

# Research on the Optimal Asset Allocation Decision of Chinese Retirees from the Perspective of Health

Busheng Li<sup>#</sup>, Qi Li<sup>\*,#</sup>, Yidan Wu, Haiyue Chen, Zehang Zhang

School of Economics and Finance, South China University of Technology, Guangzhou, Guangdong, 510006

\*Corresponding author: ylj546438@163.com

<sup>#</sup> These authors contributed equally.

**Abstract.** This paper constructs an optimal asset allocation decision model for retirees based on the characterization of the CRRA utility function and health stock, and uses the Ramsey equation to obtain the optimal intertemporal asset allocation under steady-state conditions. Combining with the actual situation of Chinese retirees, the theoretical model is numerically simulated and analyzed. The results show that different health shock intensities lead to different asset allocation decisions. Based on the research results, this paper puts forward relevant suggestions on how to make optimal asset allocation decisions for retirees and how the government formulates social policies to cope with the aging society.

**Keywords:** Health shock; retirees; utility function; asset allocation decision.

## 1. Introduction

The asset allocation decision of retirees mainly takes into account their own income and makes consumption and investment decisions according to the principle of utility maximization. After discovering that health status has an impact on retirees' investment decisions, domestic and foreign scholars have conducted in-depth research on its impact mechanism and path, and regard health impact as an important factor affecting asset allocation decisions.

Asset decisions such as consumption and investment of retirees are affected by many factors. In terms of consumption, the author believes that urban retirees are still in the primary stage of meeting basic survival needs, and food expenditures and medical expenditures account for a large proportion of total expenditures [1]. Wang Shaohui believes that the consumption structure of urban and rural elderly groups in my country is similar, and they are all concentrated in clothing, housing, and medical care [2]. Yang Fan et al. believe that the consumption types of the elderly can be divided into three categories: basic type, medical type and development type. When the elderly face health shocks and generate demand for medical consumption, there is a possibility of transforming into a medical consumption structure [3]. Empirical research conducted by Rsoen H.S. found that health is an important determinant of portfolio allocation [4]. Using dynamic panel data to test causality. Michaud P.C. et al. confirmed that health has a clear impact on wealth domination of elderly couples [5]. Lei Xiaoyan used the Tobit model to study that the deterioration of health risk status will significantly reduce the proportion of risk assets held by urban residents [6]. Sui Lirui and others found through research that the elderly with poor health and many family interactions will increase investment in risk-free financial products [7]. In the research of the utility function of the consumption model, Dardanoni V. believes that there is uncertainty in the static consumption model, and greater uncertainty will lead to an increase in medical demand [8]. On this basis, Picone G. et al. obtained the impact mechanism of uncertainty in the dynamic framework on the increase of medical demand through the family production model [9].

In terms of the mechanism of health impact on retirees' asset decision-making, He Yangping and others believed that the proportion of non-healthy members of the family's self-assessment affects the risk preference of the head of the household, which in turn affects the proportion of risky financial assets investment [10]. Wu Weixing et al. believed that health status affects the proportion of households' investment in stocks and risk assets through risk preference and bequest motives, while

medical insurance only affects the proportion of risk assets held [11]. Fu Qiannan influences elderly households' financial asset portfolio decision through risk preference and the cross marginal utility of health and consumption [12]. With the objective of maximizing the objective function, through numerical simulation or dynamic programming, we will obtain the consumption and investment decision results that maximize the utility of retirees. There are significant differences in the investment risk preferences of different retirees, the degree of response to health expenditures and changes in consumption under health shocks. Becker G.S. et al. proposed that health can be regarded as a form of human capital [13].

Health shocks have a huge impact on the economic activities of families. The health status of the retired elderly deteriorates with age, and they face a higher probability of health shocks. Although social medical insurance in my country can alleviate the medical expenditure pressure of retired elderly to a certain extent, serious health shocks will consume a large amount of asset stock, change their future expectations, and then affect their assets in consumption, investment, health expenditure, etc. allocation decision. Therefore, this paper deeply studies the impact of health shocks on retirees' asset allocation decisions, and provides decision-making basis for retirees in large and medium-sized cities to formulate reasonable asset allocation plans to deal with health risks, improve the quality of retirement life, and reduce financial risks during retirement.

## 2. Optimal asset allocation decision model for retirees

This paper draws on the research results of [9] and selects the relative risk scale utility function (CRRA function) to describe its preference. The CRRA function considers the relative risk aversion coefficient to be a constant value, and the proportion of risky investment in asset wealth remains unchanged. In addition, it is assumed that the investor's consumption  $C$  and health stock  $H$ . From the Cobb-Douglas preference, the utility function is obtained in the form:

$$U_t(C_t, H_t) = \frac{(C_t^\gamma H_t^{1-\gamma})^{1-\rho} - 1}{1-\rho} \quad (1)$$

where  $\gamma, \rho > 0$  are given parameters,  $\rho$  is the relative risk aversion coefficient. This paper sets  $\gamma=0.6$ .

For the health status of retirees, set the health stock  $H_t$  as a quantitative indicator, which is affected by the health expenditure  $M_t$  and the health shock  $\varepsilon_t$ , that is, the health stock in each period will increase with the increase of health expenditure, and with the health shock. The initial health stock  $H$  when retirees retire is taken as a given value. Therefore, this paper sets the initial health stock of retirees when they retire  $H_0 = 22$ .

In this paper, referring to [14], the health investment income is setting  $L_t$ , and the uncertainty of health expenditure on the increase of health stock is described by the probability of health shock occurrence  $\theta$ . The functional relationship between health stock and health expenditure is as follows:

$$H_{t+1} = H_t(1 - \delta) + \varepsilon_t + L_{t+1} \quad (2)$$

$$L_{t+1} = M_{t+1} \exp(r_{h,t} + \theta \varepsilon_t) \quad (3)$$

Among them, it is assumed that the health depreciation coefficient  $\delta$  is related to the age of retirees and takes a value in  $(0,1)$ ,  $\theta$  is a constant and satisfies  $\theta > 0$ , and  $r_{h,t}$  is the health investment rate of return.  $\varepsilon_t$  is health shock. Health shock refers to the health impairment of retirees due to physical illness, accidental injury, physical disability, and life dysfunction, which will have a serious and lasting impact on the family's economic decision-making.

Under the premise of given initial wealth level, health level and bequest willingness, retirees maximize their utility through asset allocation decisions of consumption, investment and health expenditure. The objective function has the following expression:

$$V = \max_{C_t, M_t, K_t} \sum_{t=0}^{\infty} \beta U_t(C_t, H_t) \quad (4)$$

Satisfying the intertemporal budget constraint is:

$$W_t = C_t + M_t + K_t \quad (5)$$

$$W_{t+1} = K_t + K_t^\alpha \quad (6)$$

$$H_{t+1} = H_t(1 - \delta) + \varepsilon_t + L_{t+1} \quad (7)$$

$$L_{t+1} = M_{t+1} \exp(r_{h,t} + \theta \varepsilon_t) \quad (8)$$

$C_t > 0, M_t > 0, W_{t+1} > 0, H_{t+1} > 0$ . where  $\beta$  is the time discount factor, assumed to be constant.  $W_{t+1}$  is the total wealth of financial assets at the beginning of the t+1 period, by adding the total wealth of financial assets  $K_t$  in the t period plus the current financial asset income  $K_t^\alpha$ ,  $W_0$  is the total amount of disposable assets of the retirees at the beginning of the retirement period. This paper uses the 2000-2020 China A-share market investment return to measure the return on risky assets, and the calculated value is 18.61%. Using the 2000-2020 Chinese market risk-free rate to measure the risk-free asset return, the calculated value is 2.33%. According to the 2018 data from the China Health and Retirement Tracking Survey (CHARLS), the proportion of retirees investing in risky assets is 6.23%, and the calculated investment return  $\alpha=3.34\%$ .

### 3. Model solution

This paper refers to the Ramsey model idea to solve the model, starting from the micro-foundation of retirees' intertemporal consumption and investment behavior, to determine steady-state savings.

The total lifetime utility obtained by retirees at time t can be attributed to the current consumption and expenditure utility and the discounted value of the future utility generated by the current savings. Therefore, the problem of intertemporal utility maximization can be transformed into the following form: given  $K_t$ , choose the next period  $K_{t+1}$  to maximize utility:

$$V(K_t) = \max_{M_{t+1}, K_{t+1}} [\beta V(K_{t+1}) + U_{t+1}(C_{t+1}, H_{t+1})] \quad (9)$$

By transforming, we can get:

$$V(K_t) = \max_{M_{t+1}, K_{t+1}} \left[ \beta V(K_{t+1}) + \frac{(SP)^{1-\rho}-1}{1-\rho} \right] \quad (10)$$

$$S = (K_t + K_t^\alpha - M_{t+1} - K_{t+1})^\gamma \quad (11)$$

$$P = H_t(1 - \delta) + \varepsilon_t + M_{t+1} \exp(r_{h,t} + \theta \varepsilon_t)^{1-\gamma} \quad (12)$$

At this time, there are two decision variables  $K_{t+1}$  and  $M_{t+1}$  in the equation at the same time, and we only hope to solve the optimal result by determining  $K_{t+1}$ , so we will eliminate  $M_{t+1}$  by analyzing the relationship between  $K_{t+1}$  and  $M_{t+1}$ .

By adjusting  $M_{t+1}$  to maximize the following formula, the overall utility can be maximized:

$$(SP)^{1-\rho} \quad (13)$$

Differentiating  $M_{t+1}$ , so that it equals 0, the result is as follows:

$$\frac{\partial (C_{t+1}^\gamma H_{t+1}^{1-\gamma})}{\partial (M_{t+1})} = -\gamma C(M_{t+1})^{\gamma-1} H(M_{t+1})^{1-\gamma} + C(M_{t+1})^\gamma (1 - \gamma) H(M_{t+1})^{-\gamma} = 0 \quad (14)$$

Solving the above equation to get:

$$\frac{C(M_{t+1})}{H(M_{t+1})} = \frac{\gamma}{1-\gamma} \frac{\gamma}{2-\gamma} \quad (15)$$

Assuming  $x = \frac{\gamma}{1-\gamma} \frac{\gamma}{2-\gamma}$ , we have:

$$x = [H_t(1-\delta) + \varepsilon_t + M_{t+1} \exp(r_{h,t} + \theta \varepsilon_t)] = K_t + K_t^\alpha - M_{t+1} - K_{t+1} \quad (16)$$

In the optimal solution equation after solving by the above formula, the decision variable is only  $K_{t+1}$ :

$$C_t^\gamma H_t^{1-\gamma} = H_t(1-\delta) + \varepsilon_t + G \quad (17)$$

$$G = \frac{\exp(r_{h,t} + \theta \varepsilon_t) [K_t + K_t^\alpha - K_{t+1} - x(H_t(1-\delta) + \varepsilon_t)]}{\exp(r_{h,t} + \theta \varepsilon_t) + 1} \quad (18)$$

Based on the above formula, numerical simulation analysis can be carried out by computer. At this time, it is necessary to determine the reasonable range of  $K_t$ , and use the steady-state conditions of the Ramsey model to solve the value of  $K_t$  under steady-state conditions.

Assuming  $F(K_t, K_{t+1}) = C_t^\gamma H_t^{1-\gamma}$ , After processing, the objective function becomes:

$$V(K_t) = \max_{K_{t+1}} \left[ \beta V(K_{t+1}) + \frac{F(K_t, K_{t+1})^{1-\rho} - 1}{1-\rho} \right] \quad (19)$$

Differentiate  $K_{t+1}$  so that the result of the derivation is 0, we can get:

$$V'(K_{t+1}) = \frac{1}{\beta} \cdot [F(K_t, K_{t+1})]^{-\rho} \cdot x^\gamma \cdot \frac{\exp(r_{h,t} + \theta \varepsilon_t)}{\exp(r_{h,t} + \theta \varepsilon_t) + 1} \quad (20)$$

Further, we have:

$$V'(K_t) = [F(K_t, K_{t+1})]^{-\rho} \cdot x^\gamma \cdot \frac{\exp(r_{h,t} + \theta \varepsilon_t)(\alpha \cdot K_t^{\alpha-1} + 1)}{\exp(r_{h,t} + \theta \varepsilon_t) + 1} \quad (21)$$

$$V'(K_{t+1}) = [F(K_{t+1}, 2)]^{-\rho} \cdot x^\gamma \cdot \frac{\exp(r_{h,t} + \theta \varepsilon_t)(\alpha \cdot K_t^{\alpha-1} + 1)}{\exp(r_{h,t} + \theta \varepsilon_t) + 1} \quad (22)$$

When  $K$  tends to steady state,  $K_t = K_{t+1} = K_{t+2}$ , we have:

$$K_t = K_{t+1} = \frac{1 - \beta^{\frac{1}{\alpha-1}}}{\alpha \beta} \quad (23)$$

#### 4. Numerical simulation analysis

By solving the optimal asset decision-making model for retirees' investment and consumption, this paper finds the impact of health shocks on retirees' asset allocation decisions. Next, MATLAB software is used to further verify the results of the model.

Under the optimal solution of the model based on the premise of maximizing the effect, the various expenditures in different time periods obtained by the model simulation in this paper, as shown in Figure 1, the various expenditures of retirees are in order of savings investment, consumption expenditure, Healthy investment, which is in line with the current social phenomenon and national conditions of the high savings rate of retirees in China.

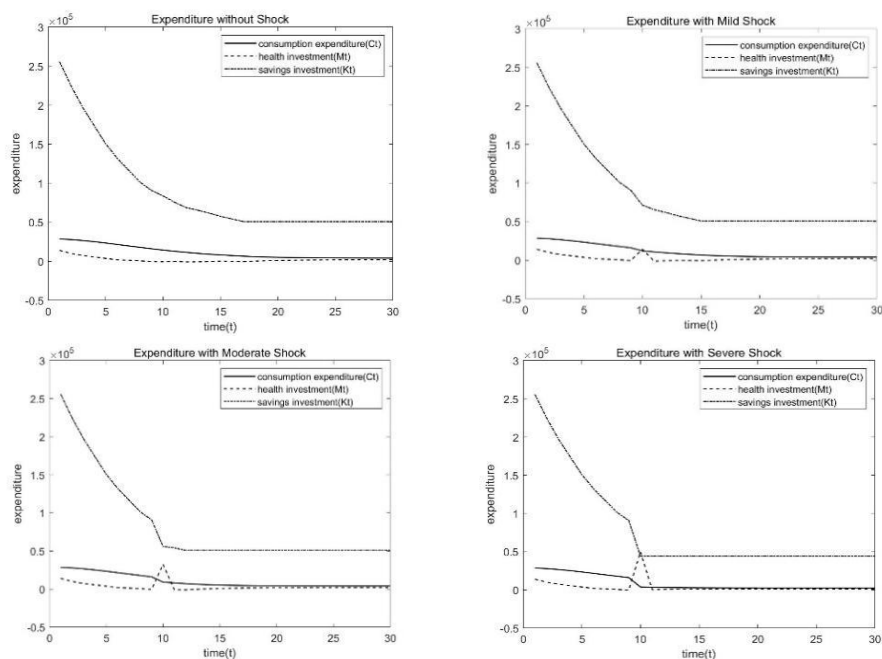


**Figure 1.** Simulation of the evolution of various expenditures over time under four health shocks

As shown in Figures 2 and 3, as the degree of health shocks suffered by retirees increases, the rate of decline in total assets also increases gradually, and the time point for reaching the termination steady state situation gradually advances. In the steady state, the number of total assets held by retirees also decreases with the magnitude of the health shock, which is consistent with the idea that retirees with major illnesses should increase their savings.

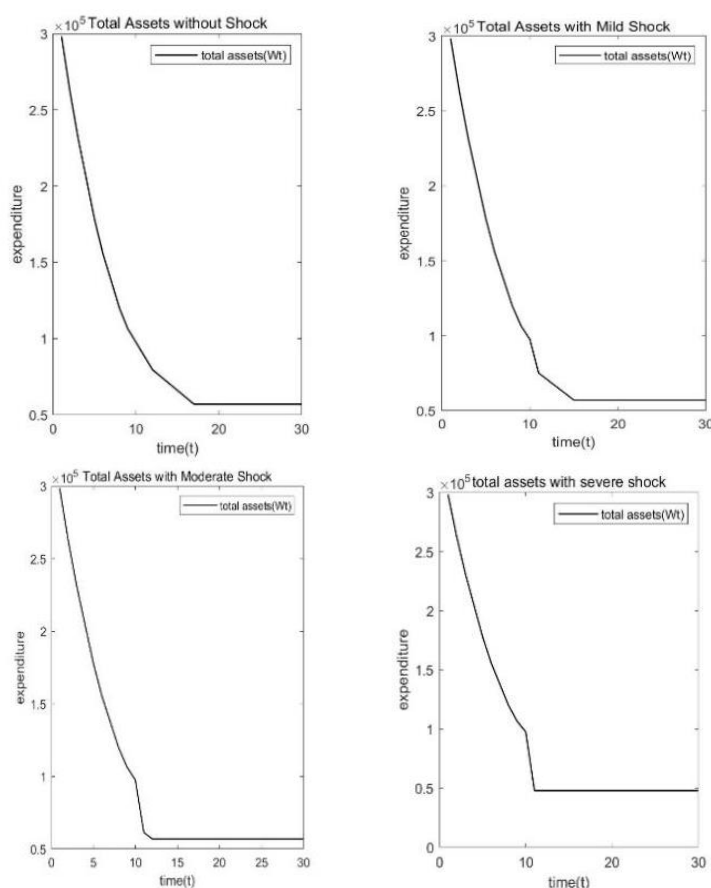


**Figure 2.** Simulation of the evolution of total assets over time under four health shocks

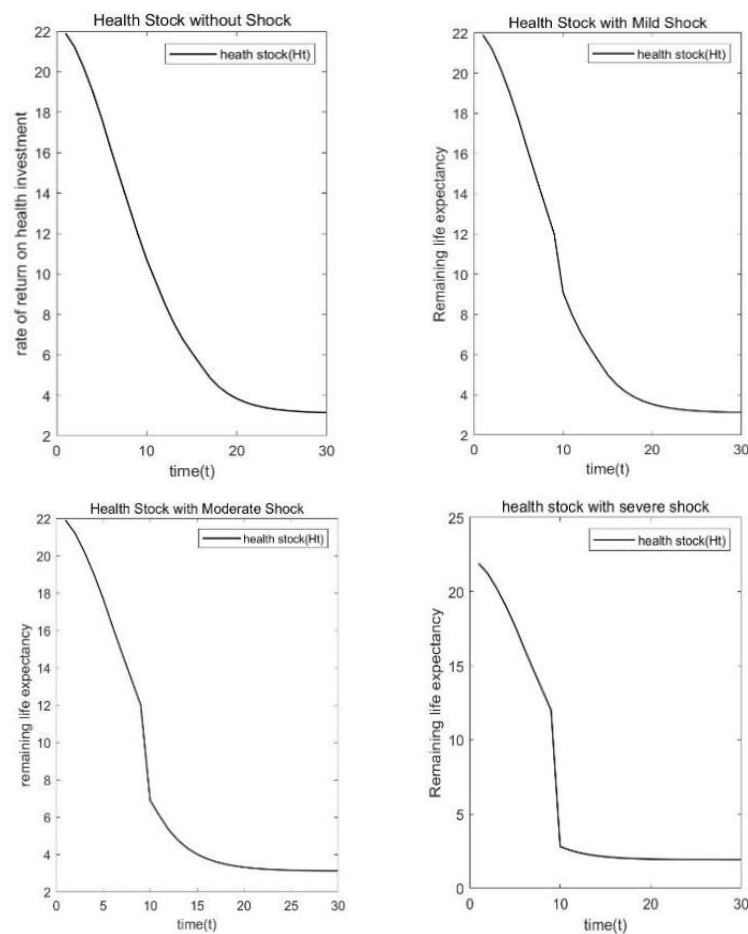


**Figure 3.** Simulation of health stock evolution over time under four health shocks

By assigning different values of savings investment yields and solving the model multiple times, we can obtain the changing trends of the optimal proportions of consumption expenditure, health investment, and savings investment. As shown in Figure 4, under the condition that other factors remain unchanged and optimized, as the rate of return on savings investment increases, the proportion of consumption expenditure shows a trend of slowly decreasing. The proportion of healthy investment showed a trend of rising first and then decreasing, and the overall change was small, and the proportion of savings investment had a positive relationship with the rate of return on savings investment.

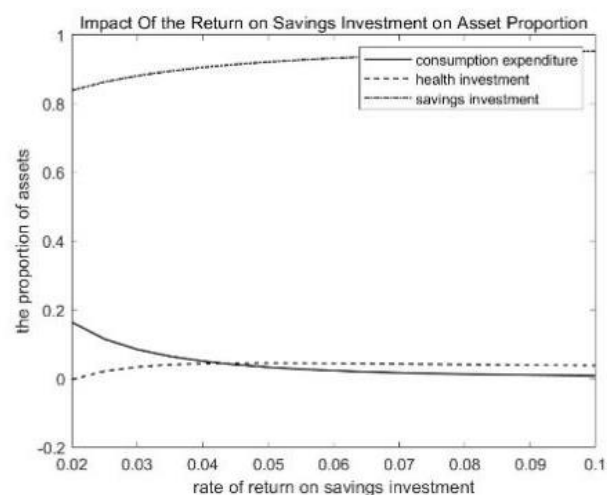


**Figure 4.** Simulation of the impact of savings investment yield on optimal asset allocation

By assigning different assignments to the health stock and solving the model multiple times, the changing trends of the optimal proportions of consumption expenditure, health investment, and savings investment can be obtained. As shown in Figure 5, under the condition that other factors remain unchanged and optimized, the proportion of savings investment has nothing to do with the expected remaining life; the proportion of consumption expenditure has a positive relationship with the expected remaining life, the proportion of health investment is related to the expected remaining life shows an inverse relationship. When retirees suffer from health shocks, the health stock of retirees, that is, the expected remaining life expectancy, decreases, the proportion of consumption expenditure decreases, and the proportion of health investment increases.

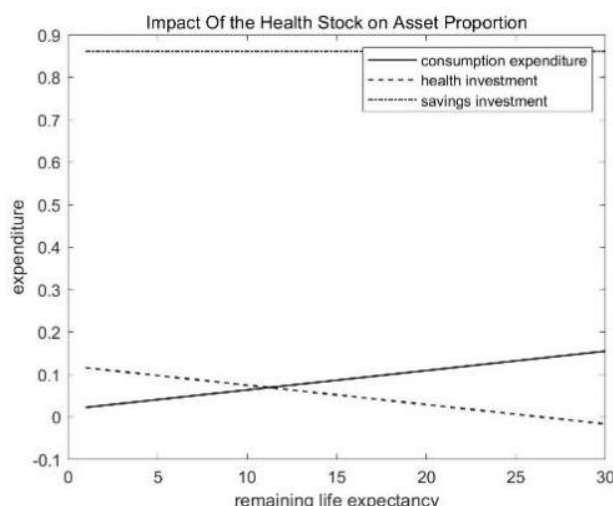


**Figure 5.** Simulation of the impact of healthy stock on optimal asset allocation

By solving the model multiple times for different assignments of the rate of return on health investment, the changing trend of the optimal proportions of consumption expenditure, health investment and savings investment can be obtained. As shown in Figure 6, under the condition that other factors remain unchanged and optimized, the proportion of consumption expenditure has nothing to do with the rate of return on healthy investment, the proportion of savings investment and the rate of return on healthy investment show a positive relationship, the proportion of healthy investment and health shows an inverse relationship.

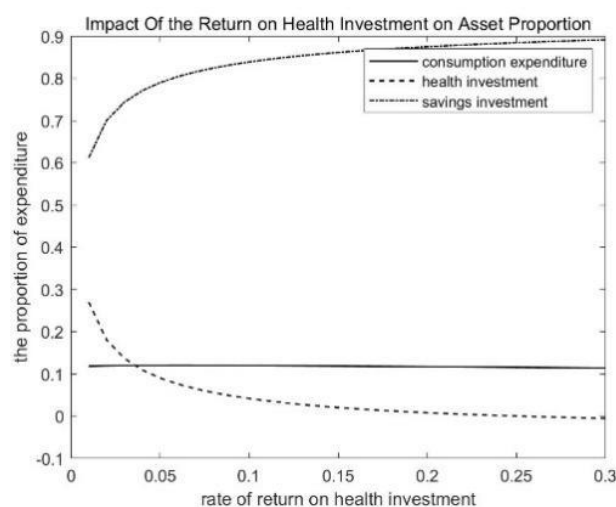


**Figure 6.** Simulation of the impact of healthy investment returns on optimal asset allocation

## 5. Conclusion

This paper selects the relative risk scale utility function and the health stock function, and sets the objective function that satisfies the constraints of the budget condition. Referring to the Ramsey model, the model is solved, and the intertemporal optimal allocation of assets is studied. Further through numerical simulation analysis, the impact of health shock on retirees' asset allocation decision was studied, and the changing trend of the optimal proportion of consumption expenditure, health investment and savings investment was obtained: Health shock affects retiree's asset allocation decision through health investment expenditure, retirees' consumption expenditure and health expenditure show opposite trends with age, and asset decisions are adjusted with changes in health and savings investment returns.

## References

- [1] Cao Rongrong, Hao Lei. Research on the consumption characteristics of urban retirees in China [J]. Research on technical economy and management research, 2021(05): 66-70.
- [2] Wang Shaohui, Li Fu. Consumption structure, demand characteristics and behavioral decision-making of the elderly in China [J]. Beijing Social Sciences, 2021(08): 119-128.
- [3] Yang Fan, Pan Yue, Huang Yingjiao. The consumption structure of the elderly in China and the influencing factors of consumption upgrading [J]. Population Research, 2020, 44(05): 60-79.
- [4] Rosen H S, Wu S. Portfolio choice and health status [J]. Journal of Financial Economics, 2004, 72 (3): 457-484.
- [5] Michaud P C, Soest A. Health and wealth of elderly couples: causality tests using dynamic panel data models [J]. Journal of Health Economics, 2008, 27(5): 1312-1325.
- [6] Lei Xiaoyan, Zhou Yuegang. Asset portfolio selection of Chinese households: health status and risk appetite [J]. Financial Research, 2010, (1): 31-44.
- [7] Sui Lirui, Li Yuanyuan, Shan Chengyun. A study on the influencing factors of financial behavior heterogeneity of the elderly in my country: An empirical analysis based on CHARLS data [J]. Price Theory and Practice, 2021, (01): 136-139.
- [8] Dardanoni V, Wagstaff A. Uncertainty, inequalities in health and the demand for health, Journal of Health Economics 6, 283-290.
- [9] Picone G, Uribe M, Wilson R M. The effect of uncertainty on the demand for medical care, health capital and wealth [J]. Journal of Health Economics, 1998, 17(2): 171-185.
- [10] He Yangping, He Xingqiang. Health and family risk financial assets investment participation degree [J]. Journal of East China Normal University (Social Science Edition), 2018, (4): 135-142.
- [11] Wu Weixing, Rong Apple, Xu Qian. Health and family asset choice [J]. Economic Research, 2011, (S1): 43-54.
- [12] Fu Qiannan, Qu Shiyong, Tian Boping, Li Yanpeng. Research on financial asset decision-making of Chinese elderly households from the perspective of health impact [J]. Forecast, 2020, 39(6): 83-89.
- [13] Bramhananda, Baljit, Singh, et al. 1. Becker, G.S. Human Capital – A theoretical and empirical analysis - with special reference to education, Columbia University Press, 1964.
- [14] Grossman M. On the concept of health capital and the demand for health [J]. The Journal of Political Economy, 1972, 80(2): 223-255.