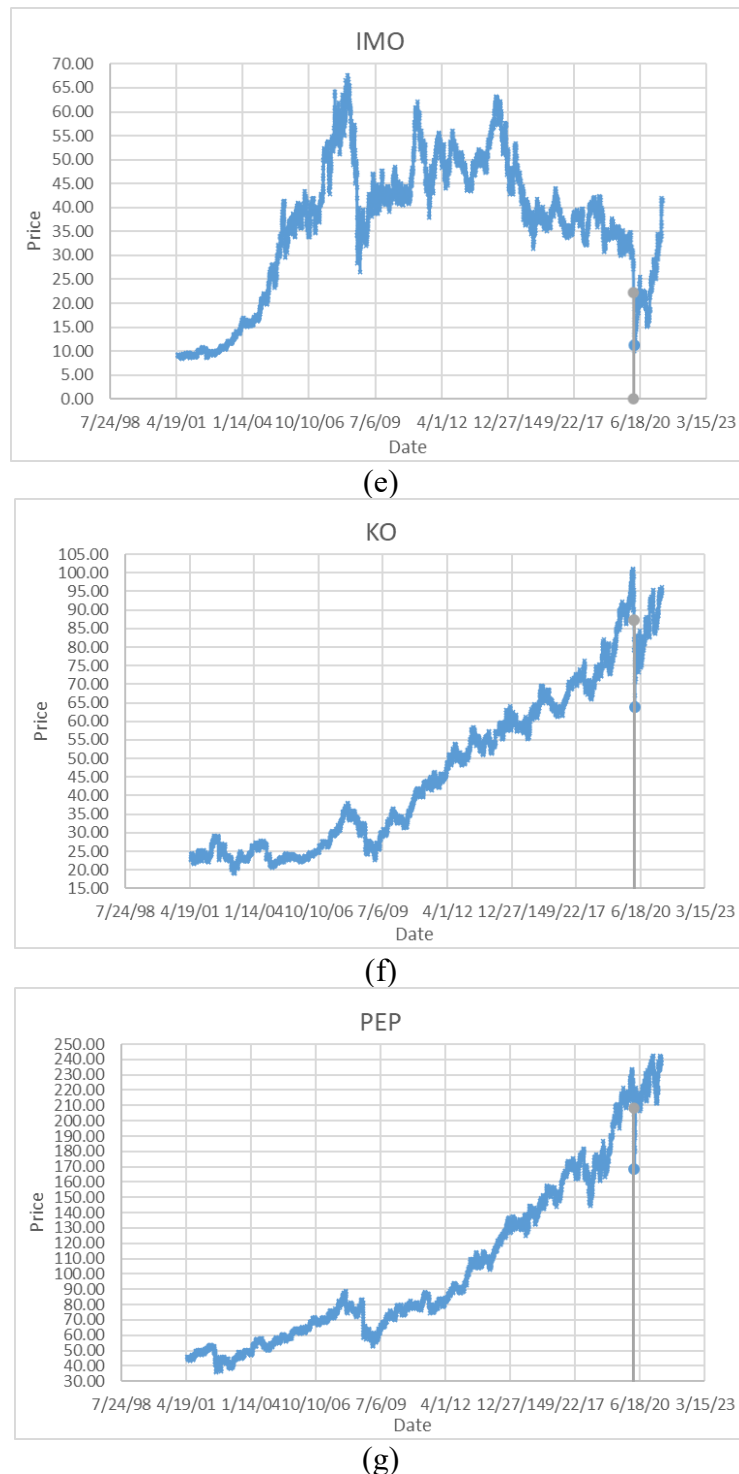


Figure 1. Stock prices for individual stocks and SPX. (a)SPX500 (b)AKAM (c)ORCL (d)XOM (e)IMO (f)KO (g)PEP

Table 1. Correlation between Stock Returns before and after Covid-19

Before	SPX	AKAM	ORCL	XOM	IMO	KO	PEP
SPX	1.000	0.410	0.554	0.544	0.450	0.452	0.491
AKAM	0.410	1.000	0.245	0.084	0.157	0.087	0.093
ORCL	0.554	0.245	1.000	0.313	0.231	0.031	0.182
XOM	0.544	0.084	0.313	1.000	0.611	0.312	0.213
IMO	0.450	0.157	0.231	0.611	1.000	0.217	0.075
KO	0.452	0.087	0.031	0.312	0.217	1.000	0.547
PEP	0.491	0.093	0.182	0.213	0.075	0.547	1.000
After	SPX	AKAM	ORCL	XOM	IMO	KO	PEP
SPX	1.000	0.321	0.544	0.708	0.835	0.779	0.769
AKAM	0.321	1.000	0.143	-0.025	0.048	0.096	0.448
ORCL	0.544	0.143	1.000	0.419	0.449	0.671	0.659
XOM	0.708	-0.025	0.419	1.000	0.872	0.481	0.394
IMO	0.835	0.048	0.449	0.872	1.000	0.631	0.620
KO	0.779	0.096	0.671	0.481	0.631	1.000	0.821
PEP	0.769	0.448	0.659	0.394	0.620	0.821	1.000

Before March 9th, 2020, when the COVID-19 pandemic has not yet affected the stock market, the correlation between the selected stocks is relatively low, suggesting that the portfolio formed in the paper has low risk, and associates with a better diversification effect. However, during COVID-19 pandemic, the correlation between some stocks has increased substantially. For example, the correlation between SPX and IMO has grown from 0.450 to 0.835.

3. Model

The portfolio in this paper is formed as:

$$\text{portfolio} = w_{SPX} * SPX + w_{AKAM} * AKAM + w_{ORCL} * ORCL + w_{KOM} * KOM + w_{IMO} * IMO + w_{KO} * KO + w_{PEP} * PEP \quad (1)$$

Where w_{SPX} , w_{AKAM} , w_{ORCL} , w_{KOM} , w_{IMO} , w_{KO} and w_{PEP} are the weights to the selected stocks respectively.

Two models, the Markowitz model and the index model, have been investigated in this model.

Markowitz model, a portfolio optimization model, is also known as the mean-variance model. In 1952, Harry Markowitz [1] proposed the venture capital model. It aims to create the most return-to-risk efficient portfolio by analyzing various portfolio combinations based on expected returns and standard deviations of the assets. Markowitz model is based on the method of mathematical statistics and there are a lot of prerequisites. The basic assumption of Markowitz Model theories are following these. Firstly, investors consider each investment choice based on the probability distribution of the return on the security over a certain holding time. Secondly, the risk of a portfolio is estimated based on the expected rate of return and variance of the security. Thirdly, investors' decisions only depend on the risk and return of the security. Fourthly, for the same level of risk, investors expect maximum return, and for the same level of return, investors expect minimum risk. According to Markowitz Model, the return of portfolio is calculated as:

$$R_p = \sum w_i * R_i \quad (2)$$

And the variance of portfolio is calculated as:

$$\sigma_p^2 = \sum \sum w_i * R_i * \text{Cov}(R_i, R_j) \quad (3)$$

Where w_i represents the investment ratio of stock i , R_i and R_j represent the return of stock i and j .

Next is the index model which is a statistical model of security returns. The most widely used technique is the single-index model which uses a market index such as the S&P 500 to represent common or systematic risk factors. It specifies systematic uncertainty and unique uncertainty. After the Markowitz model was established, it was found that the Markowitz model had some drawbacks. Therefore, the index model was set up to simplify the portfolio analysis process and enhance the analysis of security risk premiums. The model represents the return of stock i as:

$$R = \alpha_i + \beta_i * R_{\text{market}} + e_i \quad (4)$$

Where α_i is the excess return, β_i is the sensitiveness to the market return, R_{market} is the return to the market portfolios, e_i is the residual return.

To examine the performance of the portfolio, this paper applies one benchmark and four constraints.

Benchmark: A “free” problem, which illustrates how the area of permissible portfolios in general look like if there are no constraints.

No Constraint

Constraint 1: This additional optimization constraint is designed to simulate the Regulation T by FINRA(<https://www.finra.org/rules-guidance/key-topics/margin-accounts>), which means the sum of the absolute values of all weights is less than or equal to two.

$$\sum_{i=1}^7 |w_i| \leq 2 \quad (5)$$

Constraint 2: This additional optimization constraint is designed to simulate some arbitrary “box” constraints on weights, which means the absolute value of any weight is less than or equal to one.

$$|w_i| \leq 1, \text{ for } \forall_i \quad (6)$$

Constraint 3: This additional optimization constraint is designed to simulate the typical limitations existing in the U.S. mutual fund industry: a U.S. open-ended mutual fund is not allowed to have any short positions, for details see the Investment Company Act of 1940, Section 12(a)(3) (<https://www.law.cornell.edu/uscode/text/15/80a-12>), which means any weight is greater than or equal to zero.

$$w_i \geq 0, \text{ for } \forall_i \quad (7)$$

Constraint 4: Lastly, the constraint is designed to see if the inclusion of the broad index into the portfolio has positive or negative effect, which this paper made the weight of SPX equal to zero.

$$w_{\text{SPX}} = 0 \quad (8)$$

4. Results

In this section, portfolio performance was analyzed in two different types of investment portfolios, minimum variance portfolio and maximum Sharpe ratio portfolio.

First, this paper discussed the minimum variance portfolio which means only consider risks under one benchmark and four constraints in the Markowitz model and the index model before and after COVID-19.

Table 2. Weights of Minimum Variance Portfolios under the Markowitz Model before and after COVID-19

Before	SPX	AKAM	ORCL	XOM	IMO	KO	PEP
Benchmark	0.232	-0.010	0.037	0.152	0.040	0.219	0.332
Constraint 1	0.232	-0.010	0.037	0.152	0.040	0.219	0.332
Constraint 2	0.232	-0.010	0.037	0.152	0.040	0.219	0.332
Constraint 3	0.204	0.000	0.037	0.161	0.038	0.221	0.339
Constraint 4	0.000	0.002	0.078	0.197	0.059	0.264	0.398
After							
Benchmark	0.687	-0.069	0.235	0.238	-0.321	-0.253	0.484
Constraint 1	0.628	-0.022	0.240	0.230	-0.306	-0.173	0.401
Constraint 2	0.687	-0.069	0.235	0.238	-0.321	-0.253	0.484
Constraint 3	0.165	0.270	0.565	0.000	0.000	0.000	0.000
Constraint 4	0.000	0.125	0.262	0.323	-0.266	0.050	0.506

Table 3. Weights of Minimum Variance Portfolios under the Index Model before and after COVID-19

Before	SPX	AKAM	ORCL	XOM	IMO	KO	PEP
Benchmark	0.281	-0.028	-0.020	0.109	0.019	0.282	0.357
Constraint 1	0.281	-0.028	-0.020	0.109	0.019	0.282	0.357
Constraint 2	0.281	-0.028	-0.020	0.109	0.019	0.282	0.357
Constraint 3	0.213	0.000	0.000	0.112	0.020	0.290	0.366
Constraint 4	0.000	-0.017	0.022	0.161	0.034	0.352	0.448
After							
Benchmark	0.427	0.150	0.321	-0.070	-0.164	0.080	0.255
Constraint 1	0.427	0.150	0.321	-0.070	-0.164	0.080	0.255
Constraint 2	0.427	0.150	0.321	-0.070	-0.164	0.080	0.255
Constraint 3	0.000	0.218	0.454	0.000	0.000	0.035	0.293
Constraint 4	0.000	0.180	0.400	-0.036	-0.130	0.191	0.395

In this case, the two models showed almost the same results. Under the benchmark and the first and second constraint, the weight of PEP was the greatest before COVID-19. However, after COVID-19, the weight of SPX became the maximum accounting for more than half of the total. The weight of PEP accounted for the largest proportion before COVID-19 under the third constraint, and the weight of ORCL accounted for half of the total after COVID-19, which was the largest. Under the fourth constraint, whether before or after COVID-19, the weight of PEP was the maximum in the selected stocks.

Table 4. Minimum Variance Portfolios under the Markowitz Model before and after COVID-19

Before	Return	Standard Deviation	Sharpe Ratio
Benchmark	6.392%	11.614%	0.550
Constraint 1	6.392%	11.614%	0.550
Constraint 2	6.392%	11.614%	0.550
Constraint 3	6.612%	11.629%	0.569
Constraint 4	6.875%	11.768%	0.584
After			
Benchmark	18.968%	12.537%	1.513
Constraint 1	18.891%	12.568%	1.503
Constraint 2	18.968%	12.537%	1.513
Constraint 3	33.192%	15.937%	2.083
Constraint 4	12.667%	13.857%	0.914

Table 5. Minimum Variance Portfolios under the Index Model before and after COVID-19

Before	Return	Standard Deviation	Sharpe Ratio
Benchmark	5.996%	11.196%	0.536
Constraint 1	5.996%	11.196%	0.536
Constraint 2	5.996%	11.196%	0.536
Constraint 3	6.690%	11.343%	0.590
Constraint 4	6.460%	11.499%	0.562
After			
Benchmark	9.703%	14.243%	0.681
Constraint 1	10.923%	16.033%	0.681
Constraint 2	9.703%	14.243%	0.681
Constraint 3	9.216%	13.591%	0.678
Constraint 4	9.449%	13.908%	0.679

First of all, this paper analyzed the minimum variance of the Markowitz model and index model under one benchmark and four constraints before and after COVID-19. Before COVID-19, in Markowitz model, portfolios under the benchmark and the first and second constraint have exactly the same result with the smallest standard deviation. Portfolio with the constraint that the weight of SPX is zero has the greatest return and the highest standard deviation. However, in index model, return of portfolio under the third constraint has the highest value and middle standard deviation. Portfolios under the benchmark and the first and second constraint still share the same results. While after COVID-19, in Markowitz model, the return and standard deviation of minimal variance under the third constraints are the biggest. Portfolios under the benchmark and the second constraint share the same return and standard deviation. Then come to the index model, portfolio with the sum of the absolute values of all weights is less than or equal to two has the greatest return and the highest standard deviation. Portfolios under the benchmark and the second constraint still share the same results.

Second, this research investigated the maximum Sharpe ratio portfolio which means consider both risks and returns under one benchmark and four constraints in the Markowitz model and the index model before and after COVID-19.

Table 6. Weights of Maximum Sharpe Ratio Portfolios under the Markowitz Model before and after COVID-19

Before	SPX	AKAM	ORCL	XOM	IMO	KO	PEP
Benchmark	-0.424	0.127	0.204	-0.117	0.232	0.374	0.604
Constraint 1	-0.392	0.124	0.196	-0.108	0.223	0.365	0.592
Constraint 2	-0.424	0.127	0.204	-0.117	0.232	0.374	0.604
Constraint 3	0.000	0.098	0.098	0.000	0.114	0.243	0.447
Constraint 4	0.000	0.099	0.125	-0.183	0.190	0.289	0.480
After							
Benchmark	1.716	0.124	1.897	-0.563	0.201	-1.087	-1.288
Constraint 1	0.565	0.066	0.868	-0.002	-0.059	-0.438	-0.001
Constraint 2	1.000	0.134	1.000	-0.137	-0.023	-0.570	-0.404
Constraint 3	0.040	0.173	0.787	0.000	0.000	0.000	0.000
Constraint 4	0.000	1.011	3.374	-0.907	0.840	-0.642	-2.676

Table 7. Weights of Maximum Sharpe Ratio Portfolios under the Index Model before and after COVID-19

Before	SPX	AKAM	ORCL	XOM	IMO	KO	PEP
Benchmark	-0.115	0.090	0.085	-0.026	0.042	0.376	0.548
Constraint 1	-0.130	0.102	0.096	-0.029	0.047	0.423	0.617
Constraint 2	-0.115	0.090	0.085	-0.026	0.042	0.376	0.548
Constraint 3	0.000	0.081	0.061	0.000	0.033	0.334	0.491
Constraint 4	0.000	0.084	0.067	-0.043	0.036	0.347	0.509
After							
Benchmark	1.298	0.149	0.757	-0.100	-0.099	-0.596	-0.409
Constraint 1	0.680	0.145	0.675	-0.054	-0.081	-0.308	-0.057
Constraint 2	1.000	0.166	0.775	-0.078	-0.084	-0.491	-0.289
Constraint 3	0.053	0.156	0.790	0.000	0.000	0.000	0.000
Constraint 4	0.000	0.257	1.171	0.015	0.044	-0.391	-0.095

In this case, the two models showed similar results. Both in the Markowitz model and in the index model, the weight of PEP was the largest under the benchmark and four constraints before COVID-19. However, in the Markowitz model, the weight of ORCL accounted for the largest proportion after COVID-19. In the index model, after COVID-19, the weight of SPX became the maximum under the benchmark and the first and second constraint. As for the third and fourth constraint, the weight of ORCL was the largest.

Table 8. Maximum Sharpe Ratio Portfolios under the Markowitz Model before and after COVID-19

Before	Return	Standard Deviation	Sharpe Ratio
Benchmark	11.482%	15.565%	0.738
Constraint 1	11.329%	15.361%	0.738
Constraint 2	11.482%	15.565%	0.738
Constraint 3	9.680%	13.648%	0.709
Constraint 4	10.430%	14.495%	0.720
After			
Benchmark	100.008%	28.788%	3.474
Constraint 1	45.210%	15.888%	2.846
Constraint 2	57.235%	17.695%	3.234
Constraint 3	35.853%	16.563%	2.165
Constraint 4	143.647%	46.665%	3.078

Table 9. Maximum Sharpe Ratio Portfolios under the Index Model before and after COVID-19

Before	Return	Standard Deviation	Sharpe Ratio
Benchmark	9.703%	14.243%	0.681
Constraint 1	10.923%	16.033%	0.681
Constraint 2	9.703%	14.243%	0.681
Constraint 3	9.216%	13.591%	0.678
Constraint 4	9.449%	13.908%	0.679
After			
Benchmark	53.092%	19.903%	2.668
Constraint 1	40.262%	15.812%	2.546
Constraint 2	49.179%	18.556%	2.650
Constraint 3	35.986%	16.745%	2.149
Constraint 4	50.866%	22.147%	2.297

Next, this paper analyzed the maximum Sharpe ratio of the Markowitz model and index model under one benchmark and four constraints before and after COVID-19. Before COVID-19, in Markowitz model, portfolios under the benchmark and the second constraint have exactly the same result with the greatest return and the highest standard deviation. However, in index model, return of portfolio under the first constraint has the highest value and biggest standard deviation. Portfolios under the benchmark and the second constraint still share the same results. While after COVID-19, whether in the index model or in the Markowitz model, there is a significant growth both in return and standard deviation, especially the return in the Markowitz model. In Markowitz model, the return and standard deviation of maximum Sharpe ratio under the fourth constraints are the biggest. Then come to the index model, portfolio with no constraint has the greatest return. Portfolio under the fourth constraint has the biggest standard deviation.

5. Conclusions

This paper investigates the impact of the COVID-19 outbreak on the stock market between different portfolio types and different constraints by the Markowitz model and the index model. It has been shown that the outbreak of COVID-19 had a significant impact on portfolio performance. Before COVID-19, the return of minimum variance portfolio was the greatest when the weight of SPX equal to zero in the Markowitz model, and when any weight is greater than or equal to zero in the index model. After COVID-19, the return of minimum variance portfolio became the maximum when any weight is greater than or equal to zero in the Markowitz model, and when the sum of the absolute values of all weights is less than or equal to two in the index model. When it comes to maximum Sharpe ratio portfolio, before COVID-19, it had the largest return when there is no constraint or the absolute value of any weight is less than or equal to one in the Markowitz model, and when the sum of the absolute values of all weights is less than or equal to two in the index model. After COVID-19, the maximum Sharpe ratio portfolio had the greatest return when the weight of SPX is equal to zero in the Markowitz model, and when there is no constraint in the index model. After COVID-19, whether in the Markowitz model or in the index model, there is a growth both in return and standard deviation. In a conclusion, statistics after COVID-19 revealed that the portfolio under the constraint that the weight of SPX is zero has the best performance in the Markowitz model, and the portfolio under the constraint that the sum of the absolute values of all weights is less than or equal to two has the best performance in the index model. The findings of the results obtained from this paper can guide investors on how to make better investment portfolios choices through two different models after the outbreak of COVID-19.

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