

The compare of fitting between Markowitz model and single index model in different risk degree

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Abstract. COVID-19 outbreak impose a major threat on the economy. For such a major public event, government always take some measure to deal with them, and it will cause great impact to market. The paper tries to use 20 years of historical daily total return data for ten stocks, which belong in groups to three different sectors Internet Technology Industry, The financial industry, Consumer goods business, one (S&P 500) index (a total of eleven risky assets) and a proxy for risk-free rate (1-month Fed Funds rate). This paper will explore practical degree of MM model and IM model in different risk degree by choosing one market index and ten leading enterprises in all walks of life and try to use expect return to divide degree of risk. The research show that MM model will have better degree of fitting in medium risk degree, and IM will be more suitable for low-risk degree. Furthermore, when expect return above 40% to the high return level, the limiting condition will cause great influence.

Keywords: Mean-Variance Model, index model, Portfolio Optimal.

1. Introduction

In investment, an investor does not certainly know the return. Moreover, they must be ready to face the future investment risk [1], thus investors can't get around the public event, because it may cause great influence on stock market.

COVID-19 Pandemic be found in China, and all the world gradually found it. Causing great influence on finance market, during the discovery of a global epidemic, the market had huge fluctuation. Within three months of discovery COVID-19, the SPX fall 20.77% and the SSE fall 3.76%. Investment always along with the risk, there are two finance model have tried to quantify the risk. One is Markowitz model (MM), other one is the index model (IM).

MM attempts to quantify risk of portfolio and using variance to describe risk of market. MM need to calculate every covariance between each individual, but it also must to go through complicated calculations. For simple it, IM holds a hypothesis that each single covariance between all securities is zero, only through some combination of factors. It greatly simplifies process of collection and reduce the computational difficulty.

The number of enter (example of 100 stocks)

Model	Return	Variance	Covariance	Total
MM	100	100	4950	5150
IM	101	101	100	302

Although IM is more practical, which model is fitter in different risk degree is a problem for people, this paper will research the performance of different portfolios which be made by MM and IM in different risk degrees.

2. Literature preview

Fisher, Lawrence and James H. Lorie chosen U.S. stock market dates have been chosen as sample, they tried to select Variance quantities stocks to build a portfolio [2]. Sunariyah found a portfolio not only belong to an individual but also belong to an institution [3]. Then, Husnan pointed that investor will tend to build diversification to decrease the risk without reducing expected returns [4], at same time Gurrib and Alshahrani found that portfolio diversification is great method to help investor to

decrease the associated risks [5]. What's more, Tandelilin found that as the increasing of the stocks, the downtrend of risks will decrease, so an optimal portfolio must be efficient. Tandelilin also pointed that investor need to have a maximum expected return which will have the highest return. According to Tandelilin, investors have to consider two factors: Variance, covariance and the portfolio of each security. William Sharpe built index model. It simplified the method which be used to express the covariance between each security. It can decrease estimated data volume when you calculate [6]. For it, the index model is more practical in our work. Afterward, there are many scholars carry out empirical experiments by IM. Rahmadin [7] and Husnan [8] used IM to complete optimal portfolio. What's more, Mahadwartha and Gunawan shows their optimal portfolio they had built by using index model, pointed that it is can be used only for six months period, and supplement that the mostly best portfolios have higher volatility or aggressive portfolios [9]. Furthermore, Setiawan used 26 shares, be conducted from 2013 until 2016, as sample of the research and introduce Excess To beta and Ci to help model to fitter. What's more, this research also proves that investors will use index model to chooses stocks to achieve the optimal portfolio to a certain degree [10].

Although index model is practical and be validated, it also has less veracity of risk in quick-reading flow sheet, ignoring non-market factor may cause inaccurate result, therefore influence judge of investors when crisis events such as COVID-19 happen and resulted in risk of market increase. Which model have better return and less risk?

3. The basic fundamental of Mean-Variance Model and Single-Index Model

3.1 the structure of MM Model

H.M. Markowitz won the 1990 Nobel Prize in Economics by researching portfolio theory which had been published in 1952. Then Markowitz had putted forward to use the risky assets' expected return and standard deviation to represent risky [11]. It built 5 limits conditions to help the results having higher degree of fitting.

(1) All return of invest can be shown by numerical value and all investors have ability to know the probability distribution of all the portfolios. (2) the Variance of return reactions investors' estimations. (3) the investors only use two tools: expected return and Variance as the base of decision. (4) investors can modification their assets at will. (5) the assets and liabilities have perfect liquidity. It means that the assets and liabilities have infinite elasticity of supply, so that the behavior of investors won't influence market's price and market's expected return.

Based on the above assumptions:

$$\sigma_{ij} = \text{cov}(R_i, R_j) = E\{[R_i - E(R_i)][R_j - E(R_j)]\} \quad (1)$$

$$E_p = \sum_{i=1}^n X_i E_i \quad (2)$$

$$\sigma_p^2 = \sum_{i=1}^n X_i^2 \sigma_i^2 + 2 \sum_{i=1}^n \sum_{j \geq 1} X_i X_j \sigma_{ij} \quad (3)$$

Which:

R_i = Rate of return stock

$E(R_i)$ = The expected rate of return on stock i

3.2 the structure of IM Model

Sharpe made the index model to solve the complex calculation. He putted forward simplified solution techniques for efficient combinations by analysis of the function relation between return of stocks and return of stock market index, thus the practical value of portfolio theory is improved to a certain extent.

This model has one important point that ignoring the relationship between the stocks,

The single asset:

$$R_i = \alpha_i + \beta_i R_m + e_i \quad (4)$$

The all expect of the assets:

$$R_P = \vec{w} * \vec{\beta}^T \quad (5)$$

$$\sigma_P = \sqrt{(\sigma_M \beta_P)^2 + \sum_{i=1}^n w_i^2 \sigma^2(\varepsilon_i)}, \quad \beta_P = \vec{w} * \vec{\beta}^T \quad (6)$$

4. Results

4.1 data

The paper chose some experience date from leader companies come from different realms which have a relatively high correlation of the market. Therefore, we are able to research overview of the overall market.

We select three different sectors, NVIDIA Corporation, Intel and Cisco Systems are symbol of high-tech industry. NVIDIA is the world leader in programmable graphics processing technology, focused on building products that enhance the human-computer interaction experience for personal and professional computing platforms. Cisco is the world's leading network solutions provider. Intel is a global leader in semiconductor industry and computing innovation. Now Intel is transforming itself into a data-centric company. Then, the Goldman Sachs, U.S. Bancorp, Toronto-Dominion Bank and Allstate are symbol of the finance industry. Goldman Sachs is a leading global investment bank, U.S. Bancorp is a holding company for financial services, Td Bank, headquartered in Toronto, Canada, has offices worldwide, and Allstate is the second largest personal insurance and casualty insurance company in the United States. They all offer finance services to place where they are location. To a certain extent, they can represent the situation of the financial industry around the country.

Lastly, we set the consumer goods business sector. The Procter & Gamble, Johnson & Johnson and Colgate-Palmolive are be in it. Procter & Gamble is one of the world's leading consumer goods companies. Johnson & Johnson is the world's largest and diversified medical and health care products and consumer care company. Colgate-Palmolive manufactures Care products. It also provides high quality consumer goods for the public in oral care, personal care, home care and pet food.

4.2 constraint

The Sharpe ratio is a performance indicator that describes a portfolio:

$$sharpe\ ratio = \frac{\text{average rate of return} - \text{average rate of return}}{\text{standard deviation}} \quad (7)$$

It means that the bigger Sharpe ratio will have higher the risk reward, so we try to use to find the maximum Sharpe ratio to find the best weights to have the best portfolio.

Also, the standard deviation is mean as the risk of the portfolio, therefore, we try to find which standard deviation is the smallest.

Constraint:

1. This additional optimization constraint is designed to simulate the Regulation T by FINRA, which allows broker-dealers to allow their customers to have positions, 50% or more of which are funded by the customer's account equity:

$$\sum_{i=1}^{11} |w_i| \leq 2 \quad (8)$$

2. This additional optimization constraint is designed to simulate some arbitrary “box” constraints on weights, which may be provided by the client:

$$|w_i| \leq 1, \text{ for } \forall i \quad (9)$$

3. A “free” problem, without any additional optimization constraints, to illustrate how the area of permissible portfolios in general and the efficient frontier in particular look like if you have no constraints;

4. This additional optimization constraint is designed to simulate the typical limitations existing in the U.S. mutual fund industry: a U.S. open-ended mutual fund is not allowed to have any short positions, for details see the Investment Company Act of 1940, Section 12(a)(3):

$$w_i \geq 0, \text{ for } \forall i \quad (10)$$

5. Lastly, we would like to see if the inclusion of the broad index into our portfolio has positive or negative effect, for that we would like to consider an additional optimization constraint:

$$w_1 = 0 \quad (11)$$

4.3 result

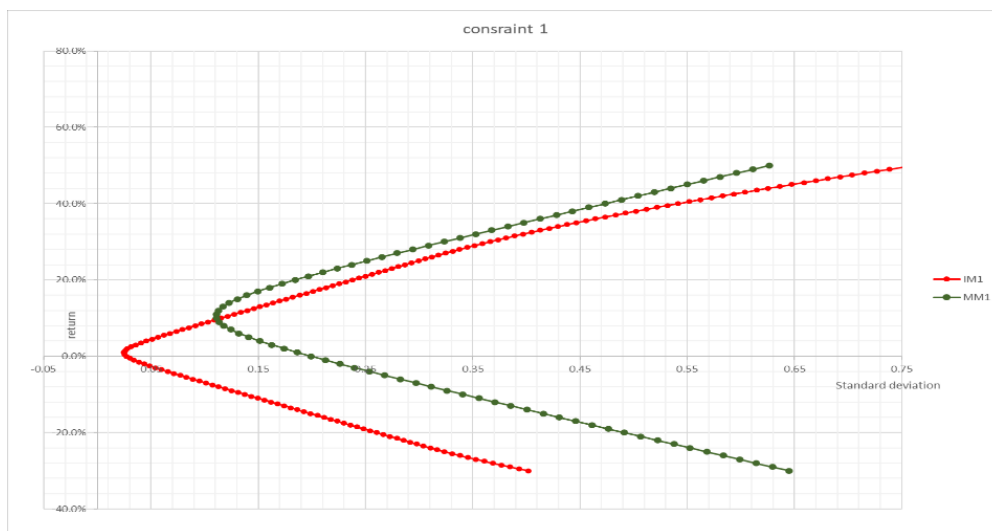


Figure 1 constraint 1

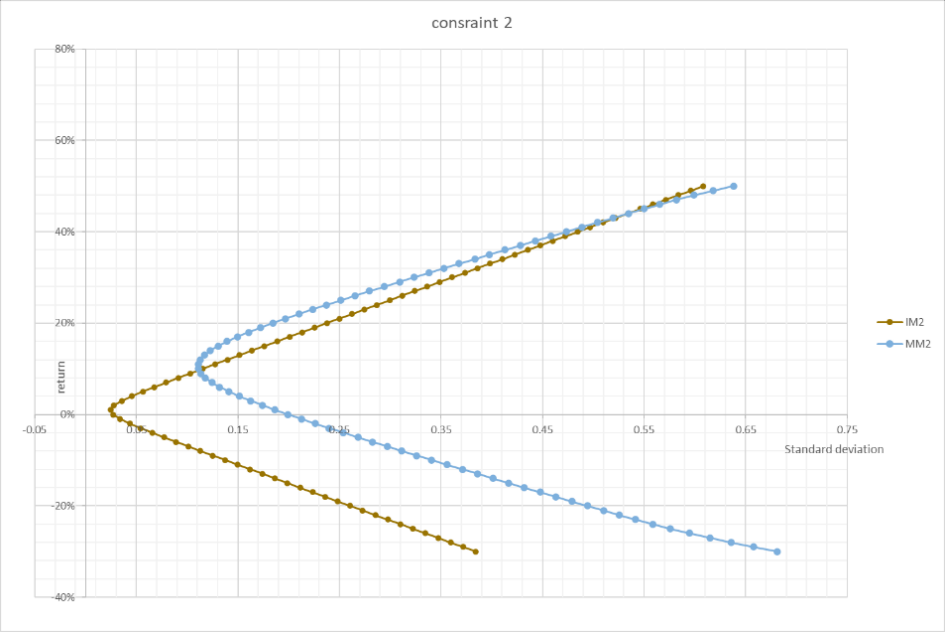


Figure 2 constraint 2

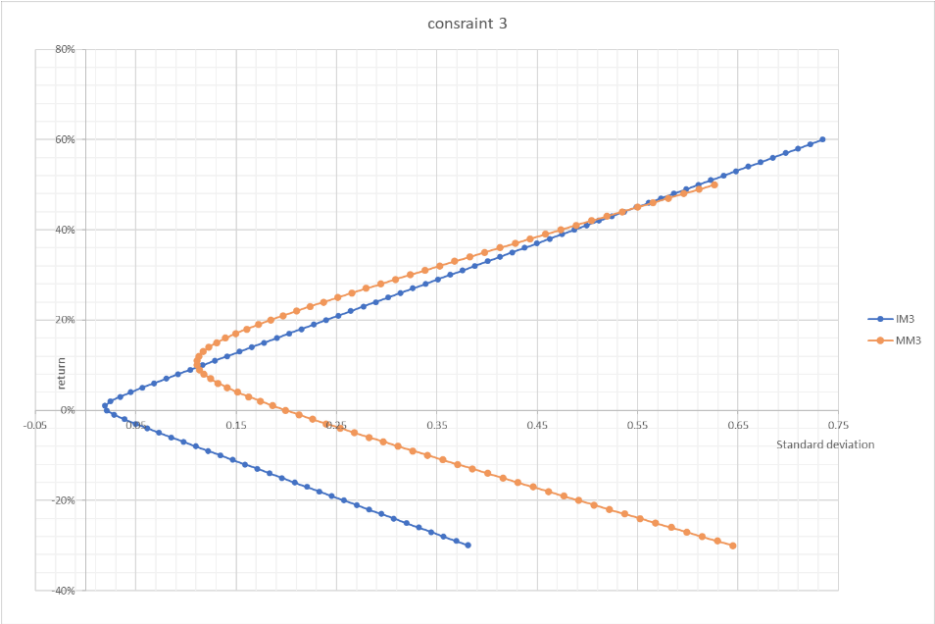


Figure 3 constraint 3

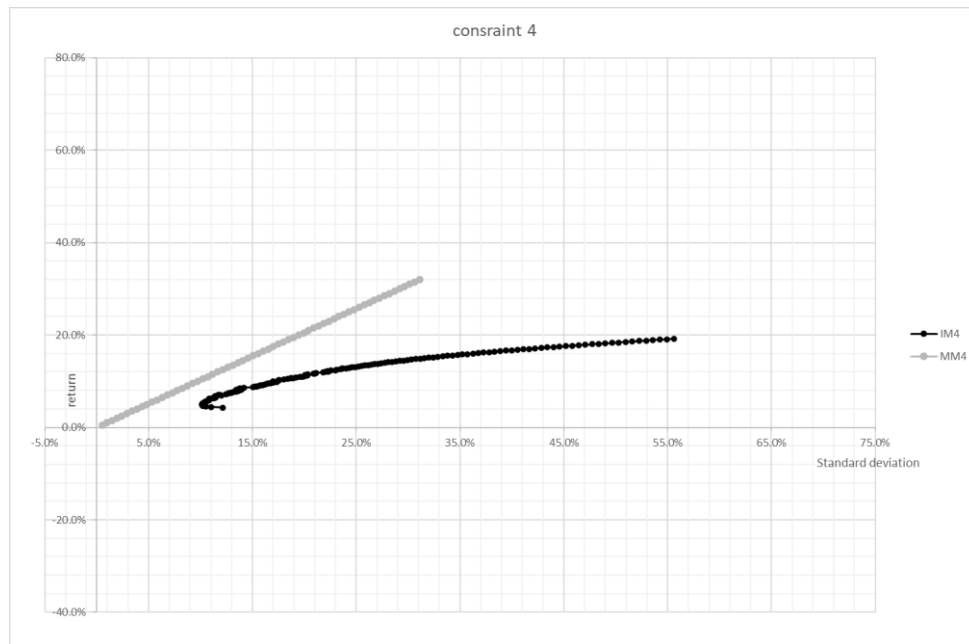


Figure 4 constraint 4

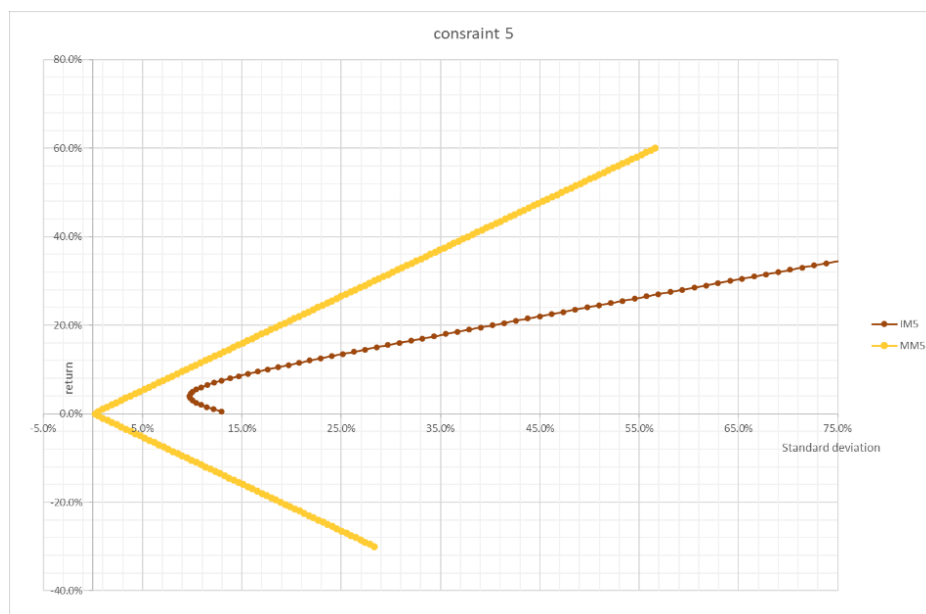


Figure 5 constraint 5

Table 1 Better Model chosen

Constraint	1	2	3	4	5
Low expect return (<12%)	IM	IM	IM	MM	IM
Medium expect return (12%~40%)	MM	MM	MM	MM	MM
High expect return (>40%)	MM	IM	IM	MM	MM

Based on the results described previously, it can be concluded that the optimal portfolio what you tend to choose, less risk or higher return. Investor need to make their own benchmark rules and try

which portfolio they choose. The study finds that using Sharpe ratio is a good way to help investor to choose. The portfolio which be made by maximum Sharpe ratio has less standard deviation than portfolio which be built based on maximum return, and has higher return than minimum standard deviation portfolio. It is easy to know that expect return has directly proportional relationship with risk. Therefore, expect return can accurate assessment the degree of the risk, and we can use expect return to divide the degree of risk.

About above 10% expect return, IM model have more efficient, less risk and higher return in the market. The study thinks that in minimum expect return, the investor has more choice to resist risk, they will get return which they expect earlier in the market, thus using the IM model is a better choice for market, it has less counting process. What's more, the IM will consider less factors of risk, when you just want to less return ($\leq 10\%$) you could get higher return by using IM to build portfolio.

Between 10% and 40%, the MM model is the best choice, whatever which constraint MM model always give us better portfolio that having less standard Variance as same expect return. MM will consider the impact of more non-market risks, it will be more efficiently when expect return over 40%, Over 40%, constraints are the important factors to affect benefit of portfolio. Experimental data indicate that IM model is appropriate for market of weak force of supervision, such as the constraint 2 and 3. On the other hand, MM model has better performance in strongly regulated market, because MM model include more non-market factors and they can make result be suitable for real world.

More than that, history shows that government would not to do nothing when major public event happens. It must to take so steps to control market. The free market will become the strongly control market. It means that investor need to use MM model when their expect return over 10%, for it, investor must to get more information about risk to help them built MM model.

Table 2. MM Model Proportion of constituent shares based on Minimum Variance

MM (Minimum Variance):	SPX	NV DA	CSC O	INT C	GS	US B	TDC N	ALL	PG	JNJ	CL
Constraint1	- 0.73 %	0.04 %	0.03 %	0.11 %	0.08 %	0.13 %	0.04 %	- 0.11 %	- 0.38 %	- 0.40 %	1.55 %
Constraint2	- 0.94 %	0.05 %	0.04 %	0.14 %	0.10 %	0.17 %	0.05 %	- 0.14 %	- 0.49 %	- 0.52 %	2.00 %
Constraint3	- 0.94 %	0.05 %	0.04 %	0.14 %	0.10 %	0.17 %	0.05 %	- 0.14 %	- 0.49 %	- 0.52 %	2.00 %
Constraint4	0.00 %	0.00 %	0.00 %	0.00 %	0.00 %	0.00 %	0.00 %	0.00 %	0.00 %	0.00 %	2.00 %
Constraint5	0.00 %	0.00 %	- 0.04 %	0.12 %	- 0.02 %	0.08 %	- 0.06 %	- 0.24 %	- 0.57 %	- 0.70 %	2.00 %

Table 3 MM Model Result data based on Minimum Variance

MM (Minimum Variance):	Return	Standard deviation	Sharpe
Constraint1	0.03%	0.002	0.161
Constraint2	0.04%	0.002	0.161
Constraint3	0.04%	0.002	0.161
Constraint4	0.17%	0.003	0.557
Constraint5	0.02%	0.002	0.082

In the MM model, we set up that the minimum Variance to get this result. We are able to know the Constraint 4 has the highest return and also it has the highest standard deviation. It shows the positive correlation between risk and return. The minimum standard deviation is the constraint 1 and constraint 2 and 3, but the constraint 2 and 3 has higher Sharpe ratio, so it is better than constraint 1.

Table 4 MM Model Proportion of constituent shares based on Maximum Variance

MM (Maximum Sharpe):	SPX	NVD A	CSC O	INT C	GS	USB	TDC N	ALL	PG	JNJ	CL
Constraint1	- 7.20 %	1.55 %	0.01 %	- 0.56 %	0.81 %	0.93 %	3.56 %	0.41 %	4.12 %	3.29 %	1.55 %
Constraint2	- 9.33 %	2.01 %	0.02 %	- 0.73 %	1.05 %	1.20 %	4.61 %	0.54 %	5.34 %	4.26 %	2.00 %
Constraint3	- 9.31 %	2.00 %	0.02 %	- 0.73 %	1.05 %	1.19 %	4.60 %	0.54 %	5.33 %	4.26 %	2.00 %
Constraint4	0.00 %	2.12 %	0.00 %	0.00 %	0.00 %	0.00 %	5.46 %	0.00 %	9.20 %	3.99 %	2.00 %
Constraint5	0.00 %	3.03 %	- 1.25 %	- 2.00 %	- 0.77 %	0.72 %	7.40 %	- 0.63 %	9.77 %	5.92 %	2.00 %

Table 5 MM Model Result data based on Maximum Variance

MM (Maximum Sharpe):	Return	Standard deviation	Sharpe
Constraint1	1.43%	0.013	1.122
Constraint2	1.85%	0.017	1.122
Constraint3	1.85%	0.016	1.122
Constraint4	2.97%	0.029	1.032
Constraint5	3.35%	0.032	1.062

In the MM model, we set up that the maximum Sharpe ratio to get this result. We are able to know the Constraint 2 and 3 has the highest return and also it has the highest standard deviation. And constraint 2 is same as the constraint 3. The minimum standard deviation is the constraint 4. What's more, the highest Sharpe ratio is the constraint 2 and 3.

Table 6 IM Model Proportion of constituent shares based on Minimum Variance

IM (Minimum Variance):	SPX	NV DA	CSC O	INT C	GS	US B	TDC N	ALL	PG	JNJ	CL
Constraint1	26.2 8%	- 4.06 %	- 5.32 %	- 2.82 %	- 8.79 %	0.72 %	10.2 4%	- 1.34 %	30.9 7%	27.6 4%	26.4 9%
Constraint2	26.2 8%	- 4.06 %	- 5.32 %	- 2.82 %	- 8.79 %	0.72 %	10.2 4%	- 1.34 %	30.9 7%	27.6 4%	26.4 9%
Constraint3	26.2 79%	- 4.06 3%	- 5.32 4%	- 2.82 2%	- 8.78 8%	0.71 9%	10.2 35%	- 1.33 8%	30.9 72%	27.6 41%	26.4 88%
Constraint4	0.00 0%	0.00 0%	0.00 0%	0.01 8%	0.00 0%	0.00 0%	9.55 9%	0.00 0%	33.2 15%	28.9 78%	28.2 30%
Constraint5	0.00 0%	- 3.34 9%	- 3.28 5%	- 1.11 5%	- 6.00 7%	3.29 5%	14.3 00%	1.24 5%	33.9 12%	31.6 06%	29.3 98%

Table 7 IM Model Result data based on Minimum Variance

IM (Minimum Variance):	Return	Standard deviation	Sharpe
Constraint1	4.2%	0.10	0.44
Constraint2	4.2%	0.10	0.44
Constraint3	4.187%	0.096	0.436
Constraint4	4.911%	0.101	0.484
Constraint5	3.987%	0.097	0.410

In the IM model, we set up that the minimum Variance to get this result. We are able to know the Constraint 4 has the highest return and also it has the highest standard deviation. It shows the positive correlation between risk and return. The minimum standard deviation is the constraint 2,3, but the constraint 4 has higher Sharpe ratio, so it is better than others. So, constraint 4 is better portfolio.

Table 8 IM Model Proportion of constituent shares based on Maximum Variance

IM (Maximum Sharpe):	SPX	NVD A	CSC O	INT C	GS	USB	TDC N	ALL	PG	JNJ	CL
Constraint1	4.719 %	- 1.11 0%	- 1.47 3%	0.16 1%	1.37 9%	3.03 9%	1.383 %	6.79 3%	2.979 %	4.328 %	1.549 %
Constraint2	73.83 3%	1.34 2%	- 0.24 4%	- 0.10 0%	- 0.18 6%	1.34 6%	5.144 %	1.01 0%	7.502 %	6.040 %	4.313 %
Constraint3	73.84 1%	1.34 2%	- 0.24 5%	- 0.10 1%	- 0.18 8%	1.33 5%	5.159 %	1.01 8%	7.460 %	6.065 %	4.313 %
Constraint4	72.42 1%	1.36 9%	0.00 0%	0.04 8%	0.09 2%	0.72 2%	4.827 %	1.22 9%	7.720 %	6.681 %	4.891 %
Constraint5	0.000 %	3.86 9%	6.16 5%	5.14 6%	8.68 8%	8.99 5%	16.74 2%	8.81 8%	14.34 9%	16.05 4%	11.17 3%

Table 9 IM Model Result data based on Maximum Variance

IM (Maximum Sharpe):	Return	Standard deviation	Sharpe
Constraint1	4.091%	0.046	0.887
Constraint2	8.260%	0.135	0.612
Constraint3	8.261%	0.135	0.612
Constraint4	8.229%	0.134	0.612
Constraint5	7.988%	0.138	0.580

In the IM model, we set up that the maximum Sharpe ratio to get this result. We are able to know the Constraint 3 has the highest return and also it has the highest standard deviation. The minimum standard deviation is the constraint 4. What's more, the highest Sharpe ratio is constraint 1. MM model has higher Sharpe ratio than IM, is proper for risk averse investors who prefer to higher expect return without risk. Furthermore, when expect return above 40% to the high return level, the limiting condition will cause great influence.

5. Conclusions

As the result, under medium degree include low and medium, MM model has better degree of fitting in medium degree, IM has better in low degree. So that, investors can choose which model is fitter for them by setting what expect degree they want. It also shows that, although MM model has more complicated counting process, it has better description of market risk. Furthermore, when expect return above 40%, to a high-risk level, constraint of market will have significant influence for result. The general trend is MM model will be more suitable for market conditions with more restrictive, namely MM model will more fitness when government may do something to control the development of economy because of major public events, such as the trade war or COVID-19, which may cause great effect for market.

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