

An Empirical Investigation of Markowitz and Index model in the US Capital Market

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Abstract. This paper uses Markowitz's portfolio theory and index model, combined with the five most commonly used constraints in the market, to conduct portfolio analysis on ten stocks, in order to provide individual investors with a more scientific investment portfolio construction method, seeking the smallest risk and maximum return and providing relevant investment advice.

Keywords: Markowitz Model, Index Model, Portfolio Management, Financial Markets.

1. Introduction

In 1952, American economist Harry Markowitz, first proposed to use the arithmetic mean of the returns of the past holding periods to represent the expected return of an asset, using its variance to represent the risk of the asset, showing quantitatively why the portfolio can effectively diversify the risk. In the securities market, both large securities investment institutions and retail investors are faced with the problem of how to allocate assets so that they can invest with less risk under the condition of a certain expected return level. In Markowitz's portfolio theory, every investor has to make a trade-off between risk and return. Assuming that there is no risk-free arbitrage opportunity in the market, and investors in the market are rational, although their risk preferences are different, they all hope that the investment portfolio can obtain the maximum return under a certain risk level. There is a rule which implies both that the investor should diversify and that he should maximize expected return. The rule states that the investor does (or should) diversify his funds among all those securities which give maximum expected return. The law of large numbers will insure that the actual yield of the portfolio will be almost the same as the expected yield [1]. Therefore, this diversified investment method that minimizes investment risks has become the mainstream investment method in modern times.

There is now a large amount of research on investment portfolios. They have a focus on research about asia-pacific markets, such as Georgia and Ziobrowski [2] research about Asia-pacific markets, and Širůček and Křen [3] targeted on building optimal portfolio on the US stock market and Ivanova, Karandikar, Sinha [4] on non-Gaussian world. Others researched certain industries, like Lee, Stevenson [5] wrote an article about real estate in the mixed - asset portfolio and Dong Jichang, Wang Shouyang, Xu Shanying [6] wrote selections on the Internet. But it did not focus on the asset allocation of multi - industry portfolios. However, Diversified investment baskets are good for diversifying risks, and industries of technology, consumer cyclical, financial services, industrials, consumer defensive and energy are currently popular investment industries, and they are industries that most investors cannot avoid when they choose to invest in the "basket". And the companies selected in this article are the industry leaders. Through the empirical analysis of this article, it is helpful for the majority of investors to make decisions.

This article begins with a recent 20 years (2001.05.11-2021.05.12) of historical daily total return data for ten stocks, which belong in groups to three-four different sectors (according to Yahoo finance), which are technology, consumer cyclical, financial services, industrials, one (S&P 500) equity index (a total of eleven risky assets) and a proxy for risk-free rate (1-month Fed Funds rate). In order to reduce the non-Gaussian effects, this article aggregated the daily data to the monthly observations, and based on those monthly observations, this article calculated all proper optimization inputs for the full Markowitz Model ("MM"), alongside the Index Model ("IM"). Then, concerning the different common constraints in the markets, it selected out 5 most common cases for these

optimization inputs for MM and IM for regions of permissible portfolios (efficient frontier, minimal risk portfolio, optimal portfolio, and minimal return portfolios frontier). Then this article finds that under these five constraints, the Markowitz Model and the Index Model are not much different, and the tighter the constraints, the fewer possible cases of the resulting efficient frontier. Since then, this article has extracted the companies that generally have larger shares when the maximum Sharpe ratio is down under different conditions and formed a conclusion.

The rest of this paper is organized as follows. Section 2 briefly summarizes the data used in this paper. Section 3 and Section 4 show the method and results in this paper, respectively. Section 5 and 6 are the overall analysis and conclusion, respectively.

2. Data

In order to obtain the necessary data for the research, this paper grabbed daily data from Yahoofinance (<https://finance.yahoo.com/>). 10 typical companies in these top-heat industries are Apple Inc., Amazon.com Inc., Citrix Systems Inc, JPMorgan Chase & Co, Berkshire Hathaway Inc., The Progressive Corporation, United Parcel Service Inc, FedEx Corporation, J.B. Hunt Transport Services Inc, Landstar System Inc. respectively. This article transforms these daily closing prices to daily returns, and forms the basic data as shown in Table 1:

Table 1. Descriptive statistics of the selected assets

	SPX	AMZN	AAPL	CTXS	JPM	BRK/A
Average Return	7.542%	33.796%	34.016%	15.636%	11.865%	9.002%
Anl StDev	14.850%	41.410%	34.452%	41.514%	29.013%	16.243%
beta	100.000%	135.128%	125.689%	122.058%	136.081%	57.214%
Anl alpha	0.000%	23.604%	24.536%	6.430%	1.601%	4.687%
Resi StDev	0.000%	36.223%	28.957%	37.348%	20.818%	13.844%
	PGR	UPS	FDX	JBHT	LSTR	
Average Return	15.406%	9.850%	12.952%	22.525%	17.387%	
Anl StDev	21.059%	21.437%	26.690%	30.654%	23.924%	
beta	71.204%	82.962%	110.363%	107.627%	79.754%	
Anl alpha	10.036%	3.592%	4.628%	14.407%	11.372%	
Resi StDev	18.212%	17.543%	21.066%	26.158%	20.786%	

By visualizing the data, this paper can find that the annualized return of "AAPL" and "AMZN" is far ahead of other companies, but at the same time the standard deviation is also quite high, while the "CTXS" has a rather high standard deviation and a relatively low return, and thus having an unsatisfying performance. This article uses SPX as a benchmark, with "JPM" and "UPS" having the lowest risk compensation returns. So, it is clear that the performance of "AAPL" and "AMZN" are better, while the performance of "JPM" and "UPS" are unsatisfying.

3. Method

3.1 Correlation

The correlation coefficient is a statistical indicator used to reflect the closeness of the correlation between variables.

$$r = \frac{\sum (X - \bar{X})(Y - \bar{Y})}{\sqrt{\sum (X - \bar{X})^2 \sum (Y - \bar{Y})^2}} \quad (1)$$

The absolute value of the r value is between 0 and 1. Generally speaking, the closer r is to 1, the stronger the correlation between the two quantities of x and y . On the contrary, the closer r is to 0, the weaker the degree of correlation between the two quantities of x and y .

3.2 Markowitz Mean-variance Model

This article assumes that security buyers want to achieve two goals when purchasing a "stock basket": the highest possible yield and the lowest possible uncertainty risk.

It assumes that there are n risky assets in the market, and the return for the assets are $r_1, r_2, \dots, r_n$, the weights investors allocated to the risky assets are $\omega_1, \omega_2, \dots, \omega_n$, thus the return for the portfolio is $r_p = \sum_{i=1}^n \omega_i r_i$, when $\sum_{i=1}^n \omega_i = 1$

So, the expected return for the portfolios is as follows:

$$E(r_p) = \sum_{i=1}^n w_i E(r_i), \quad (2)$$

The variance is shown as:

$$\sigma^2 = \text{var}(\sum_i x_i r_i) = \sum_{ij} x_i x_j \text{cov}(r_i, r_j), \quad (3)$$

4. Results

The five for the following five cases of the additional constraints:

The first constraint is simulating the Regulation T by FINRA (<https://www.finra.org/rules-guidance/key-topics/margin-accounts>). In this constraint, broker-dealers can have 50% or more positions using the equity in customer's account.

$$\sum_{i=1}^{11} |w_i| \leq 2 \quad (4)$$

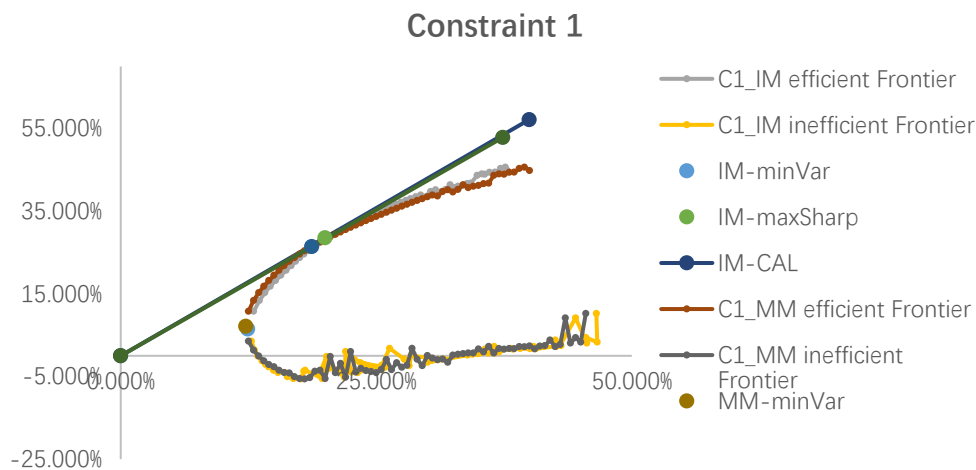


Figure 1. Efficient frontier and CAL under constraint 1

Table 2. Weights for mean-variance and index model under constraint 1

MM	SPX	AMZN	AAPL	CTXS	JPM	BRK/A	PGR
MinVar	72.24%	-2.35%	-3.85%	-1.04%	-18.47%	36.21%	13.91%
maxSharp	-48.25%	16.40%	30.02%	-0.10%	-0.09%	41.31%	32.96%
	UPS	FDX	JBHT	LSTR	Return	StDev	Sharpe
MinVar	3.43%	-10.28%	-0.55%	10.75%	7.15%	12.24%	0.584
maxSharp	-0.02%	-1.46%	12.50%	16.73%	26.42%	18.69%	1.413
IM	SPX	AMZN	AAPL	CTXS	JPM	BRK/A	PGR
MinVar	65.79%	-4.14%	-4.74%	-2.44%	-12.87%	34.51%	13.42%
maxSharp	-47.30%	17.84%	30.52%	0.50%	-2.68%	22.44%	31.53%
	UPS	FDX	JBHT	LSTR	Return	StDev	Sharpe
MinVar	8.56%	-3.61%	-1.72%	7.24%	6.46%	12.43%	0.520
maxSharp	1.17%	0.11%	19.20%	26.66%	28.57%	19.98%	1.430

This article finds that the standard deviation for investment portfolio under minimum variance is 12.24% and the hyperbolic area to the right of this point is the portfolio with the highest return with a standard deviation greater than 12.24%.

In this limit, the maximum Sharpe ratio that can be formed by the Markowitz model and the Index Model is basically the same. Under this combination of maximum Sharpe ratios, a large amount of short selling of nearly 50% of SPX is required, and a large amount of holding is required. (About 30%) of AAPL, BRK/A and PGR stocks to achieve.

Arbitrary “box” constraints are simulated in this additional constraint.

$$|w_i| \leq 1, \text{ for } \forall i \quad (5)$$

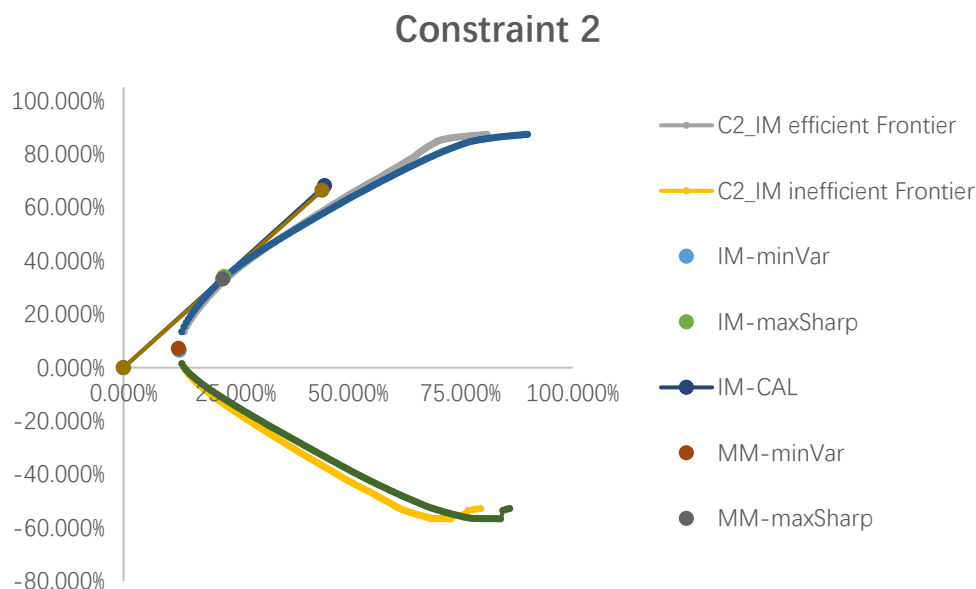


Figure 2. Efficient frontier and CAL under constraint 2

Table 3. Weights for mean-variance and index model under constraint 2

MM	SPX	AMZN	AAPL	CTXS	JPM	BRK/A	PGR
MinVar	72.24%	-2.35%	-3.85%	-1.04%	-18.47%	36.21%	13.91%
maxSharp	-100.00%	22.33%	39.76%	-1.20%	-0.50%	62.48%	46.01%
	UPS	FDX	JBHT	LSTR	Return	StDev	Sharpe
MinVar	3.43%	-10.28%	-0.55%	10.75%	7.15%	12.24%	0.584
maxSharp	-3.11%	-10.56%	20.88%	23.91%	33.18%	22.11%	1.501
IM	SPX	AMZN	AAPL	CTXS	JPM	BRK/A	PGR
MinVar	65.79%	-4.14%	-4.74%	-2.44%	-12.87%	34.51%	13.42%
maxSharp	-100.00%	21.84%	36.62%	3.09%	-8.79%	33.32%	39.98%
	UPS	FDX	JBHT	LSTR	Return	StDev	Sharpe
MinVar	8.56%	-3.61%	-1.72%	7.24%	6.46%	12.43%	0.520
maxSharp	9.52%	5.62%	24.97%	33.83%	34.07%	22.40%	1.521

This article finds that the standard deviation for investment portfolio under minimum variance is 6 to 7% and the hyperbolic area to the right of this point is the portfolio with the highest return with a standard deviation greater than 12.24%.

In this limitation, the maximum Sharpe ratio that can be formed by the Markowitz model and the Index Model is basically the same, about 1.5. Under this combination of maximum Sharpe ratios, all short-sold SPX are required, and through a large amount of holding AAPL (about 1.5). 40%) and PGR (40%), BRK/A stocks are also heavy, but there is a big difference between the positions in the Markowitz Model and the Index Model. In the Markowitz Model, it is 62%, which is twice that in the Index model.

Besides, under this constraint, comparing two models, it can be seen the index model can include more possibilities of portfolio.

The condition in a “free” state, this illustrates the condition when no constraints are set.

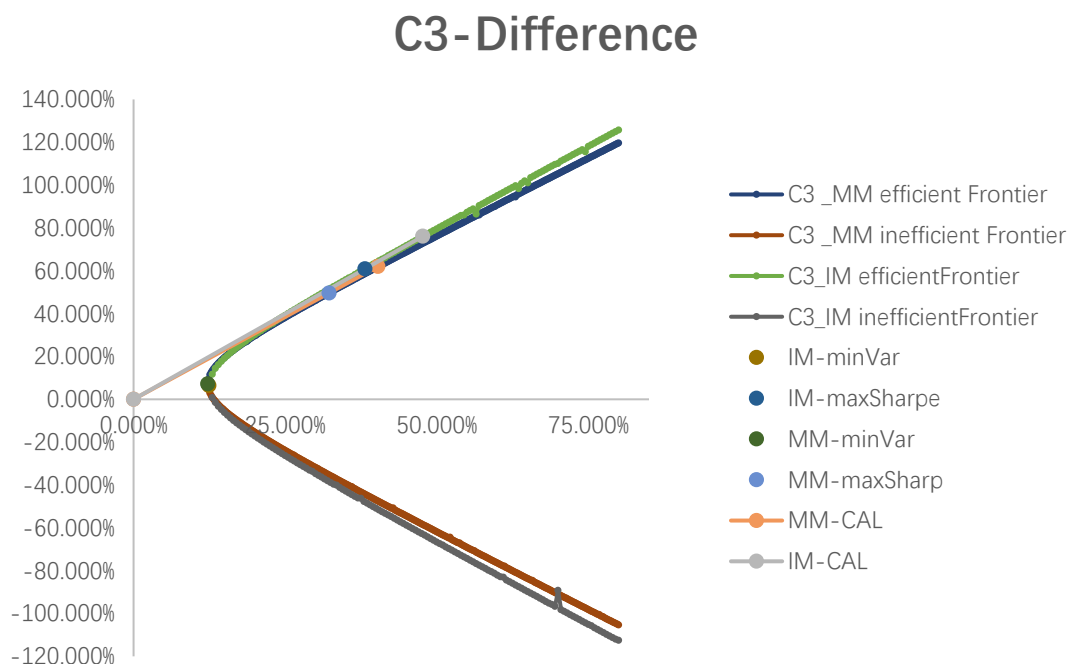


Figure 3. Efficient frontier and CAL under constraint 3

Table 4. Weights for mean-variance and index model under constraint 3

MM	SPX	AMZN	AAPL	CTXS	JPM	BRK/A	PGR
MinVar	72.24%	-2.35%	-3.85%	-1.04%	-18.47%	36.21%	13.91%
maxSharp	-48.25%	16.40%	30.02%	-0.10%	-0.09%	41.31%	32.96%
	UPS	FDX	JBHT	LSTR	Return	StDev	Sharpe
MinVar	3.43%	-10.28%	-0.55%	10.75%	7.15%	12.24%	0.584
maxSharp	1.27%	-8.69%	30.97%	34.02%	49.61%	32.25%	1.539
IM	SPX	AMZN	AAPL	CTXS	JPM	BRK/A	PGR
MinVar	65.80%	-4.14%	-4.74%	-2.44%	-12.88%	34.51%	13.42%
maxSharp	-329.78%	43.02%	69.96%	11.02%	8.83%	58.47%	72.35%
	UPS	FDX	JBHT	LSTR	Return	StDev	Sharpe
MinVar	8.56%	-3.61%	-1.72%	7.24%	6.46%	12.43%	0.520
maxSharp	27.91%	24.94%	50.35%	62.93%	60.91%	38.16%	1.596

Under constraint 3, the efficient and inefficient frontier of Index model are bigger than Markowitz model. Two minimum variance point nearly overlap. This article finds that the standard deviation for investment portfolio under minimum variance is 12% and the hyperbolic area to the right of this point is the portfolio with the highest return with a standard deviation greater than 12%.

In this limitation, the maximum Sharpe ratio that can be formed by the Markowitz model and the Index Model is basically the same, which is about 1.5. Under this combination of maximum Sharpe ratios, a large amount of short-selling SPX is required, and through heavy positions AMZN, AAPL, BRK /A, PGR, JBHT and LSTR to reach.

This additional optimization constraint does not allow short selling. This aims to simulate the typical limitations in U.S. mutual fund industry whose open-ended mutual fund is not allowed to have any short positions. Details can be found in the Investment Company Act of 1940, Section 12(a)(3) (<https://www.law.cornell.edu/uscode/text/15/80a-12>).

$$|w_i| \geq 0, \text{ for } \forall i \quad (6)$$

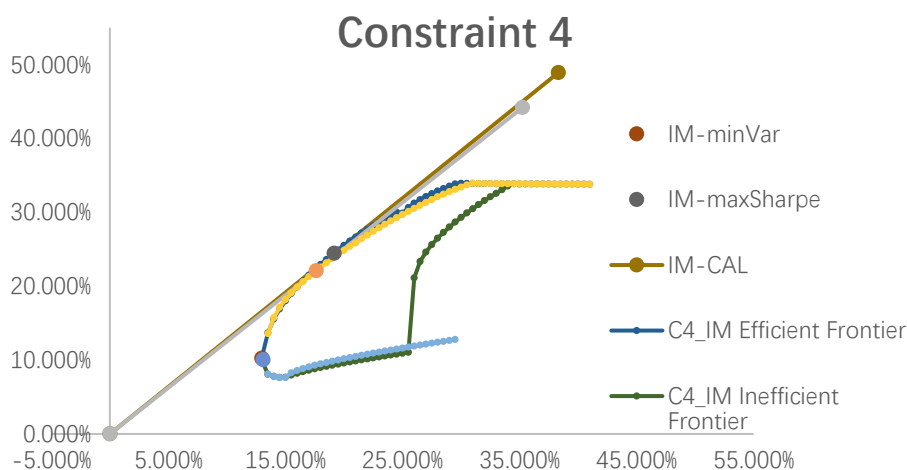


Figure 4. Efficient frontier and CAL under constraint 4

Table 5. Weights for mean-variance and index model under constraint 4

MM	SPX	AMZN	AAPL	CTXS	JPM	BRK/A	PGR
MinVar	38.47%	0.00%	0.00%	0.00%	0.00%	38.49%	14.23%
maxSharp	0.00%	12.96%	25.21%	0.00%	0.00%	19.26%	22.72%
	UPS	FDX	JBHT	LSTR	Return	StDev	Sharpe
MinVar	0.72%	0.00%	0.00%	8.09%	10.04%	13.09%	0.767
maxSharp	0.00%	0.00%	8.81%	11.04%	22.09%	17.62%	1.254
IM	SPX	AMZN	AAPL	CTXS	JPM	BRK/A	PGR
MinVar	30.46%	0.00%	0.00%	0.00%	0.00%	37.65%	14.64%
maxSharp	0.00%	15.46%	27.12%	0.00%	0.00%	3.26%	21.36%
	UPS	FDX	JBHT	LSTR	Return	StDev	Sharpe
MinVar	9.34%	0.00%	0.00%	7.90%	10.24%	12.99%	0.788
maxSharp	0.00%	0.00%	13.98%	18.82%	24.45%	19.16%	1.277

Furthermore, it finds that the curve is most affected under Constraint Four, where short selling is not allowed, which excludes portfolios with negative returns. There is a big different between two models about inefficient frontier under constraint four. There is a jump at twenty five percent in Index model. But after twenty five percent, two efficient frontier almost overlap. This article finds that the standard deviation for investment portfolio under minimum variance is 10% and the hyperbolic area to the right of this point is the portfolio with the highest return with a standard deviation greater than 10%. In this limitation, the maximum Sharpe ratio that can be formed by Markowitz model and Index Model is basically equal, about 1.2. Under this combination of maximum Sharpe ratio, SPX, CTXS, JPM, UPS, FDX all need to close positions, and the remaining share of stocks under the restrictions is relatively average compared to the weighted shares of stocks under the other restrictions.

Lastly, the weight of SPX is set as zero, by which constraint the effects of the broad index have on the portfolios can be analysed.

$$w_1 = 0 \quad (7)$$

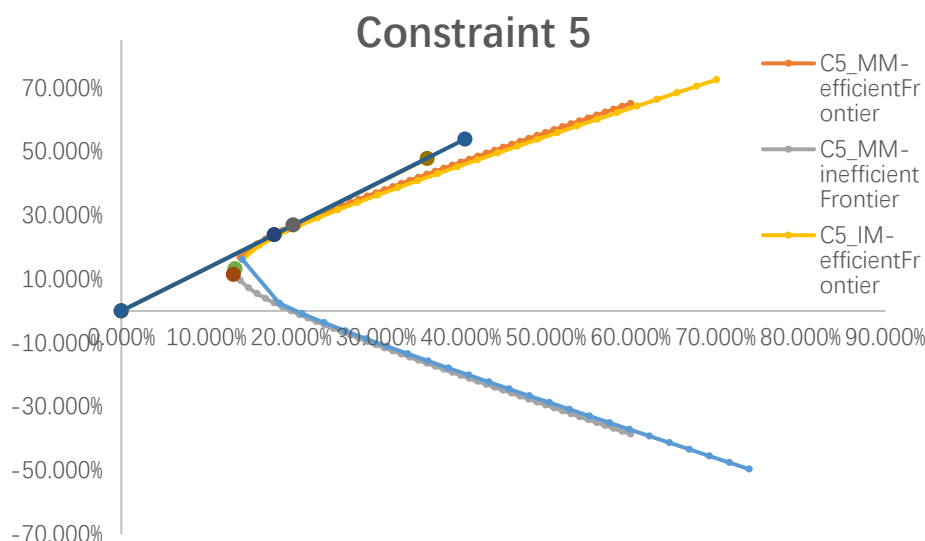


Figure 5. Efficient frontier and CAL under constraint 5

Table 6. Weights for mean-variance and index model under constraint 5

MM	SPX	AMZN	AAPL	CTXS	JPM	BRK/A	PGR
MinVar	0.00%	2.45%	4.19%	0.84%	-7.09%	56.31%	23.52%
maxSharp	0.00%	14.65%	26.73%	-3.44%	-15.54%	36.35%	31.99%
	UPS	FDX	JBHT	LSTR	Return	StDev	Sharpe
MinVar	11.41%	-8.05%	0.60%	15.82%	13.20%	13.39%	0.986
maxSharp	-12.17%	-13.21%	17.83%	16.81%	23.89%	18.02%	1.326
IM	SPX	AMZN	AAPL	CTXS	JPM	BRK/A	PGR
MinVar	0.00%	-1.55%	-0.81%	-0.11%	-5.02%	47.99%	21.65%
maxSharp	0.00%	18.56%	31.69%	-0.47%	-21.23%	11.67%	27.66%
	UPS	FDX	JBHT	LSTR	Return	StDev	Sharpe
MinVar	17.82%	3.45%	2.82%	13.76%	11.47%	13.21%	0.869
maxSharp	-5.30%	-5.40%	18.57%	24.25%	26.95%	20.24%	1.331

In the last comparison, the results of the two models are very similar. This article finds that the standard deviation for investment portfolio under minimum variance is more than 10% and the hyperbolic area to the right of this point is the portfolio with the highest return with a standard deviation greater than 10%. In this limit, the maximum Sharpe ratio that can be formed by the Markowitz model and the Index Model is basically the same, about 1.3. Under the combination of this maximum Sharpe ratio, SPX needs to close the position, while the relative buying shares of BRK/A and PGR are relatively high. many.

Besides, compared these two models, the curves are very similar in the distribution and the trend. So, it believes that no matter which model, the constraints will make the feasible set and the effective set smaller. Besides, under the five constraints, there is little difference in the labeling difference of the minimum variance combination, about 12% to 13%, and the range of the maximum Sharpe ratio is about 1.2 to 1.5. Among them, the weight of SPX is the most volatile. In the case of the highest return under each limit, the positions of AAPL, BRK/A and PGR are relatively heavy, which means that investors should consider the above three when choosing a portfolio, whether it is due to better profitability. Or companies with better market expectations. On the contrary, CTXS, JPM, UPS and FDX have less shares and sometimes even need to short sell for higher interest.

5. Overall analysis

First, the constraint four is the most restrictive. To be more specific, banning short selling reduces maximum returns and maximum risk-taking. That is like reducing leverage. Investing in this constraint is suitable for the risk averse, not the risk seeking. Second, the constraint 5 which means investors cannot invest in SPX also has a great restriction power. As people all know, SPX is a market-value-weighted index which is indicator to measure the performance of the stock market. People can use it to track average returns and comparing performance of managers. Because it is a stock index which is broadly based on many firms, so it can make a huge difference to our portfolios. THIRD, the constraint 1 is less restrictive than the constraint 4, but also has a great limited ability. The intrinsic mean of constraint 1 is the restriction of maintenance margin. The percentage margin is defined as the ratio of net worth, or “equity value”, of the account to the market value of the securities, which is fifty percent under the constraint 1. The investor can borrow part of the purchase price of the stock from the broker and securities purchased on margin are used as collateral from the loan. It can get great yield and risk. That is why this constraint can get bigger return and risk than the constraint 4 which is no short selling. This article made a conclusion about degree of restriction that: C4>C5>C1>C2>C3

6. Conclusions

Both the Markowitz Model and the Index Model are return models that take into account market risk. Generally speaking, the results obtained are not very different. The situation where short selling is not allowed is the most restrictive and the smallest range of returns available. The unrestricted case has the highest achievable maximum Sharpe ratio of the five common restrictions, at 1.596, but also has the highest risk, at 38.16%. Overall, regardless of the restrictions, AAPL, BRK/A and PGR are recommended stocks to buy, and they will bring relatively high investment returns.

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