

Comparison of the Markowitz Model and Index Model for Portfolio Optimization in the Chinese Stock Market

Chaoran Qin

Department of Economics and Management, Tongji University, Shanghai, China

1952935@tongji.edu.com

Abstract. Markowitz Model (MM) and Sharpe's Single Index Model (SIM) are two classical and practical models in portfolio theory. Currently the Chinese stock market is booming, therefore, it is worth testing whether MM and SIM can effectively spread the risk in Chinese security market. This paper establishes the MM and SIM based on the monthly observations of 10 stocks and CSI300. During the analysis process, there is an additional optimization constraint that selling short is not allowed. Finally, some conclusions are obtained: first, the constraint has a significant impact on the optimal portfolio, while the minimum-variance portfolio is very little affected. What's more, although both MM and SIM can effectively disperse the specific risk, SIM have better performance in the Chinese stock market by providing higher rewards given the same risk level and bearing lower risk given a certain return. These conclusions are of great significance to investors in the Chinese security market, which can help them obtain the maximum utility through asset diversification.

Keywords: Markowitz Model; Single Index Model; Portfolio Optimization.

1. Introduction

Throughout the research on modern financial theory, portfolio optimization, or asset allocation, has a pivotal position. As a pioneer, Markowitz firstly proposed portfolio theory and his famous mean-variance model [1]. Markowitz introduced quadratic programming method to calculate risk and return of an investment strategy. Since then, a bunch of scholars have tried to propose enhanced models pursuing the same goal of minimal variance under a given level of expected rate of return. Sharpe proposed single index model (SIM) based on Markowitz model [2]. SIM was regarded as an attracting candidate because it simplified the calculation process of Markowitz model to a great extent. These two models laid the solid foundation in the arena of portfolio optimization and were still widely used by financial practitioners and individual investors all around the world.

It is undeniable that China's stock market is booming, the amount of Chinese investors is rising, and the Chinese people's demand for wealth management is growing. Since the reform and opening-up, China has established a systematic and complete financial market. According to the operating data of securities companies in 2021 released by China Securities Association, by the end of 2021, the number of A-share capital accounts was 298 million, a year-on-year increase of 14.89%. According to the annual survey report of Chinese investors released by China Securities Investor Protection Co., Ltd, most investors' losses are due to their inability to choose stocks and wrong investment ideas. Based on this, the use of practical portfolio models may help the public investors spread the risk and improve the risky assets yield.

So far, some scholars have done empirical studies on the practical application effect of Markowitz model and SIM in the stock market in different stock markets worldwide. Since forecasting the parameters is fundamental for institutional and other investors to apply portfolio theory, Board and Sutcliffe used a large set of data in London Stock Exchange and concluded that when short selling is permitted, SIM performed well in estimating the covariance matrix [3]. Zhan did an empirical study on Markowitz model using 12 stocks in the US stock market and drew a conclusion that if an investor wants to use Markowitz model to choose an ideal portfolio, taking short selling and risk-free asset into account is fundamental [4]. Mandal's paper was based on 21 sample securities, and it turned out that the total risk of the optimal portfolio is 2.87% (in terms of SD) under SIM [5]. Sen and Fattawat used 30 stocks and BSE Sensex and found out that the total risk of the optimal portfolio calculated under SIM and Markowitz model was 2.21% and 1.3874%, respectively, which showed a significant

difference [6]. Patel and Chakraborty also did an empirical study on optimal portfolio in Indian Market under SIM and Markowitz model except that they used Nifty50 stocks. It is found that only six stocks out of Nifty50 stocks are chosen for inclusion in optimal portfolio [7]. Similarly, Putra and Dana tested the effectiveness of Markowitz model and SIM in Indonesian Stock Exchange using LQ45 Stocks, but there is no significant difference in the average return of the single index model with the Markowitz model [8].

In conclusion, both Markowitz Model and Single Index Model still have application value on portfolio optimization. Though numerous empirical studies have been done, there is few empirical papers aimed at testing the effectiveness of the two models in the Chinese stock market. Also, the comparison results of Markowitz model and Single index model vary in different country's stock market. Consequently, this paper will compare the optimal portfolio results for Markowitz model and single index model in the Chinese stock market using CSI300 as a market index and 10 stocks from 4 sectors. The empirical process is summarized as follow. First, this paper selects 10 representative stocks through five indicators and extracts the return of a market index and a risk-free asset. Then, the paper obtains the monthly excess return of 10 stocks through data preprocessing and makes a descriptive statistical analysis of the data. Next, this paper uses Markowitz Model and Sharpe's Single Index Model to obtain five outputs: the minimum variance frontier, the efficient frontier, the minimum-variance portfolio, the optimal portfolio and the capital allocation line under both constrained and unconstrained situations. Finally, by analyzing these results, this paper draws a conclusion that both MM and SIM can effectively spread risk in the Chinese stock market, but the SIM has better performance in comparison.

This paper is constructed as follows. Section 2 shows the data. Section 3 presents the methods while Section 4 refers to the results. Section 5 concludes the paper.

2. Data

The data in this article is derived from the Wind database, a commonly used financial database in China. This paper selects a recent 17 years of historical daily closing price data for eleven risky assets from April 18,2005 to April 15, 2022, including 10 stocks and CSI 300 which is an equity index. Also, this paper extract the daily interest rate of 10-year Chinese Treasury Bond as a proxy for risk-free rate.

The closing price can be translated to the rate of return of each risky asset using the following formula:

$$r_i = \frac{p_i - p_{i-1}}{p_{i-1}} \quad (1)$$

Where r_i stands for the rate of return of the i-th observation, and p_i refers to the corresponding price.

Then the excess rate of return can be obtained by the following formula:

$$R_i = r_i - r_f \quad (2)$$

Where R_i is the excess return of the i-th observation, r_i is the rate of return of the i-th observation, and r_f stands for the rate of return of the risk-free asset.

Table 1. Descriptive statistics of the excess return for 10 stocks

	BXJC	WHHX	HLHS	DZJG	SYKJ
Avg	21.534%	16.719%	14.430%	13.791%	12.881%
Std	49.304%	45.157%	46.236%	50.272%	44.519%
Max	56.362%	46.189%	31.953%	32.839%	46.526%
Min	-50.872%	-34.818%	-38.704%	-48.013%	-36.213%
	CDKJ	CCGX	XHC	TSL	JCYY
Avg	19.368%	31.363%	14.891%	3.926%	16.417%
Std	51.527%	53.182%	49.389%	37.800%	46.986%
Max	53.245%	60.194%	51.386%	29.850%	90.701%
Min	-48.030%	-36.059%	-46.977%	-43.394%	-35.920%

Note: The Avg and Std are both annualized for the sake of the following study.

Table 1 shows the descriptive statistics of selected stocks. The results reveal that the 'XHC' has the highest average rate of return as well as the largest standard deviation. 'JCYY' 's average return ranks the lowest, while 'BXJC' has the smallest standard deviation among the 10 sample stocks. Moreover, 'BXJC' has the lowest max return of 27.93% and the highest min return of -25.85%, which means the return of this stock is relatively more concentrated. 'XHC' has the highest max return, which is 60.41%.

Table 2. Basic information of the ten representing company stocks

#	Acronym	Code	PE (time)	ROE (%)	PB (time)	NAPS (¥)	Listing Date	Sector
1	BXJC	000786.SZ	14.10	19.71	2.61	11.21	1997-06-06	Materials
2	WHHX	600309.SH	10.23	42.03	3.68	21.82	2001-01-05	Materials
3	HLHS	600426.SH	9.46	38.46	3.06	10.54	2002-06-20	Materials
4	DZJG	002008.SZ	17.38	18.67	3.28	10.37	2004-06-25	IT
5	SYKJ	600183.SH	13.68	24.62	2.95	5.67	1998-10-28	IT
6	CDKJ	600584.SH	14.27	17.21	2.01	11.80	2003-06-03	IT
7	CCGX	000661.SZ	16.55	29.46	4.27	36.01	1996-12-18	Healthcare
8	XHC	002001.SZ	18.00	21.02	3.57	8.45	2004-06-25	Healthcare
9	TSL	600535.SH	8.60	18.91	1.57	8.58	2002-08-23	Healthcare
10	JCYY	600566.SH	11.95	20.37	2.28	10.15	2001-08-22	Healthcare

Note: The PE index in the table takes the Trailing Twelve Months (TTM) algorithm in order to exclude seasonality change in financial analysis. The PB index in the table use the Last File (LF) net asset per share

Table 2 shows the basic characteristics of the 10 companies to show the rationality of the selection. As shown above, Condition 1 shows that the Listing Date is before 2007, which is to ensure that there is sufficient data to prevent the freedom problem in subsequent analysis. Conditions 2 to 4 are that the P/E ratio is no more than 20 times, the ROE is at least 15%, and the P/B ratio is no more than 5 times. These three conditions are respectively corresponding to the three key indicators of mainstream stock selection. The stocks that meet these three conditions have high investment value in both bull and bear markets. In addition, condition 5 considers that the net asset per share is greater than ¥5, to some extent indicating the "true value" of these stocks is high. Some other information of the selected assets is shown below. Based on the above conditions, it is sufficient to show that the selected stocks are rational.

Table 3. The histogram results of CSI return

bin	daily	weekly	monthly
-20%	0	0	1
-18%	0	0	1
-16%	0	0	1
-14%	0	1	3
-12%	0	4	5
-10%	0	5	6
-8%	4	10	7
-6%	24	21	9
-4%	61	51	15
-2%	250	97	21
0%	1584	179	28
2%	1827	215	36
4%	323	137	28
6%	47	53	18
8%	7	29	13
10%	4	13	5
12%	0	5	3
14%	0	2	2
16%	0	2	1
18%	0	1	1
20%	0	0	1
other	0	0	0
SUM	4131	825	205
Kurtosis	3.837	2.717	1.452

Note: The kurtosis formula in Excel has subtracted 3, so the smaller the value is, the closer the distribution is to Gaussian distribution.

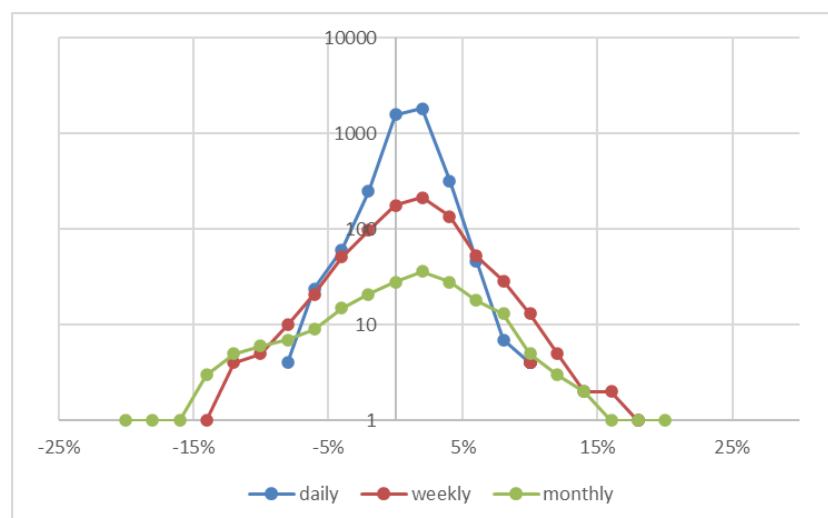


Fig. 1 The distribution of CSI 300 return

In order to reduce the non-Gaussian effects, this paper aggregates the daily data to the weekly and monthly observations. Table 3 shows the histogram statistical results of the daily, weekly and monthly return of CSI 300. The kurtosis of daily, weekly and monthly return is 3.837, 2.717, and 1.452 respectively, showing a significant approach to 0, which means that the monthly data's non-Gaussian effect is the weakest. Figure 1 shows the distribution of CSI300 return, in which the y-axis is a

logarithmic scale. It can be seen more directly from the figure that the monthly data is the closest to the normal distribution.

3. Methodology

The portfolio theory aims at solving the problem of efficient diversification, which is adjusting the investment proportion of various assets under the condition of risk uncertainty and effectively spreading the risk to maximize the investment efficiency. Markowitz model (MM) is effective in identifying the efficient set of portfolios, or the efficient frontier of risky assets. Sharpe's single index model (SIM) only considers the market factor when calculating the covariance, thus simplifying the drawing of efficient frontier [9].

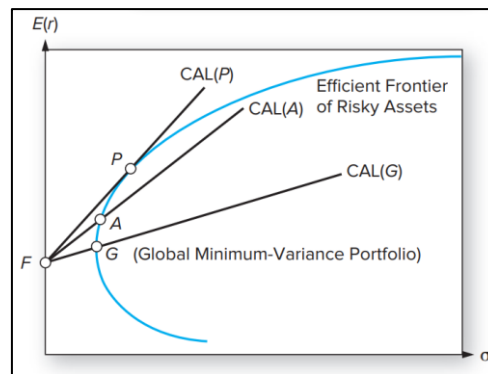


Fig. 2 The efficient set of portfolios

Figure 2 shows the efficient portfolio set with three capital allocation lines. The procedure of constructing an optimal portfolio should be separated into several steps. The first step is calculating the risk-return combinations and drawing the minimum-variance frontier (the entire blue curve in figure 2), which represents the lowest possible variance that can be attained given an expected return. Point G is the global minimum-variance portfolio, which has the smallest standard deviation among all the available portfolios. The portion of the frontier that lies above the global minimum-variance portfolio G is called the efficient frontier. The second part of the optimization plan is searching for the optimal risky portfolio. The best risky portfolio is the one that results in the steepest capital allocation line (CAL), because steeper CALs provide greater rewards for bearing any level of risk, which means the Sharpe ratio is higher. Therefore, CAL (P) dominates all alternative feasible lines, which means Portfolio P is the optimal risky portfolio. The last step is capital allocation, that is, the selection of the desired point along the CAL (P). This step is where the investor's risk aversion comes into play. The more risk-averse investors will put more weights on risk-free assets and less weights on the optimal risky portfolio P. The whole result is known as the separation property [9]. The basic formula of Markowitz Model is as follows:

$$\min \sigma^2(r_p) = \sum_{i=1}^n \sum_{j=1}^n \omega_i \omega_j \text{cov}(r_i, r_j) \quad (3)$$

$$\text{s. t. } \sum_{i=1}^n \omega_i = 1; \quad (4)$$

$$r_p = \sum_{i=1}^n \omega_i r_i; \quad (5)$$

Where r_p is the rate of return of portfolio, r_i is the rate of return of the i -th security, ω_i refers to the weight for the i -th security, $\sigma^2(r_p)$ is the variance of portfolio, and $\text{cov}(r_i, r_j)$ presents the covariance between r_i and r_j . The formula of Single Index Model is as follows:

$$\min \sigma^2(r_p) = \sum_{i=1}^n \sum_{j=1}^n \omega_i \omega_j \beta_i \beta_j \sigma_M^2 + \sum_{i=1}^n \omega_i \sigma^2(e_i) \quad (6)$$

$$s. t. \sum_{i=1}^n \omega_i r_i - \beta_i = \beta_p; \quad (7)$$

$$\sum_{i=1}^n \omega_i = 1; \quad (8)$$

$$\sum_{i=1}^n \omega_i \alpha_i + \beta_p R_M = R_p; \quad (9)$$

Where M denotes the common macroeconomic factor, or the market index. R_p is the excess return of the portfolio. R_M refers to the excess return of the market index, and $R_M = r_M - r_f$. σ_M^2 is the variance of the market index. ω_i refers to the weight for the i-th security, α_i is the stock's expected return if the market is neutral, that is, if $R_M = 0$. β_i is the security's responsiveness to market movements, and $\beta_i = \frac{\text{cov}(r_i, r_m)}{\sigma^2(r_m)}$. Both α and β follow the equation $r_i = \alpha_i + \beta_i r_m + e_i$. e_i is the residual, which means the unexpected return due to unexpected events that are relevant only to the firm-specific uncertainty. $\sigma^2(e_i)$ is the variance attribute to firm-specific uncertainty.

4. Result

This paper applies the Markowitz model and Single index model in the Chinese stock market. Each model finally obtains five outputs, including the minimum variance frontier, efficient frontier, minimum variance portfolio, optimal portfolio and capital allocation line respectively under unconstrained and constrained conditions. This additional optimization constraint is that the weight of each risky asset cannot be negative, which is in line with China's relevant policies and regulations that the stock market is not allowed to sell short. The constraint formula is as follows:

$$\omega_i \geq 0, \text{ for } \forall i \quad (10)$$

This paper first analyzes the results of MM and SIM to demonstrate their effectiveness, and then compares the outputs under constrained and unconstrained conditions of the same model to reflect the differences in the application of the models in the Chinese stock market. Finally, this paper also compares the outputs under the same condition of different models to see which model has a better performance in Chinese security market. For all of the following graphs, the x-axis is the annualized standard deviation which represents the risk level, and the y-axis refers to the annualized rate of return.

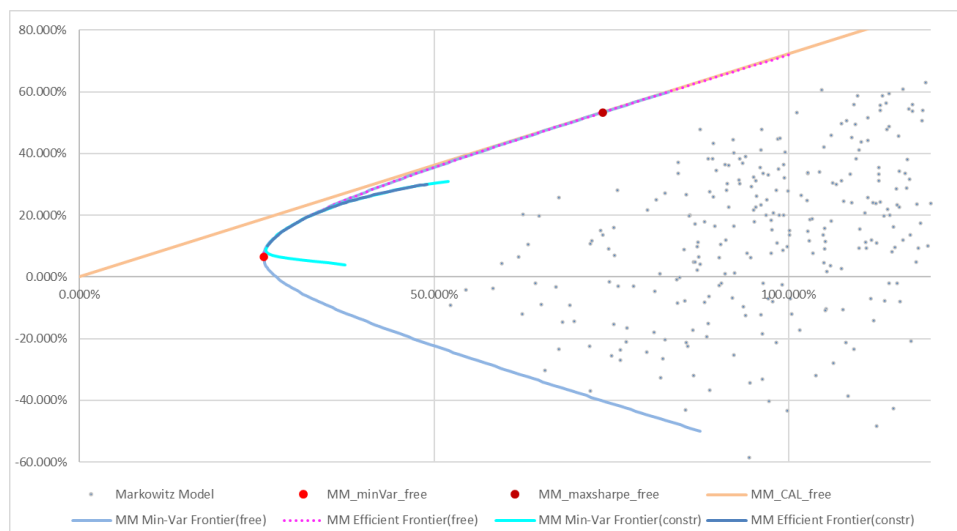


Fig. 3 The outputs of Markowitz Model (MM)

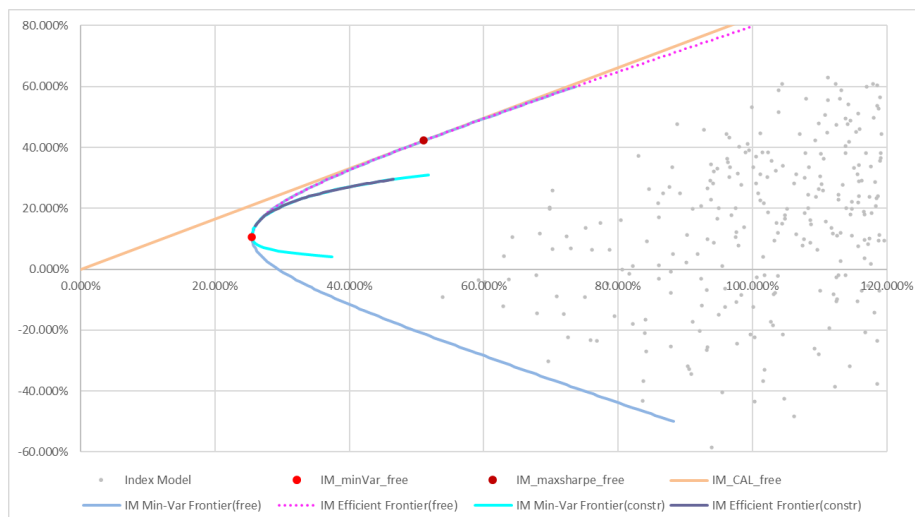


Fig. 4 The outputs of Single index model (SIM)

Figure 3 shows the minimum variance frontier and efficient frontier without additional optimization constraint and those under constraint of Markowitz model. The range of the constrained minimum-variance frontier is significantly narrowed compared with that under unconstrained condition. To elaborate, given the same level of risk, the maximum return and the maximum loss of constrained portfolio are relatively small, especially the maximum loss is reduced more. This result reveals that the constraint that Chinese stock market does not allow to sell short is not only a restriction but more importantly a kind of protection for those individual investors who have weak risk tolerance. Figure 4 is the outputs of Single index model, which also shows that the constrained minimum-variance frontier and efficient frontier is narrowed as well as the inefficient frontier.

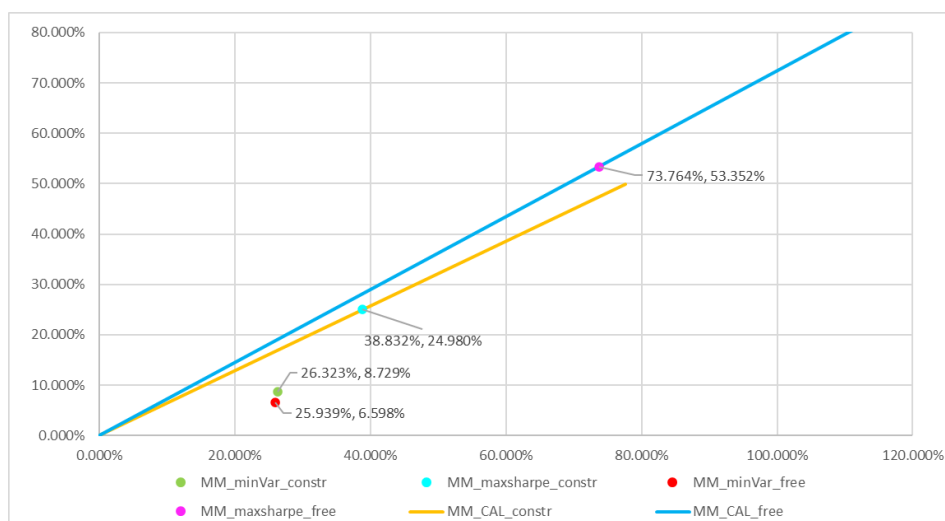


Fig. 5 Comparison of the MM outputs under unconstrained and constrained conditions

Figure 5 mainly compares the capital allocation line, the optimal portfolio and the minimum-variance portfolio under constrained and unconstrained situations of Markowitz Model. The blue line is the CAL without extra constraint and the orange line shows the CAL with additional constraint. The slope of the blue line is greater than the orange one, which means that under the same risk level, the return under short sale constraint is lower than that without any additional constraint. The abscissa of the purple point is 73.764%, which represents the standard deviation of the optimal portfolio obtained by MM without additional constraints. The ordinate of the purple dot is 53.352%, which represents the return of this portfolio. Similarly, the bright blue dot indicates the maximal Sharpe ratio portfolio with additional optimization constraint, whose standard deviation is 38.832% and the return is

24.980%. This shows that the constraint of not allowing short selling avoids the risk of about 35% at the cost of losing nearly 30% yield. As for the minimum-variance portfolio, the coordinates of the red dot and green dot are not much different, indicating that this constraint has little impact on investors pursuing minimum-variance portfolio.

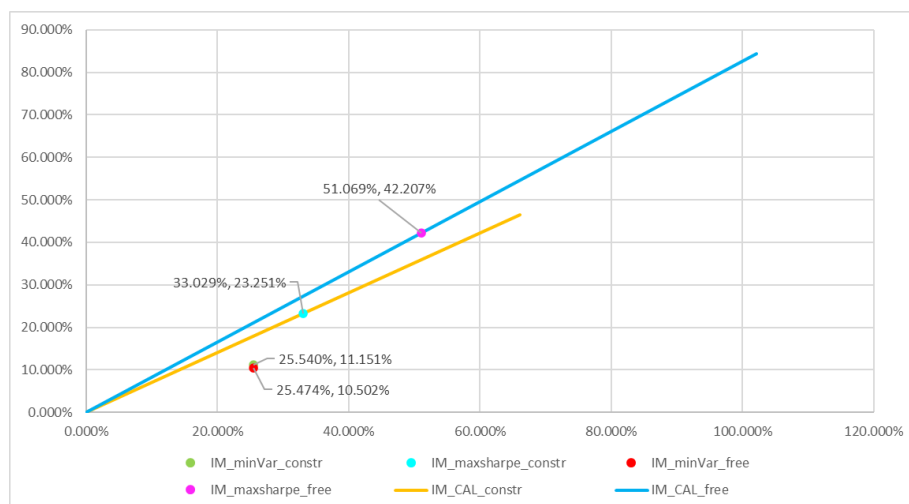


Fig. 6 Comparison of the SIM outputs under unconstrained and constrained conditions

Figure 6 mainly depicts the capital allocation line, the optimal portfolio and the minimum-variance portfolio under constrained and unconstrained situations of SIM. The conclusion drawn from this graph is very similar to Figure 5, but there are slight differences. First, the two minimum-variance portfolios with or without constraints under the single index model almost coincide. Second, the difference between the two optimal portfolios is also smaller than that of Markowitz model.

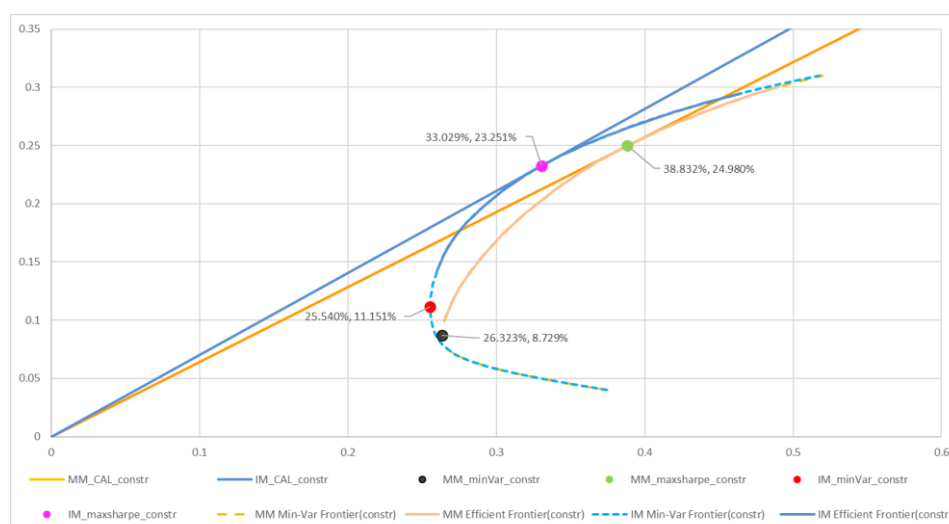


Fig. 7 Comparison of the MM and SIM outputs under constrained conditions

Finally, as shown in Figure 7, this paper compares which model produces better outputs under the additional optimization constraint of not allowing short selling, which is exactly in line with the actual situation of the Chinese stock market. Firstly, the blue dotted line represents the minimum-variance frontier of the SIM and the orange dotted line is that of the MM. It is found that the lower half of these two lines almost coincide, while the portion above the minimum-variance portfolio of SIM always gives a higher return than the MM under a given risk level. Secondly, the blue real curve and orange real curve represent the efficient frontier of SIM and MM respectively. Given the same standard deviation, the ordinate of blue curve is always higher than the orange curve. Thirdly, the blue and

orange lines are the CAL of SIM and MM. Obviously, the slope of blue line is greater than that of the orange line. As mentioned before, steeper CAL represents higher rewards for bearing any level of risk. Fourthly, the abscissa and ordinate of the purple point are 33.029% and 23.251% respectively, so the Sharpe ratio for the optimal portfolio of SIM is 0.704. The abscissa of the green point is 38.832% and the ordinate is 24.980%. Therefore, the Sharpe ratio for the optimal portfolio of MM is 0.643. Consequently, SIM's optimal portfolio has a higher Sharpe ratio. Last but not least, the red dot on the figure represents the minimum-variance portfolio of the SIM. The abscissa and ordinate of the red point are 25.540% and 11.151% respectively. The black dot is the minimum-variance portfolio of the MM, whose coordinates are 26.323% and 8.729% respectively. In a nutshell, whether looking at the efficient frontier, the minimum variance frontier, the minimum-variance portfolio, the optimal portfolio, or the CAL, the outputs calculated by the SIM are all better than the MM, which shows that the single index model has a better performance in the Chinese stock market.

5. Conclusion

Different from the existing academic research, this paper focuses on the application of portfolio theory in the Chinese stock market. This paper extracts the daily data of 10 representative stocks, an equity index and a 10-year T-Bond interest rate for 17 years through Wind database and reduces the non-Gaussian effect through data aggregation. This paper establishes MM and SIM and obtains the minimum variance frontier, efficient frontier, optimal portfolio, minimum-variance portfolio, and the CAL respectively. Besides analyzing these outputs, the paper further compares the difference between the results of the same model with or without additional optimization constraints, and which of the two models performs better in Chinese stock market. Finally, the following conclusions are obtained: First, the constraint of not allowing short selling in Chinese stock market has an obvious impact on the optimal portfolio, while the minimum-variance portfolio is very little affected by this constraint. In Markowitz Model, the influence of whether there are constraints on the two portfolios is more obvious; Second, both Markowitz Model and Single Index Model can effectively spread the specific risk, as the standard deviation of their constrained minimum-variance portfolio is less than that of any single asset; Third, whether looking at the CAL, minimum variance frontier, efficient frontier, minimum-variance portfolio or optimal portfolio with the additional optimization constraint, the results of SIM are better than MM.

This paper uses CSI300 as the market factor and a 10-year Chinese Treasury Bond as a proxy for risk-free rate to confirm the effectiveness of MM and SIM in the Chinese stock market. Further research could use other indices, like Shanghai Securities Composite Index (SSEC) or Shenzhen Securities Component Index (SZI) and also other risk-free asset to test whether these models can apply to different situations. Plus, currently other scholars have proposed improved models to overcome the deficiency of MM and SIM, and the effectiveness of such models needs to be further tested.

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