

Portfolios under Constraints in Real Practice

Yuxi Lin

School of Finance, Nankai University, Tianjin, China

2012760@mail.nankai.edu.cn

Abstract. Portfolio optimization now has a decisive status in the financial researching. However, few research focus on several specific industries and the commonly used constraints in the realistic financial world. This paper aims to do the asset allocation for three industries, i.e., technology industry, financial service industry and industrial industry. The SPX500 and ten firms from the above industries are chosen to obtain their correlation coefficient matrix. Then Mean-Variance Model is used to calculate the maximum Sharpe Ratio, the minimum variance, the capital allocation line. Finally, solver table is applied to calculate the minimum variance frontier under each constraint. The result shows that, first, the SPX500 has a high correlation coefficient with the listed firms, and it's a good choice in a portfolio that balances risk and return; second, the minimum variance frontier and capital allocation line behave worse when adding constraints. These findings may help the investors with specific risk appetite to make their own investment decisions. Besides, investors will be aware of the price of adding constraints when making a investment decision.

Keywords: Portfolio management; Mean-Variance Model; real constraints.

1. Introduction

After Markowitz originally proposed the Mean-Variance (MV) Model in 1952, mathematical statistics was applied to the study of portfolio selection for the first time. Compared with institutional investors, individual investors have lower risk tolerance and higher expected return. Thus, it is particularly necessary for individual investors to diversify risks through investment portfolios. After all, diversification is often spoken of as the only free lunch in investing (Thomas, 2017). Therefore, how to solve the contradiction between high returns and low risks is definitely a hotspot of financial research. Furthermore, firms tend to hold diversified assets instead of a single one. Exactly as the article says, managing and selecting portfolios is regarded as a top priority strategy by most of the biggest corporations worldwide all the time (Ulrich, Harald, Matthias & Robert, 2011).

As is known that there are numerous research regarding portfolios and diversification currently. Most of the current investigations regarding portfolio management is on the basis of the entire market. For instance, João, Guilherme, and André (2016) used dynamic factor models to research portfolio optimization under the whole US bond market, and Zhu, Yu and Thomas (2019) analyzed portfolio selection based on the US stock market. In addition, Kais (2018) made comparison between TVS and the efficient frontier under the whole financial market. Meanwhile, some of the researchers focused on a specific industry or firm, such as Suleman, Muhammad, Awais and Aviral (2018), their results indicated the importance of oil assets for making an optimal portfolio consisting of manufacturing firm stocks, and Qi (2020) did the Portfolio selection based on traditional Chinese medicine enterprises. In addition, few research aim to add different constraints to find the optimal portfolio and discuss their results. Most of the existing research only focused on portfolios with no constraint. For example, Alexandre, Wai, and Bobby (2009) used Black-Litterman framework to do the non-constraint investment management. To sum up, it may be noticed that few research aim to focus on several typical industries, such as technology, financial services and industrials.

This paper's intention is to focus on several typical industries in the market, i.e., technology, financial services and industrials, and discuss the results of different constraints. The empirical process in this paper can be summarized as follows. First, this paper chooses SPX500 Index and ten typical firms from technology industry, financial services industry and industrial industry respectively, and get their stock price from May, 11, 2001 to May, 12, 2021; Second, this paper does data cleaning to match the time of the financial time series; Third, this paper calculates the correlation coefficient

among the eleven underlying assets, and it can be found that firms in the same industry have a higher correlation coefficient than that of firms in different industries. What's more, the SPX500 Index has a high correlation coefficient with the other ten assets; Fourth, this paper constructs certain portfolios by the selected securities, i.e., the minimum volatility portfolio and the maximum Sharpe Ratio portfolio, and the results show that SPX500 occupies the most weight in the portfolios; Fifth, this paper uses solver table to get the minimum variance frontier and capital allocation line under each constraint, and the results show that adding constraints will reduce the efficiency of diversification.

This paper is constructed as follows. Section 2 shows the data used in this paper. Section 3 summarizes the methods. Section 4 presents the results and Section 5 concludes the paper.

2. Data

The data in this article is derived from Yahoofinance (<https://finance.yahoo.com/>). This article selects SPX500 and ten representative firms. i.e., ADBE, IBM, SAP, BAC, C, WFC, GS, LUV, ALK, FDX for closing prices, from May 11, 2001, to May 12, 2021. The above firms are chosen because they are the representative firms in the specific industries and have been listed for over twenty years. Then, 2662 data are selected to obtain the final result. Finally, to implement further investigations, data processing is done to get the annual return and the standard deviation of the eleven assets. Some basic information of the eleven assets is shown in Table 1.

Table 1. Descriptive statistics of the selected assets

	'SPX'	'ADBE'	'IBM'	'SAP'	'BAC'	'C'	'WFC'	'GS'	'LUV'	'ALK'	'FDX'
Mean	0.090	0.210	0.062	0.134	0.125	0.025	0.103	0.123	0.113	0.189	0.144
StDev	0.148	0.318	0.232	0.339	0.393	0.424	0.281	0.296	0.317	0.377	0.266
Beta	1.000	1.425	1.017	1.486	1.600	2.012	1.053	1.414	1.146	1.172	1.102

Notes: StDev is used to represent std.dev; Beta is the slope between SPX and each asset.

Through visualizing the data, it can be discovered that 'ADBE' has the highest annual return, while 'C' has the lowest annual return. When it comes to variance, 'C' is the highest, while 'SPX' is the highest. In addition, the 'C' has the lowest annual return and the highest volatility, which is not a good choice in a portfolio. What's more, most of the annual return of the ten firms are more than ten percent, which is a pretty good return in the market. And most of the standard deviation of the ten firms is above 30%, which is with normal limits.

3. Methods

In this paper, correlation matrix and modern portfolio theory are selected to do portfolio management for the selected SPX 500 and the ten representative stocks.

3.1 Pearson correlation coefficient

The process of stock price changing is a time series data which contains both the randomness and some regular pattern. When dealing with the SPX500 and stock prices, knowledge of probability theory is applied to obtain the correlation among the eleven assets. Besides, correlation matrix are necessary details for portfolio management. The correlation coefficient is calculated by the following formula,

$$r = \frac{\sum_{i=1}^n (x_i - \bar{x})(y_i - \bar{y})}{\sqrt{\sum_{i=1}^n (x_i - \bar{x})^2} \sqrt{\sum_{i=1}^n (y_i - \bar{y})^2}} \quad (1)$$

Where $\{X_i, Y_i\}$ are the annual returns of the eleven assets.

3.2 Mean-Variance Model

Through Mean-Variance Model, mathematical statistics is applied to the study of portfolio selection for the first time. It allows investors to select the optimal portfolio, which has a higher return and a lower volatility. According to Kamil, Anton, Fei, Chin, Kok and Lee (2006), Mean-Variance Model is the main idea which used to build up the optimal portfolio in order to achieve the objective of maximize the return and minimize the risk. Wang and Zhou (2019) approached the continuous-time mean-variance (MV) portfolio selection with reinforcement learning (RL) to achieve the best tradeoff. And the precondition of Mean-Variance Model is as follows,

$$\sum_{i=1}^n Weight_i = 1 \quad (2)$$

Where $Weight_i$ is the weight of $asset_i$ in a portfolio. The portfolio return and standard deviation can be calculated as follows.

$$portfolio\ return = \sum_{i=1}^n Weight_i * asset\ return_i \quad (3)$$

Where $asset\ return_i$ is the annualized average return of $asset_i$'s monthly return.

$$portfolio\ StDev = \sqrt{\sum_{i=1}^n Weight_i^2 StDev_i^2 + \sum_{i=1}^n \sum_{j=1}^n Weight_i Weight_j StDev_i StDev_j Cov(i,j)} \quad (4)$$

Where $asset\ StDev_i$ is the standard deviation of $asset_i$, and $cov(i,j)$ denotes the covariance between $asset_i$ and $asset_j$. What's more, Sharpe Ratio is also an index to measure the quality of an asset and is one of the three classic indicators that can simultaneously consider both return and risk. The indicator measures the excess return per unit of risk. It can be calculated as follows.

$$SharpeRatio = \frac{E(R_p) - R_f}{\sigma_p} \quad (5)$$

Where $E(R_p)$ is the expected return of the portfolio, R_f is the risk-free rate, and σ_p is the standard deviation of the portfolio.

4. Results

Through the calculation of the correlation coefficient, the correlation coefficient matrix is obtained. The results are shown in Table 2 below.

Table 2. Correlation coefficient matrix of the eleven assets

	'SPX'	'ADBE'	'IBM'	'SAP'	'BAC'	'C'	'WFC'	'GS'	'LUV'	'ALK'	'FDX'
'SPX'	1.000	0.664	0.649	0.649	0.602	0.701	0.554	0.708	0.534	0.460	0.612
'ADBE'	0.664	1.000	0.454	0.533	0.422	0.462	0.296	0.437	0.387	0.231	0.486
'IBM'	0.649	0.454	1.000	0.585	0.312	0.419	0.266	0.508	0.345	0.355	0.329
'SAP'	0.649	0.533	0.585	1.000	0.330	0.433	0.297	0.445	0.316	0.280	0.318
'BAC'	0.602	0.422	0.312	0.330	1.000	0.826	0.760	0.522	0.427	0.274	0.436
'C'	0.701	0.462	0.419	0.433	0.826	1.000	0.703	0.570	0.427	0.303	0.455
'WFC'	0.554	0.296	0.266	0.297	0.760	0.703	1.000	0.438	0.404	0.345	0.428
'GS'	0.708	0.437	0.508	0.445	0.522	0.570	0.438	1.000	0.396	0.321	0.448
'LUV'	0.534	0.387	0.345	0.316	0.427	0.427	0.404	0.396	1.000	0.517	0.505
'ALK'	0.460	0.231	0.355	0.280	0.274	0.303	0.345	0.321	0.517	1.000	0.401
'FDX'	0.612	0.486	0.329	0.318	0.436	0.455	0.428	0.448	0.505	0.401	1.000

As Table 2 indicates, each underlying asset has a higher correlation with other firms in the same industry. In addition, 'SPX' has a high correlation with the other ten assets. All the coefficients are positive, which means that assets are not isolated from others. What's more, most of the correlation coefficient between any two assets is not less than 0.3, which is in favour of building a portfolio.

In this paper, certain portfolios are constructed under five constraints, and the maximize Sharpe Ratio portfolio, the minimum variance portfolio, the capital allocation line and the minimum variance frontier are calculated to find the difference. The detailed results are shown below.

Constraint 1: There is no specific limit, which is a commonly used one in the investment decisions. Its results are shown in Table 3 and Figure 1.

Table 3. Weights of each asset under constraint 1

	'SPX'	'ADBE'	'IBM'	'SAP'	'BAC'	'C'	'WFC'	'GS'	'LUV'	'ALK'	'FDX'	Return	StDev	Sharpe
minVar	1.40	(0.09)	0.06	(0.11)	0.00	(0.20)	0.13	(0.12)	0.00	(0.05)	(0.02)	7.8%	12.0%	0.65
maxSharpe	0.79	0.30	(0.19)	0.02	0.24	(0.58)	0.21	0.06	(0.09)	0.16	0.08	20.1%	19.2%	1.05

Notes: minVar is used to represent the minimum variance; maxSharpe is used to represent the maximum Sharpe Ratio.

As Table 3 indicates, 'SPX' has the maximum weight among the eleven assets, while 'C' has the minimum weight in the portfolio.

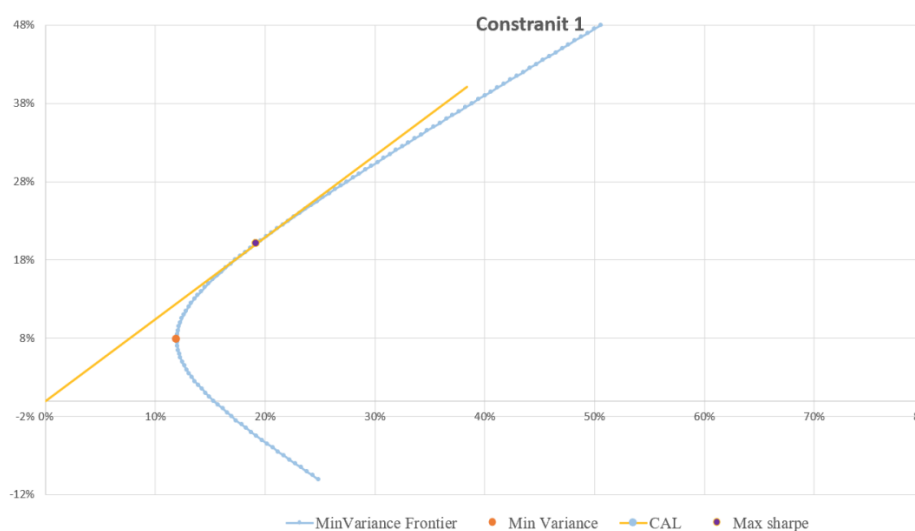


Fig. 1 Minimum variance frontier and capital allocation line under constraint 1

Constraint 2: the purpose of constraint 2 is to test what the portfolio will be like without an index. It is known that index is a passive strategy in the stock market, so a portfolio without index is designed to measure the effect of a positive investment strategy. Its results are shown in Table 4 and Figure 2.

$$Weight_1 = 0 \quad (6)$$

Table 4. Weights of each asset under constraint 2

	'SPX'	'ADBE'	'IBM'	'SAP'	'BAC'	'C'	'WFC'	'GS'	'LUV'	'ALK'	'FDX'	Return	StDev	Sharpe
minVar	0.00	0.08	0.45	(0.01)	(0.04)	(0.22)	0.41	0.08	0.06	(0.01)	(0.20)	11.8%	17.7%	0.67
maxSharpe	0.00	0.54	(0.16)	0.11	0.33	(0.75)	0.35	0.22	(0.11)	0.26	0.21	27.0%	26.8%	1.01

As Table 4 indicates, 'WFC' has a high weight in the portfolio. In addition, it can be seen from the last column that the portfolio without index performs worse than a portfolio with index.

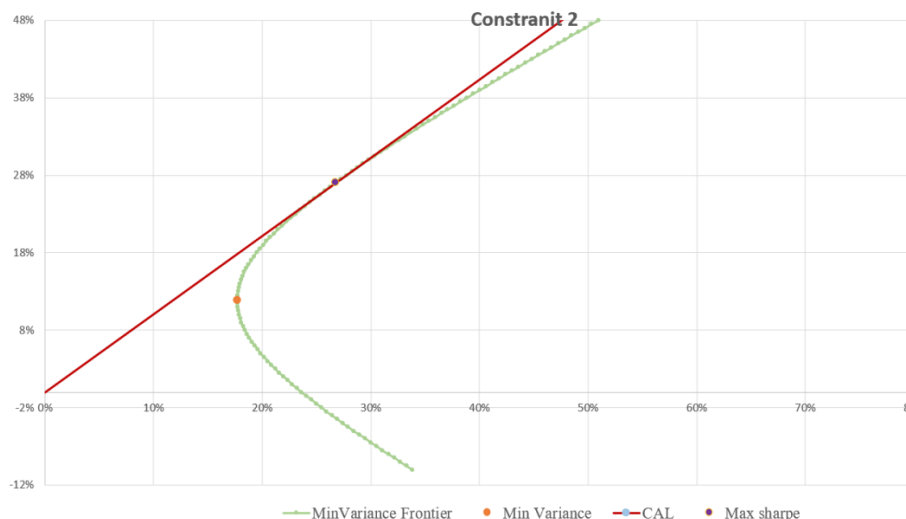


Fig. 2 Minimum variance frontier and capital allocation line under constraint 2

Constraint 3: constraint 3 is not allowed to have any short positions, which commonly exists in the U.S. mutual funds. Mutual fund raises funds from the public, and the public have a lower risk tolerance. So, mutual funds can't have any short positions. Its results are shown in Table 5 and Figure 3.

$$w_i \geq 0, \text{ for } \forall i \quad (7)$$

Table 5. Weights of each asset under constraint 3

	'SPX'	'ADBE'	'IBM'	'SAP'	'BAC'	'C'	'WFC'	'GS'	'LUV'	'ALK'	'FDX'	Return	StDev	Sharpe
minVar	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	9.0%	14.8%	0.61
maxSharpe	0.07	0.47	0.00	0.00	0.00	0.00	0.04	0.00	0.00	0.25	0.17	18.1%	23.6%	0.77

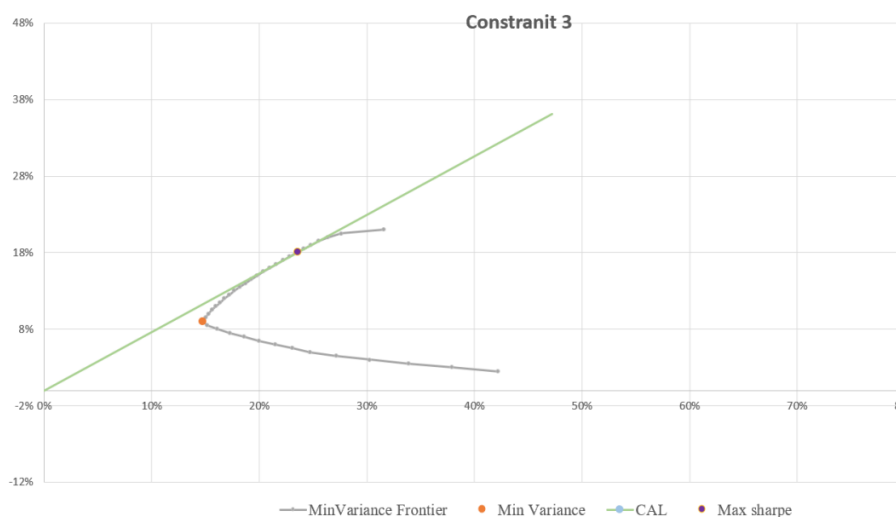


Fig. 3 Minimum variance frontier and capital allocation line under constraint 3

As Figure 3 indicates, the minimum variance frontier of constraint 3 lies in the right of that of constraint 1. Because investors cannot short some bad assets. Besides, an interesting fact may be confirmed that if short selling is not allowed, investing in the SPX to avoid risk is a preferred choice.

Constraint 4: constraint 4 is a usual constraint for a investment manager, which refers to the condition that investor could not borrow money. Its results are shown in Table 6 and Figure 4.

$$|w_i| \leq 1, \text{ for } \forall i \quad (8)$$

Table 6. Weights of each asset under constraint 4

	'SPX'	'ADBE'	'IBM'	'SAP'	'BAC'	'C'	'WFC'	'GS'	'LUV'	'ALK'	'FDX'	Return	StDev	Sharpe
minVar	1.00	(0.04)	0.17	(0.08)	(0.01)	(0.21)	0.21	(0.06)	0.02	(0.04)	0.04	8.9%	12.5%	0.71
maxSharpe	0.79	0.30	(0.19)	0.02	0.24	(0.58)	0.21	0.06	(0.09)	0.16	0.08	20.1%	19.2%	1.05

As Table 6 indicates, 'SPX' has the maximum weight in the above portfolios, and 'C' has the minimum weight in the portfolio, which is the same as constraint 1.

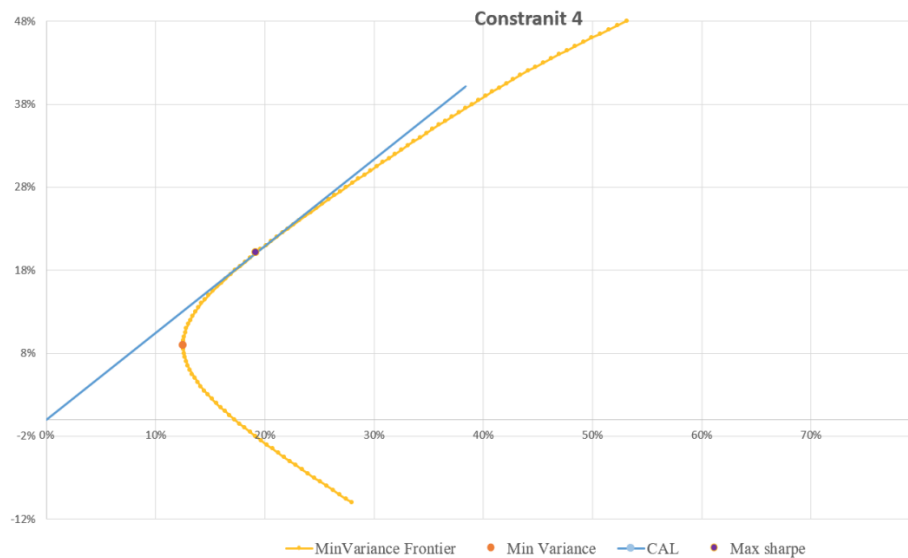


Fig. 4 Minimum variance frontier and capital allocation line under constraint 4

Constraint 5: is designed to simulate the Regulation T by FINRA, which allows broker-dealers to allow their customers to have positions, 50% or more of which are funded by the customer's account equity. Its results are shown in Table 7 and Figure 5.

$$\sum_{i=1}^n Weight_i = 1 \quad (9)$$

Table 7. Weights of each asset under constraint 5

	'SPX'	'ADBE'	'IBM'	'SAP'	'BAC'	'C'	'WFC'	'GS'	'LUV'	'ALK'	'FDX'	Return	StDev	Sharpe
minVar	1.35	(0.08)	0.04	(0.09)	(0.01)	(0.19)	0.11	(0.10)	(0.00)	(0.04)	(0.00)	8.2%	12.0%	0.68
maxSharpe	0.69	0.25	(0.03)	0.00	0.16	(0.46)	0.20	0.02	(0.01)	0.11	0.07	17.4%	17.0%	1.02

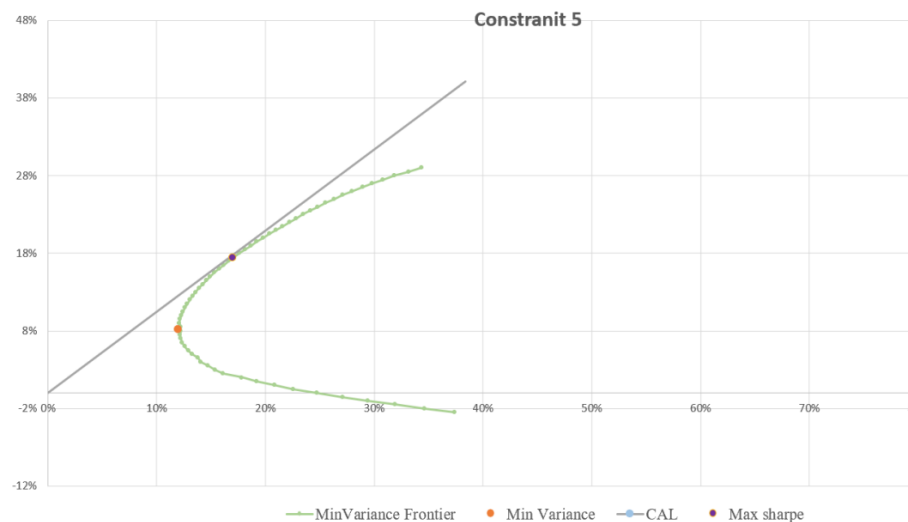


Fig. 5 Minimum variance frontier and capital allocation line under constraint 5

As is shown in the tables and figures, a portfolio without constraint (constraint 1) performs better than the portfolios under constraints in the maximum Sharpe portfolio, the minimum variance portfolio, the capital allocation line and the minimum variance frontier. For specific performance, when adding some constraints to the model, the constraints carry a price tag in the sense that an efficient frontier constructed subject to extra constraints will offer a Sharpe ratio inferior to that of a non-constrained one.

5. Conclusions

Currently, most of the portfolio research is based on the analysis of general market situations or a specific industry. In addition, few research focus on the constraints formulated by law or industry regulation. The purpose of our study is to do portfolio analysis on retail industry, technology industry, financial service industry and industrial industry to benefit the potential investors when making investment decisions under different constraints. In this paper, statistical analysis is used to calculate the correlation coefficient among the eleven assets, and the index has a high correlation coefficient with the other ten assets. Then, the Mean-Variance Model is applied to do portfolio optimization and construct minimum volatility portfolio, maximum Sharpe ratio portfolio and minimum variance frontier. The study has identified portfolios under constraints has a higher volatility and a lower return than the non-constrained portfolio. So, it is recommended that the client should be made aware of this cost and should carefully consider constraints that are not mandated by law.

However, deficiencies also exist. For instance, only a few representative firms from three industries are selected, which is not very broad. What's more, the Mean-Variance Model assumes too much, which is not that accord with reality.

References

- [1] Raffinot T. Hierarchical Clustering-Based Asset Allocation [J]. *The Journal of Portfolio Management*, 2017, 43 (2): 89-99.
- [2] Ulrich P, Harald R, Matthias K, et al. Corporate portfolio management: Theory and practice [J] *Journal of Applied Corporate Finance*, 2011, 23 (1): 63-76.
- [3] Caldeira J, Moura G V, Santos A. Bond portfolio optimization using dynamic factor models [J] *Journal of Empirical Finance*, 2016, 37: 128–158.
- [4] Zhu Y, Yu P, Mathew T. Improved Estimation of Optimal Portfolio with an Application to the US Stock Market [J]. *Journal of Statistical Theory and Practice*, 2020, 14 (1).
- [5] Dachraoui K. On the optimality of target volatility strategies. [J]. *The Journal of Portfolio Management*, 2018, 44 (5): 58-67.
- [6] Sarwar S, Shahbaz M, Anwar A, et al. The importance of oil assets for portfolio optimization: The analysis of firm level stocks [J]. *Energy Economics*, 2018.
- [7] Qi Y. Portfolio analysis and prospect of listed TCM enterprises in Guizhou Province [J]. *Journal of Guizhou Party School*, 2020 (3): 10.
- [8] Da, Silva, Alexandre, et al. The Black–Litterman Model for Active Portfolio Management [J]. *Journal of Portfolio Management*, 2009.
- [9] Kamil A A, Fei C Y, Kok L K. Portfolio analysis based on Markowitz model [J] *Journal of Statistics and Management Systems*, 2006, 9 (3) 519–536.
- [10] Wang H, Zhou X Y. Continuous-Time Mean-Variance Portfolio Selection: A Reinforcement Learning Framework [J]. 2019.