Portfolio Analysis of Stocks in Diverse Industries

Kaidong Zhang

Department of economics and management, Tongji University, Shanghai, China

k2992353767@163.com

Abstract. Portfolio optimization is a key issue in the current financial field. This paper aims to analyze the asset allocation of technology, financial services, industry, consumer defense and health. This paper selects ten representative company stocks from these five industries, then calculates the characteristic data and randomly assigns weights to them. Then, it uses the formulas of the two most effective models at present - Markowitz model and single index model to calculate the return, standard deviation and sharp ratio respectively, and randomly simulates a large number of data. Then it calculates the CAL curves under the two models, calculates the minimum variance frontier, efficient frontier and inefficient frontier under the five constraints that may be encountered in the actual investment, and draws the corresponding graphs. Finally, it finds out the differences and connections of figures under different constraints and makes a preliminary comparison between the effects of the two models. The results show that: first, there are slight differences in the results of the portfolio solved by the two models; Secondly, the two models can jointly reflect the portfolio characteristics under different constraints. These findings are useful for relevant investors who build portfolios in different industries.

Keywords: Portfolio; Markowitz Model; Index Model.

1. Introduction

Nowadays, with the increasing income of people, investment has a higher and higher status. However, venture capital without research, like playing poker and never looking at cards, is bound to fail. Therefore, the optimization of investment has always been the focus of financial research. In 1952, Markowitz proposed the efficient frontier model, and then Sharp proposed the Index model. Obviously, the risk of a single asset is far greater than that of a portfolio, so it is necessary to construct a portfolio. Its selection should be divided into two stages. The first stage is to observe and practice securities, and then evaluate the future performance of the available securities; the second stage is to carry out portfolios on the basis of the first stage. In 2018, the China Banking and Insurance Regulatory Commission put forward *the guidance on regulating the asset management business of financial institutions* [1], which requires financial institutions to conduct net value management of asset management products in accordance with the principle of fair value and shall not promise to guarantee the principal and guarantee the income. When it is difficult to cash out, financial institutions shall not be able to advance capital in any form. [2] It can be seen that the selection and research of portfolio is important for investors.

It's found that currently, there are numerous studies regarding portfolio and risk assessment. However, most of their studies are single and macro risk assessment and investment choice. For example, Jakšić analyzed the risk management system, including its early identification, evaluation, measurement and risk control [3]; Eyal analyzed portfolio selection in unstable markets [4]. At the same time, some researchers studied the portfolio and risk of individual industries. For example, Wang, Li and Zhu studied the quantitative portfolio strategy based on AI industry [5]; Zhu studied the investment risk of the coal industry and used the relevant knowledge of venture capital to moderately measure the coal industry [6]. Other researchers focused on the research of risk analysis methods. For example, David Scott, divya Anand and Andrew smithies designed the questionnaire and produced compound effects between the questions in the questionnaire, and generated compound risk scores for the project based on these compound effects [7]. It can be seen that there is few studies in the field of portfolio and risk assessment and analysis in different industries. Therefore, it is of interest to combine different industries and construct portfolios to achieve certain goals.

The empirical process in this paper can be summarized as follow. First, this paper chooses SPX and ten typical companies from technology, financial services, industry, consumer defensive and healthcare for closing prices from May 11, 2001, to May 12, 2021; Second, it calculates the Annual Average Return, Annual Standard Deviation, beta, alpha and Residual Standard Deviation of each stock and the Correlations between them. Thirdly, it uses the obtained characteristic data and random weight to calculate the return, standard deviation and sharp ratio under the two models with Markowitz model and index model respectively and finds that there are slight differences between the results under the two models. Fourth, it adds Min variance and Max sharp constraints under the two models to draw the CAL curve. Finally, under five constraints on the stock weight, this paper calculates the minimum variance frontier, effective frontier and effective frontier under the two models and drew images. In the portfolio, the weight of SPX is always slightly larger.

The rest of this paper is organized as follow. Section 2 shows the data, Section 3 presents the methods, Section 4 depicts the results and Section 5 concludes the paper.

2. Data

The data in this article is derived from Yahoofinance (https://finance.yahoo.com/). It selects the representative companies in various industries. i.e., ADBE, IBM, SAP, JPM, BRK/A, PGR, UPS, PG, JNJ and CL for closing prices, from May 11, 2001, to May 12, 2021, a total of 5219 data. At the same time, it also adds SPX data. These companies are so representative, and the time span of the data is so large, that it is enough to explain the characteristics and changes of stock prices of various industries and companies. Then, to implement further investigations, it summarizes the original data and calculates the required data separately. The calculation results are presented in Tables 1 and 2.

Table 1. Descriptive statistics of the selected stocks

	SPX	**ADBE**	**IBM**	**SAP**	**JPM**	**BRK/A**
Annual Average Return	7.54%	8.89%	9.85%	15.41%	17.39%	9.71%
Annual StDev	14.85%	28.13%	31.80%	21.06%	23.92%	30.81%
beta	1	1.052	1.150	0.712	0.798	1.321
alpha	0.00%	0.95%	1.18%	10.04%	11.37%	-0.25%
residual Stdev	0.00%	23.40%	26.82%	18.21%	20.79%	23.76%
	PGR	**UPS**	**PG**	**JNJ**	**CL**	
Annual Average Return	11.01%	9.44%	13.15%	7.04%	13.45%	
Annual StDev	18.13%	14.59%	23.30%	16.26%	18.67%	
beta	0.787	0.405	1.002	0.538	0.675	
alpha	5.07%	6.38%	5.59%	2.98%	8.36%	
residual Stdev	13.87%	13.29%	17.93%	14.16%	15.74%	

Table 2. Correlations of the selected stocks

Correlations	**SPX**	**ADBE**	**IBM**	**SAP**	**JPM**	**BRK/A**	**PGR**	**UPS**	**PG**	**JNJ**	**CL**
SPX	1	0.555	0.537	0.502	0.495	0.637	0.645	0.412	0.639	0.491	0.537
ADBE	0.555	1	0.406	0.300	0.316	0.254	0.541	0.282	0.287	0.305	0.272
IBM	0.537	0.406	1	0.355	0.354	0.328	0.442	0.226	0.273	0.261	0.246
SAP	0.502	0.300	0.355	1	0.289	0.304	0.371	0.237	0.249	0.316	0.313
JPM	0.495	0.316	0.354	0.289	1	0.354	0.374	0.197	0.268	0.221	0.234
BRK/A	0.637	0.254	0.328	0.304	0.354	1	0.410	0.220	0.480	0.194	0.292
PGR	0.645	0.541	0.442	0.371	0.374	0.410	1	0.231	0.349	0.342	0.377
UPS	0.412	0.282	0.226	0.237	0.197	0.220	0.231	1	0.193	0.452	0.350
PG	0.639	0.287	0.273	0.249	0.268	0.480	0.349	0.193	1	0.279	0.357
JNJ	0.491	0.305	0.261	0.316	0.221	0.194	0.342	0.452	0.279	1	0.499
CL	0.537	0.272	0.246	0.313	0.234	0.292	0.377	0.350	0.357	0.499	1

From the above data, we can know that the average return rate of JPM is obviously the highest, while that of JNJ is the lowest. When it comes to standard deviation, IBM is the highest, while SPX representing the whole market is the lowest. The beta value of BRK / A is the highest and UPS is the lowest, which shows their market relevance. From the correlations, we can see the relationship between the stocks of various companies. Data show that almost all of their companies are most relevant to SPX, that is, the market. Compared with SPX, the relationships with the remaining companies do not show obvious rules.

3. Methods

3.1 Stock characteristic data preparation

In order to obtain the annual average return, annual standard deviation, beta, alpha and residual standard deviation of each stock and the correlations between them, this paper uses a variety of formulas and basic methods to solve.

Annual average return, which represents the return of each stock and has important value, the calculation is shown below.

$$R = \frac{R_1+R_2+\ldots+R_n}{n} = \frac{1}{n}\sum_{n=1}^{n} R_t \tag{1}$$

R represents the average annual return.

Annual standard deviation, which represents the volatility of each stock and the risk of the stock, the calculation is shown below.

$$\sigma = \sqrt{1/N \sum_{i=1}^{N}(x_i - \mu)^2} \tag{2}$$

x represents the closing price and μ represents the average share price.

Beta, which represents the change range of each stock with the change of the market, which is calculated relative to SPX, the calculation is shown below.

$$\beta_a = \frac{Cov(r_a,r_m)}{\sigma_m^2} = \rho_{am} \cdot \frac{\sigma_a}{\sigma_m} \tag{3}$$

$Cov(r_a, r_m)$ is the co-variance between the return of stock a and the market return, ρ_{am} refers to the correlation coefficient between stock a and the market, σ_a is the standard deviation of stock a, σ_m is the standard deviation of the market.

Alpha coefficient is the difference between the absolute return of the investment and the expected risk return calculated according to the beta coefficient, the calculation is shown below.

$$\alpha = (R_i - r_f) - \beta[E(R_m) - r_f] \tag{4}$$

Absolute return is the actual return on investment minus the return on risk-free investment, which is used to measure the investment technology of investors. The expected return is the product of beta coefficient and market return, which reflects the return of investment due to the overall changes of the market.

Residual standard deviation, which is used to represent the accuracy of the estimated value. In statistics, the difference between the real value and the estimated value is called the residual, the calculation is shown below.

$$S = \sqrt{Q/f_Q} = \sqrt{\sum_{i=1}^{N} (y_i - \hat{y}_i)^2 / n - 2} \quad (5)$$

Correlation coefficient is a statistical method to reflect the correlation between two stocks, that is to tell us how closely one stock is related to another. When two stocks move up or down in the same direction, the correlation coefficient is positive. When the two stocks move in the opposite direction, the correlation coefficient is negative, the calculation is shown below.

$$P = \begin{pmatrix} \rho_{11} & \cdots & \rho_{1n} \\ \vdots & \ddots & \vdots \\ \rho_{n1} & \cdots & \rho_{nn} \end{pmatrix} \quad (6)$$

The above process calculates the data required for the next operation. Next, it uses Markowitz model and index model to calculate the return, standard deviation and sharp ratio of the portfolio.

3.2 Markowitz model

Markowitz model, namely Mean-Variance model, is a venture capital model proposed by Markowitz in 1952. He defined risk as the volatility of return and applied the method of mathematical statistics to the research of portfolio selection, so as to achieve the best balance between the goal of mutual restriction between high return and low risk. This method makes the multi-objective optimization of income and risk achieve the best balance effect. The Markowitz model is a method that formulates the elements of return and risk in an investment, and specifically the elements of risk can be minimized through diversification and combination of various investment instruments into a portfolio. By using the Markowitz method, investors can take advantage of all available information as a basis for maximizing the portfolio. [8] The following is the return and standard deviation formula of Markowitz model.

$$r_p = \vec{w} \cdot \vec{\mu}^T \quad (7)$$

In this formula, w is the weight of each stock in the portfolio, and μ is its rate of return, the calculation is shown below.

$$\sigma_p = \sqrt{\vec{v}\, P \vec{v}^T}, \ \vec{v} = \{w_1\sigma_1, w_2\sigma_2, \ldots, w_n\sigma_n\}^T \quad (8)$$

In this formula, P is the correlation coefficient.

3.3 Index model

Single index model and multiindex model are practical tools to analyze the structure and characteristics of portfolio. Modern portfolio theory was put forward by Markowitz in 1952. His "mean variance" theory of portfolio provides a feasible quantitative means for the trade-off between risk and return, but the calculation method is very complex. In 1964, sharp and others proposed the capital asset pricing model CAPM, which greatly simplified the calculation. Make the portfolio theory applied to the whole field of economics. This model has been widely used in real portfolio performance evaluation, securities valuation and public utility stock management. The single index model based on this is the empirical part of CAPM theory and can be used as a tool to test the capital asset pricing model. [9] The following is the return and standard deviation formula of Index model.

$$r_p = \vec{w} \cdot \vec{\mu}^T \quad (9)$$

It's the same as Markowitz's model.

$$\sigma_p = \sqrt{(\sigma_M \beta_p)^2 + \sum_{i=1}^{n} w_i^2 \sigma^2(\varepsilon_i)}\ ,\ \beta_p = \vec{w} \cdot \vec{\beta}^T \tag{10}$$

This formula is different from the calculation method of Markowitz model.

Later, this paper uses the planning solution and solvertable function of Excel to calculate the data of minimum variance frontier, efficient frontier and inefficient frontier under different constraints.

4. Results

According to the above methods, this paper uses the data and properties of each stock and uses the formulas under Markowitz model and index model to calculate the standard deviation and sharp ratio as shown in Table 3. The comparison found that there were slight differences between the two groups of data.

Table 3. Calculation results under the two models

SPX	**ADBE**	**IBM**	**SAP**	**JPM**	**BRK/A**	**PGR**	**UPS**
-0.810	0.864	0.382	0.207	0.554	-0.795	0.259	1.409
PG	**JNJ**	**CL**	**Return**	**MMStDev**	**MMSharpe**	**IMStDev**	**IMSharpe**
-0.789	0.033	-0.315	12.21%	47.36%	0.258	40.13%	0.304

The weight of each stock is a random number.

Then, it lists 1000 possible securities portfolios with random number function, and makes a figure with the standard deviation of each portfolio as the x-axis and the rate of return as the y-axis, as shown in Figure 1.

Fig. 1 Random portfolio of Markowitz model and index model

Through figure 1, it is found that the results of the two models are similar on the whole, but there are slight differences.

Then, it uses programming solution to calculate the minimum variance portfolio and maximum sharp ratio portfolio under Markowitz model and single index model. The data are shown in Table 4.

Table 4. The portfolio data under minVar and maxSharpe

	SPX	**ADBE**	**IBM**	**SAP**	**JPM**	**BRK/A**	**PGR**
MM minVar	0.189	-0.065	-0.042	0.087	0.071	-0.061	0.187
MM maxSharpe	-0.920	-0.040	-0.025	0.346	0.326	-0.031	0.326
IM minVar	0.031	-0.011	-0.024	0.098	0.053	-0.064	0.126
IM maxSharpe	-1.023	0.019	0.018	0.326	0.283	-0.005	0.284
	UPS	**PG**	**JNJ**	**CL**	**Return**	**StDev**	**Sharpe**
MM minVar	0.391	0.058	0.139	0.050	10.52%	11.31%	0.931
MM maxSharpe	0.454	0.322	-0.099	0.340	19.15%	15.25%	1.255
IM minVar	0.382	-0.001	0.261	0.150	10.53%	10.64%	0.989
IM maxSharpe	0.389	0.187	0.160	0.360	17.78%	13.83%	1.286

From these data, it can be seen that the weight of IBM and BRK/A stocks in the four portfolios is very small, even negative. This shows that the returns of these two stocks are relatively small and the risks are relatively large.

According to the data in Table 4, this paper calculates and makes the Capital Allocation Line (CAL) curve. As shown in Figure 2.

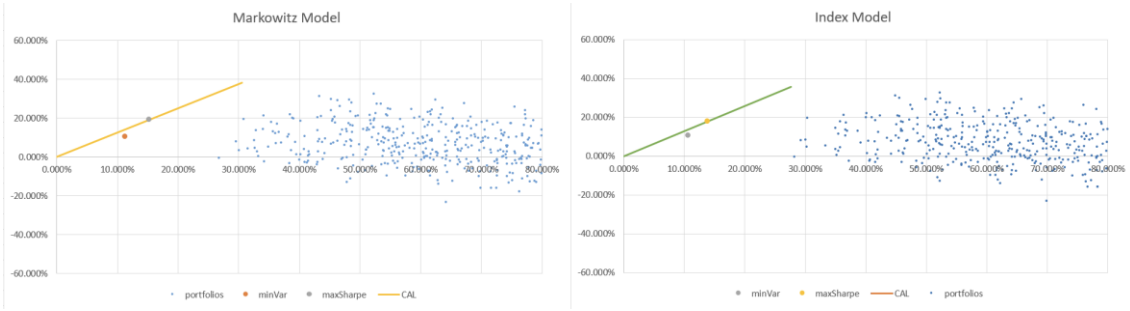

Fig. 2 CAL curves of Markowitz model and index model

CAL curve represents all feasible risk return combinations for investors. The slope s of the cal line is called the return risk ratio, or sharp ratio, which represents the market price of risk. A high risk price means a higher rate of return with the same risk, so a larger slope is the goal.

Then, it uses solver table to solve the data group of the minimum variance frontier, and uses the data to make the figure of the effective frontier, as shown in Figure 3.

Fig. 3 Minimum variance frontier of Markowitz model and index model

Risk averse pursues the minimization of risk under the condition of given return. The minimum variance frontier defines a smaller set of portfolios that investors want to invest in. Investors without risk aversion will choose the portfolio on the right side of the minimum variance frontier, because the portfolio on the minimum variance frontier can provide the same level of return with lower risk.

It uses solver table again to make the data group of efficient frontier and inefficient frontier, and makes the figure, as shown in Figure 4.

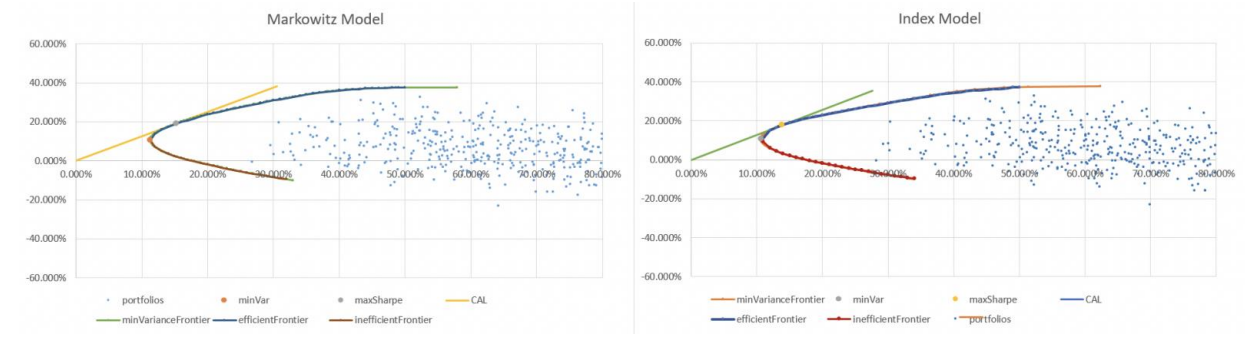

Fig. 4 Efficient frontier and inefficient frontier of Markowitz model and index model

Efficient frontier is used to describe the relationship between risk and return of a portfolio. It is expressed as a curve on the coordinates with risk as the horizontal axis and expected return as the vertical axis. All risk return portfolios falling on this curve are the maximum return that can be obtained under a certain risk or minimum risk. The inefficient frontier represents the maximum loss under a certain risk and plays a role in controlling the loss.

Then, this paper simulates five possible constraints for portfolio in reality.

4.1 Constraint

4.1.1 Constraint 1

This additional optimization constraint is designed to simulate the *Regulation T by FINRA* [10]. This allows brokers to allow their customers to hold positions, 50% or more of which are funded by the customer's account interests:

$$\sum_{i=1}^{11} |w_i| \leq 2 \tag{11}$$

4.1.2 Constraint 2

This optimization constraint is a portfolio with arbitrary weight, which often depends on the requirements of customers:

$$|w_i| \leq 1, for \forall i; \tag{12}$$

4.1.3 Constraint 3

This is a "free" portfolio without any additional constraints. This constraint can explain what the portfolio area and especially the efficient frontier are like without constraints.

4.1.4 Constraint 4

This optimization constraint is to simulate the situation that no short positions are allowed. American open-end mutual funds are not allowed to have any short positions, and China's A-share trading is not allowed to be short directly, for details see *the Investment Company Act of 1940*, *Section 12* (a) (3) [11]:

$$w_i \geq 0, for \forall i; \tag{13}$$

4.1.5 Constraint 5

Finally, this constraint can show us the impact, both positive and negative, if broad index is included in the portfolio. Take SPX as an example:

$$w_1 = 0 \tag{14}$$

Then, using the same method as above, this paper constructs five minimum variance boundaries, effective boundaries and invalid boundaries under different constraints, as shown in Figure 5.

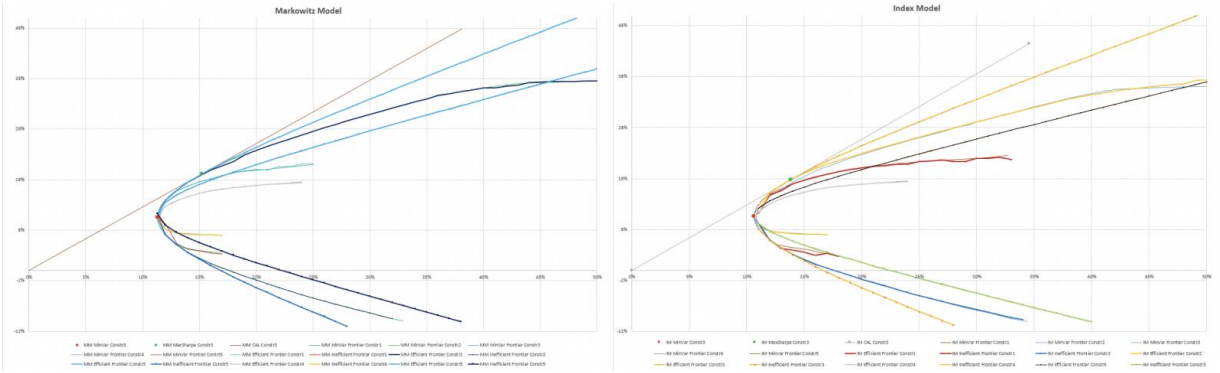

Fig. 5 Minimum variance frontier, Efficient frontier and inefficient frontier of Markowitz model and index model under five different constraints

4.2 Constraint analysis

Overall, Constraint1, 2, 3, 5 present a near coincidence of CAL, from the picture, you cannot see the blue line and the red line because they are covered with the orange line (constraint 3), their slope, which is the Sharpe Ratio, is all 1.03 approximately which is greater than that of Constraint 4. This means that although the short sales constraint plays an important role in reducing investment risks and diluting the excessive speculative atmosphere of investors, it reduces the overall return rate meanwhile. In the example in the figure, the return corresponding to the short sales constraint drops by about 28% compared with that corresponding to the other four constraints.

In addition, pay attention to the two specific lines: Constraint 3 and 5. One is no constraints, the other is setting the weight of SPX to zero. As depicted in the line chart, even though the two lines overlap almost completely, with slopes of 1.03 and 1.02 respectively, it can be infered that whether or not it includes inclusion of the broad index in our portfolio has very little impact on our returns.

On the other hand, it considers the minimal variance point. The three points of constraint 1, 2, 3 almost coincide and also the Sharpe Ratio of all these constraints except Constraint 4 are basically the same. As it can be seen, the four points are almost in the same line if we draw a line to connect them. it shows that this indicate the same result just as the CAL above: the return of the portfolio that corresponds to the sales constraint for unit of risk is reduced.

From the overall view of the chart, constraint 4 has the strongest constraint ability, and its range is significantly narrowed compared with other constraints. Under the same risk, the maximum return and maximum loss are relatively small, which is not only a kind of protection, but also a kind of restriction. Therefore, buyers with weak strength and low risk tolerance can choose mutual funds. As a market index, SPX has an obvious role in expanding the choice in the low-risk stage, but it is no longer obvious in the pursuit of higher stage returns.

It analyzes the range of efficient frontier from large to small.

1) From the efficient frontier chart, we can see that constraint 3 is the condition that includes the most securities portfolio, and it is also a curve tangent to CAL. Therefore, it is also the condition of maximum return under the same investment risk (standard deviation).

2) Constraint 2 is the frontier with the largest degree of fit with constraint 3. Of course, this doesn't mean much. After all, this constraint depends on the client's personal requirements and preferences. At least under the condition of wi less than or equal to 1, its efficient frontier is slightly lower than constraint 3, which is almost fitted.

3) Constraint 5 is the portfolio without SPX, whose maximum return in the low standard deviation stage (< 20%) is evidently lower than that of the portfolio with SPX. It can be seen that as a indicator of the whole stock market, the index is good for people who choose low-risk investment portfolio. However, when the standard deviation is greater than 20%, the difference between constraint 5 and constraint 3 is not very large.

4) Constraint 1 has a high degree of fit with constraint 3 when the standard deviation is low (< 25%), but the maximum return is significantly less than constraint 3 when the standard deviation is greater than 25%. It shows that in the case of buying stocks with deposit, the higher the risk, the more additional deposit so that the lower the maximum return. So as to control the leverage ratio of the market and maintain the stable development of the market.

5) Constraint 4 is when wi greater than or equal to 0, that is, every stock cannot be short. Obviously, the range of constraint 4 is the smallest. This situation is a protective measure for relatively weak buyers who buy mutual funds. The protection is strong, and the risk is low, so the maximum return of the same risk is also low.

6) Both constraint 1 and constraint 4 are due to the official protection measures for the market and buyers. Similarly, the range of both is significantly narrowed compared with constraint 3. This shows the effectiveness of official methods.

7) Even without constraints, constraint 3 still cannot achieve zero risk, which is the inevitable system risk at work.

Inefficient frontier and efficient frontier are basically relative. An obvious phenomenon is that constraint 5 and constraint 1 intersect at a standard deviation of about 22%. It shows that in the stage of more than 22%, the maximum loss of constraint 1 is less than constraint 5, which also reflects the significance of market protection.

When comparing different models horizontally, under constraint 1, the CAL curve of Markowitz model is steeper, and the steeper CAL will provide greater return under certain risk. Investors will naturally want to cooperate with the venture capital portfolio with high sharp ratio. For constraints 2, 3 and 5, its conclusion is very similar to constraint 1.

For constraint 4 often used in mutual funds, the efficient frontier of Markowitz model and Index model actually coincide, and their CAL coincides, which means that the results of the two models are the same under this constraint.

5. Conclusion

At present, most portfolio studies are based on the analysis of general market conditions or specific industries. The purpose of this paper is to analyze the investment portfolio of ten stocks of different companies in different industries, including technology, financial services, industry, consumer defensive and health, so that potential investors can benefit from making investment decisions. In this paper, it first calculates the characteristic data of ten stocks, and then randomly assigns weights to ten stocks. It uses the formulas of Markowitz model and single index model to calculate the rate of return, standard deviation and sharp ratio in two cases, and randomly simulates a thousand groups of data and draws figures. It is found that the results of the two models are slightly different. Then it uses the programming solution to calculate the portfolio with the maximum sharp ratio and the minimum standard deviation under the two models, so as to make the CAL curve. Finally, it uses solver table to find the minimum variance frontier, efficient frontier and inefficient frontier under five constraints that may be encountered in real investment and draw the figures. Through comparison, this paper finds the differences and relations between the minimum variance frontier, efficient frontier and inefficient frontier under different constraints, and preliminarily compares the performance of the two models. However, there are also deficiencies. For example, the number of constraint assumptions is not enough to apply to all conditions, and only invest in the stock market, which is too single. Considering the impact of other financial products on returns, it is worth further study.

References

[1] http://www.gov.cn/zhengce/zhengceku/2018-12/31/content_5433072.htm

[2] Du Qiang Research on the transformation and development of trust industry under the background of new asset management regulations [J] North China finance, 2019 (5): 5

[3] Jakšić M. Risk management of portfolio securities [J]. Ekonomski horizonti, 2012, 14 (3): 151-164.

[4] Kenig E. Portfolio selection in non-stationary markets [J]. Algorithmic Finance, 2021, 9 (1-2): 35-47.

[5] Wang Gangzhen, Li Wenbo, Zhu Jiaming Quantitative portfolio strategy based on AI industry [J] Journal of Shenyang University: Social Science Edition

[6] Zhu Mengting Research on prediction of return rate of investment portfolio of China's coal mining and beneficiation industry -- Based on CAPM and Fama French three factor model [J] 2021 (2019-5): 41-47.

[7] Scott D, Anand D, Smithies A. PORTFOLIO AND PROJECT RISK ASSESSMENT: US, US20090070188 A1 [P]. 2009.

[8] Hanif A, Hanun N R, Febriansah R E. Optimization of Stock Portfolio Using the Markowitz Model in the Era of the COVID-19 Pandemic [J]. TIJAB (The International Journal of Applied Business), 2021, 5 (1): 37-50.

[9] Yu Junli, Han Wenxiu Application of index model in investment analysis [J] System engineering theory and practice, 2001

[10] https://www.finra.org/rules-guidance/key-topics/margin-accounts

[11] https://www.law.cornell.edu/uscode/text/15/80a-12