An empirical study of down-and-out put option pricing based on Geometric Brownian Motion and Monte Carlo Simulation: evidence from crude oil and E-mini Nasdaq-100 futures

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Abstract. Option, an instrument of significant financial values in the modern market, is of growing importance. In the case of pricing the option, pricing exotic option remains the problem, since none of the practical methods have been developed as a corresponding way of solution. In order to address the existing issue, this paper examines the feasibility of down-and-out put option pricing based on Geometric Brownian Motion and Monte-Carlo Simulation. Specifically, the stock prices will be calculated through the Geometric Brownian Motion certain while the underlying asset price and down-and-out put option price will be obtained by Monte-Carlo simulations. Two typical underlying assets are selected as the investigation target to validate the pricing feasibility: Crude Oil Futures and E-mini Nasdaq-100 futures. According to the analysis, the barrier option price is lower than a European option, and the barrier option is always cheaper than a European option with the same parameters. These results shed light on the seeking of a method in pricing exotic options.

Keywords: Monte Carlo simulation; Geometric Brownian Motion barrier option.

1. Introduction

Option is the right for the buyer to buy or sell a certain amount of underlying asset at a predetermined price at a certain time in the future [1]. Contemporarily, option has become an important tool in the financial market, and it is an inevitable issue to pricing options. There have been many useful methods to price the option, e.g., binary tree model and Black-Scholes model (BS model) [2-4]. However, the problem is that almost all the popular methods can only be used to determine European options or American options. In other words, there hasn't been an accepted method like the BS model for pricing some exotic options, which are also important. Thus, finding a model to simulate exotic options' prices becomes a challenging task.

This paper tries to investigate the feasibility of down-and-out put option pricing based on Geometric Brownian Motion and Monte-Carlo Simulation. On this basis, data of two assets (E-mini Nasdaq-100 futures, crude oil futures) is collected from the real market. Geometric Brownian motion (GBM), also known as exponential Brownian motion, is a random process in continuous time in which the logarithm of a random variable follows Brownian motion [5 6]. GBM is used in financial mathematics to simulate stock prices in the Black-Scholes model. Monte Carlo methods, or Monte Carlo experiments, are a broad class of computational algorithms that rely on repeated random sampling to obtain numerical results. The underlying concept is to use randomness to solve problems that might be deterministic in principle. They are often used in physical and mathematical problems. Besides, they are most helpful when it is difficult or impossible to use other approaches. Monte Carlo methods are used primarily in three problem classes: optimization, numerical integration, and generating draws from a probability distribution [7, 8].

With real data of risk-free rate, alpha, spot price, volatility, and time to maturity, the underlying asset price in the future and the down-and-out put option price can be simulated by Excel. In order to verify whether the results are in line with market rules, comparisons between the simulations and European put option traded on the market (with the same strike price, time to maturity with the down-and-out put option price) have been presented. In detail, both the real option price as well as the option price change as a function of barrier price and strike price are compared.

The rest of the paper is organized as follow: Sec. 2 introduces the data collection, model descriptions as well as simulation methods; Sec. 3 presents the results for three assets, respectively; Sec. 4 discusses the pros and cons of the GBM model and Monte-Carlo simulations as well as the limitations of the studies; Sec. 5 gives a summary eventually.

2. DATA & METHOD

2.1 Data collection

To investigate the option pricing feasibility of the GBM model and Monte-Carlo simulations, two different underlying assets are chosen to carry out the empirical test from Refs. [9, 10]: the crude oil futures and E-mini Nasdaq-100 futures.

Table 1. Descriptive of Data collection

Product	Crude oil futures [9]	E-mini Nasdaq-100 futures [10]
Expire date	8/1/2021	6/1/2021
Price date	23/6/2021	5/20/2021
Future Spot price	\$73.32	\$13,234
European option price	\$9.85	\$570.25
Barrier price	\$30	\$12,000
Strike price	\$65	\$13,200
Volatility σ	-100.44%	52.67%
Time to maturity	38/252	10/252

As listed in Table I, the expiration date of Crude oil future is chosen as 8/1/2021 with an assumed strike price of \$65 and barrier price of \$30, while 6/1/2021 is chosen for E-miniNasdaq-100 futures with assuming strike price of \$13,200 and barrier price of \$12,000. The corresponding future spot prices and the European option price of the two futures are collected at specific data from the database accordingly. Subsequently, the volatility is acquired from the calculations based on Excel.

2.2 GBM model and Monte-Carlo Simulation

Based on Ito's theory and GBM, the model of Monte-Carlo simulation can be derived as:

$$S_T = S_0 e^{\left(\alpha - \frac{1}{2}\sigma^2\right)T + z\sigma\sqrt{T}} \tag{1}$$

where the variable definitions are listed in Table II. The equation above can be used to value the futures used in this paper at time T.

To better illustrate the advantages of this model in simulating assets, the points have been listed below:

(1) Primarily, the expectation of Geometric Brownian Motion is independent of the price of the random process (stock price), which is consistent with our expectation of the real market.

(2) In addition, the geometric Brownian motion process only considers positive prices, just like real stock prices.

(3) Besides, the geometric Brownian motion process presents the same "roughness" as the price trajectory observed in the stock market.

(4) Moreover, payoffs can occur at any time during the life of the derivative rather than all at the end.

(5) Finally, the calculation is relatively simple.

As a matter of fact, there are three hypothesizes applied in the models:

(1) Fluctuations in stock prices do not change over time. This means that σ in the formula remains unchanged no matter how long the time to maturity is.

(2) The return of stock prices satisfies a normal distribution, meaning the future price could be represented as a function of "z."

(3) The expectations of the underlying asset remain unchanged in the short term, which is called risk neutrality, which means that people can pretend that there is no risk premium for options and futures pricing.

Table 2. Descriptive of variables

Variables	Descriptions
α	equals to 0 for futures or (r-δ) for stocks
σ	Volatility, a standard deviation of an underlying asset
T	time to maturity
z	normally distributed random number
S_0	stock or futures price at time 0
S_T	stock or futures price at time T

2.3 Simulation process

The simulation process requires six parameters: spot price for futures (S_0), risk-free rate (r), time to maturity (T), α, implied volatility (σ), strike price, and. This paper takes the daily treasury yield curve rate of 1 month on 5/20/2021 as a risk-free rate, and α equals 0 when the underlying asset is futures [11]. Implied volatility can be calculated through the function "Goal Seek" in excel based on historical data of futures price. Strike and barrier prices are determined based on the spot price: strike price is set lower than the spot price, and barrier should be lower than the strike price. In order to make assumption three more reasonable, this paper simulates the futures price 1000 times a day only for ten days, so T equals 10/252. Because the payoff of Barrier Options depends on the futures price every day, a judgment on whether the price is lower than the barrier price is indispensable. If the futures price drops down below the predetermined barrier price during the ten days, then the option price of this simulation is 0. After 1000 times of simulation for ten days, calculate the average payoff on day ten and discount it to time 0. The result will show the simulated down-and-out put option price.

3. Results

3.1 Crude oil futures

Crude oil futures data on 6/23/2021 is used. The implied volatility of crude oil futures is calculated through simulation. The Monte Carlo simulation is carried out 1000 times with the spot price of crude oil on 23/06/2021 from CME ($73.32), and the implied volatility σ is -100.44%. During the process, the strike price is assumed as $65.

Fig. 1 The crude oil futures down-and-out put option price as a function of the strike price

Plugging the implied volatility and assuming that the barrier price is $30, this test simulates 1000 times. Thus, the average payoff of the down-and-out put option on crude oil futures is obtained, which is equal to $2. The standard variation is about 4.41, and the max error is about 0.2.

Fig. 2 The crude oil futures down-and-out put option price as a function of barrier price

As illustrated in Fig.1 and Fig. 2, the option price rises with the rise of strike price and drops with the increase in barrier price. Based on the results, the barrier option price is lower than a European option price with the same parameters. The closer the distance between the barrier price and 0, the higher the down-and-out put option is. All these results are in line with our expectations.

3.2 E-mini Nasdaq-100 futures

Data of E-mini Nasdaq-100 futures price ($13,234) and the cost of a European option on E-mini Nasdaq-100 futures on 5/20/2021 ($570.25) are also used.

Firstly, the implied volatility of E-mini Nasdaq-100 futures which expires in June (assume the expiration date is 6/1/2021), is calculated through simulation. The function of "Goal Seek" in Excel shows that when the European put option price is 570.25 (data from CME), σ (implied volatility) equals 52.67%.

Fig. 3 The E-mini Nasdaq-100 futures down-and-out put option price as a function of the strike price

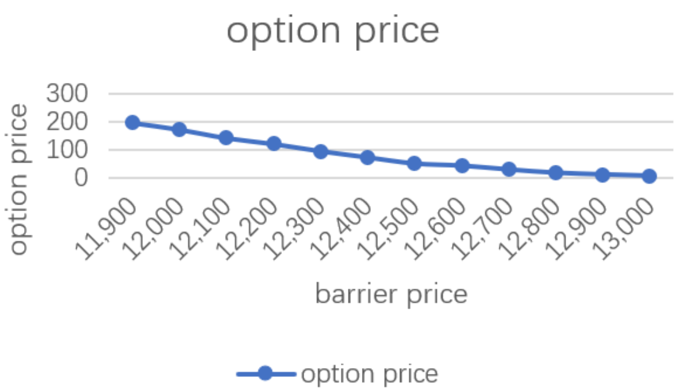

Fig. 4 The E-mini Nasdaq-100 futures down-and-out put option price as a function of barrier price

Assume the strike price is $13,200 and the barrier price is $12,000, apply the implied volatility into the Monte-Carol simulation process. This test simulates for 1000 times similarly. Thus, the average payoff of this down-and-out put option on day ten can be got, which equals $175, and the max error is about 22. This result means that the price of such a down-and-out put option should be in the interval of $153 and $197, which is lower than the European option price.

As depicted in Figs. 3 and 4, the option price rises with the rise of strike price and drops with the increase in barrier price. This result is consistent with the reality: a barrier option is always cheaper than a European option with the same parameters. The closer the barrier price is to 0, the higher the down-and-out put option (because it is closer to be a European option).

4. Discussions

Obviously, the option price is easy to calculate due to the derived Monte-Carol simulation model. Under the GBM model, the asset price is assumed to be a random variable that obeys normal distribution. Therefore, the law of large numbers and the central limit theorem can be applied to simulations. On this basis, the asset price can be settled quickly under these assumptions and simplifications.

However, there are also some limitations under such a model. First, the assumption about normal distribution requires a large sample, i.e., the simulation needs to be run many times. Second, asset price in real market does not always obey normal distribution. Hence, there may be some deviation when describing the relationship between the spot price and the strike price with the GBM model, which may affect estimation accuracy. Third, the model assumes that the asset price is a random variable, but it is a function of time in the real market. Thus, certain time trends may be neglected.

5. Conclusion

In summary, this paper uses real market data to prove that Geometric Brownian Motion and Monte-Carlo Simulation are available to price down-and-out put option. This paper selects real stock index futures data (E-mini Nasdaq-100 futures) and commodity futures data (Crude oil futures) to investigate the option pricing feasibility of the Geometric Brownian Motion Model. Crude oil futures and E-mini Nasdaq-100 futures have reached the same conclusion through model testing. Maintaining other factors, the price of barrier options is always lower than that of the European option, and the closer the barrier price is to 0, the higher the down-and-out put option is. These results offer a guideline for seeking a method in pricing exotic options.

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