A Study on the Development of Machine Learning Methods for Options Pricing Applications

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Abstract. The traditional option pricing model is of great importance, but has many defects in practical applications. With the development of machine learning theory, the powerful learning ability is applied to the option pricing problem, which can effectively solve the defects of the traditional option pricing model. This paper introduces the basic concepts of option pricing and machine learning, and describes the specific model of option pricing fused with BP neural network model and support vector machine model, expresses views on the option pricing methods of traditional model and fused with machine learning model, and presents the outlook on future research. Option pricing is an important topic in finance, and traditional option pricing models have many limitations. With the development of computers and artificial intelligence, machine learning theory can be applied to option pricing models with good results.

Keywords: Options; Option Pricing; Machine Learning.

1. Option pricing

1.1 Options

An option is a contract in which the owner of the option has the right, but not the obligation, to buy or sell a specific quantity of a specific commodity at a specific price on or before a specific date.

The earliest historical account of options is found in the biblical book of Genesis, in which Jacob was given the right to marry Laban's daughter in exchange for seven years of labor. In the history of options, there was also the famous "tulip option". In the seventeenth century in the Netherlands, tulips were a status symbol, and there was a market for them among the nobility. In order to reduce the impact of tulip price fluctuations, tulip growers and wholesalers would purchase options to hedge their risks. purchase. (Xinrui Zhao. The Application of Machine Learing in Option Pricing [D]. University of Science and Technology of China, 2019.)

In the 18th century, options were introduced into the financial market, and in 1973, the Chicago Board Options Exchange was established, and various types of options were developed to meet the needs of consumers.

Options are an important part of the overall capital market and an important risk management tool in the financial market. As a derivative, options have functions such as risk management and asset allocation; option hedging can effectively use option trading to offset investment risk and enable buyers to avoid too bad an outcome. In the 18th century, options were introduced into the financial market, and in 1973, the Chicago Board Options Exchange was established, and various types of options were developed to meet the needs of consumers.

1.2 Traditional option pricing models

The core content of option trading lies in pricing, and a good option pricing model can effectively hedge risk, so the option pricing problem is an important theory in financial mathematics. Due to its importance, option pricing models have been widely concerned by scholars, and many scholars have contributed to option pricing throughout history. 1900, Bachelier proposed the earliest option pricing model with the idea of random wandering, and later, Sprenkle proposed the idea that stock prices follow a lognormal distribution, solving the problem of negative prices in the former. 1965, Samuelsom extended the option pricing model by combining previous studies. Samuelsom combined the previous studies to generalize the option pricing model. 1973, Fischer Black and Myron Scholes derived the Black-Scholes formula, which was a milestone in the option problem, and many subsequent studies were conducted on the basis of the B-S model.

In 1973, Fischer Black, a professor at the University of Chicago, and Myron Scholes, a professor at Stanford University, published their famous paper titled "The pricing of 0ptions and Corporate Liabilities", in which they gave the Black-Scholes formula, the differential equation satisfied by the price of a security based on dividend stocks, and successfully solved this equation, thus also winning the Nobel Prize in Economics. (Li, Chunquan & Liu, Xinping. (2006). Black-Scholes model option pricing method and its application. J Chongqing Technol Business Univ (Nat Sei EdJ) (04), 351-353.)

The B-S option pricing model is based on the strict assumption: (XieLiyan & CaoYing (Beijing University, Beijing10087I, P.R. China). Black—Scholes Model of Option Pricing.. Journal of Northeast Forestry University. 2005, 33(3). 94~96, 109)

1) the underlying asset is a risky asset that can be freely traded.

2) there is no payment of any return on the underlying asset until maturity.

3) the existence of a fixed risk-free interest rate against which the investor can borrow indefinitely.

4) the absence of external factors other than price fluctuations affecting returns

5) the magnitude of price fluctuations is a known constant.

6) there are no arbitrage opportunities in the market.

7) it must be a European-style option.

8) the price movement of the underlying conforms to geometric or logarithmic Brownian motion, and the price volatility and expected return of the stock are constants.

Let S_t be the price at moment t, V be the price of the European option, r be the risk-free interest rate, σ be the volatility of the price, T be the moment of option expiration, and K be the exercise price of the option, then the differential equation of the B-S model is:

$$\frac{\partial V}{\partial t} + rS_t \frac{\partial v}{\partial S_t} + \frac{1}{2}\sigma^2 \frac{\partial^2 v}{\partial S_t^2} - rV = 0 \tag{1}$$

The pricing equation of the call option is

$$\frac{\partial V}{\partial t} + rS_t \frac{\partial v}{\partial S_t} + \frac{1}{2}\sigma^2 \frac{\partial^2 v}{\partial S_t^2} - rV = 0 \tag{2}$$

And similarly, the pricing formula for the put option is

$$P(S,T) = Ke^{-rT}N(d_2) - SN(d_1) \tag{3}$$

Where,

$$d_1 = \frac{ln\left(\frac{S}{K}\right) + \left(r + \sigma^2/2\right)T}{\sigma\sqrt{T}} \tag{4}$$

$$d_2 = d_1 - \sigma\sqrt{T} \tag{5}$$

(This equation shows that the value of the option is a combination of $SN(d_1)$ shares of the underlying stock and $Ke^{-rT}N(d_2)$ dollars of the open-end loan.)

1.3 Machine Learning

Machine learning is the science of artificial intelligence that uses data and experience to automatically improve algorithms. Machine learning has the advantages of being tireless compared to human learning, not forgetting, and being able to replicate the learning as a whole, and is gradually playing an increasingly important role in various fields. One of the more famous events was the victory of the AI player AlphaGo over many famous human players without ever learning the human game, just by playing against itself. The way the machine plays has overturned the traditional human thinking and changed the face of modern Go, and the win rate calculated by AI has become an important reference standard in teaching and professional training. (Zongqi Lou. Theoretical development and application status of machine learning [J]. China New Communication, 2019, 21(01): 60-62.)

1.4 The evolution of machine learning

The history of machine learning is not long, but it has evolved rapidly. the Turing test was introduced in the 1950s, and a machine was considered to be intelligent if it could make more than 30% of the miscalculations of the human being tested in the test. Thirty percent was then an estimate of the ability of artificial intelligence to think, and it has now far exceeded that standard. After that, various learning algorithms emerged. 70s-80s, expert system became the mainstream of development, expert system is the learning method to solve a class of problems through knowledge base and reasoning, but due to the limitation of hardware, the disadvantage of expert system which requires human to design a lot of rules through limited thinking also manifested. 80s to the beginning of 21st century, decision tree algorithm, support vector machine algorithm, random forest algorithms were proposed to make significant breakthroughs in the field of machine learning. Then, the emergence of deep learning algorithms took machine learning to a new level. (Xu Zuming. Research on the history and prospect of machine learning based on artificial intelligence [J]. Electronic World, 2018(15): 70+72.)

1.5 Machine Learning Algorithms

(a) Neural network algorithm

The neural network algorithm is a model that simulates the structure and function of the nervous system of an animal by in the animal's nervous system, external stimulus signals are transmitted into the nervous system and passed down through neurons layer by layer, and eventually the organism responds to the incoming signals. Neural network algorithms have a similar mode of action. The neural network system is generally divided into input layer, hidden layer and output layer, and the processing of the signal in the neuron is written as $f(\mathrm{y})$, y is generally expressed as

$$y = \mathrm{x}_1\theta_1 + x_2\theta_2 + \mathrm{x}_3\theta_3 \tag{6}$$

x_1, x_2, x_3 indicate data input for the input layer, $\theta_1, \theta_2, \theta_3$ can be understood as passing through different neurons.

(b) Support vector machine model

Support Vector Machine, or SVM for short, is a class of supervised learning methods that are widely used. Supervised learning is a method of training a machine to establish a connection between data and results by using given examples and known results, so that it can predict the results from the data. SVM is divided into hard interval support vector machine, soft interval support vector machine, and nonlinear support vector machine.

In option pricing problems, traditional parametric models such as the BS pricing model are based on the assumptions that the market is frictionless, no-risk and sufficiently liquid, so the model is not able to be used to fit the real market. Moreover, market participants may change their option pricing strategies from time to time, so the traditional parametric option pricing models are of very limited use, while machine learning is very good at solving such problems, and the machine-learning-based option pricing models can improve the drawbacks of the traditional models to some extent.

2. Machine learning based option pricing models

2.1 Neural network model

A neural network is a complex network system that simulates the neural network system of the brain by computer, which consists of many simple neurons interconnected according to certain rules. (Xubin Liu. Research on the application of neural network based approach to option pricing [D]. Jinan University, 2011) A neural network generally consists of an input layer, an implicit layer, and an output layer. The input layer is responsible for collecting information from the outside world and passing it down to the hidden layer. The implicit layer receives the information and processes it, and then passes it to the output layer, which will output the processed result to the outside world. Neural networks have good nonlinearity, self-learning and self-regulation.

The more widely used in option pricing models is the B-P neural network algorithm, also called reverse error reverberation propagation. the emergence of the B-P network has given the neural network powerful computing power, and the back propagation mechanism of the B-P algorithm solves the complexity and difficulty brought by the implicit layer to the learning training.

The B-P neural network has a function signal and an error signal. The function signal is a signal from the input to the output, which is considered as a function of the input variables and the relevant weights of the neuron for computation as the signal passes through each neuron. The error signal is a feedback signal from the direction of the output to the direction of the input. The error signal is generated from the output neuron in the neural network and propagates backwards along the network.

The B-P neural network algorithm is divided into two stages: forward propagation of data and backward propagation of error, in which the data is first passed from the input layer to the output layer through the implicit layer, during which the output value is obtained by the excitation function operation, and the output value is compared with the expected value by the error function. The error is continuously reduced by modifying the weights of the neurons layer by layer until the error of the output is within the acceptable range.

2.2 Option pricing using B-P neural network

Based on the previously introduced B-S option pricing model, the following factors should be considered when constructing the model for pricing: the underlying asset price, European option price, risk-free interest rate, price volatility, option expiration moment, option exercise price, etc., and set them as the input variables of the system. In general, the output variable of the system is not set directly to the price of the option, but to the ratio of the call option price to the option strike price or some other error function, and the intermediate layer uses an S-shaped function ranging from 0 to 1, and the data is processed appropriately.

Usually we need to build a two-layer BP network with five input vectors obtained from the BS model, so there are five neurons in the network input layer and only one neuron in the output layer, and the neurons in the hidden layer of the network are usually around 10, and the transfer function usually uses a sigmoid function.

After the model is built, the parameters and weights of the model will be trained and optimized. The learning function of the network is gradient descent, and then the appropriate training function, such as trainlm training function, combined with the error function, using the actual known samples for training, until the network output value and the sample value basically match, we think the algorithm training will be completed. The BP neural network model can then be practically applied to the option pricing problem. (Zhang Ling. An artificial neural network-based option pricing model [D]. Wuhan University of Technology, 2007).

2.3 Support Vector Machine Model

As mentioned earlier, although the traditional parametric option pricing model has solved the option price estimation problem well, in the face of the complex and changing market environment, the traditional option pricing model is constrained by many assumptions, and the option pricing still has a large error, which is not satisfactory in practical applications, so people expect to build non-parametric models through machine learning algorithms. One advantage of nonparametric models is that they are independent of the assumptions of continuous-time finance theory.

Support vector machines are supervised learning classifiers derived from statistical learning theory based on structural risk minimization for maximizing prediction accuracy and reducing overfitting problems. (Li Yu, Liu Li, Lu Huiyang. Comparative study of support vector machines and artificial neural networks in option price prediction [J]. Journal of Xichang College (Natural Science Edition), 2022, 36(02): 31-36. DOI: 10.16104/j.issn.1673-1891.2022.02.007.) The basic idea of support vector regression is to find a hyperplane such that the total deviation of all samples to this hyperplane is minimized.

Now assume that there is a known sample space containing the underlying asset price and the execution price, and then pick the appropriate objective function. In the actual training, we control the support vector regression model with ε such that the error between the target value of the sample and the output value obtained through computation does not exceed ε. In this case SVR optimization is considered to improve the training error through training.

The optimal combination of kernel function and loss function is determined through several experiments to jointly control the support vector regression model, which is adjusted to obtain the penalty coefficient C, maximum path epoch, and maximum learning target error ε. Then, the option pricing is performed through a hybrid model lajbcygier. (Hou Ning. Research on option pricing based on parametric and nonparametric machine learning models [D]. Northwestern University, 2021.) This hybrid model is a hybrid model based on the BS model and the SVR method, whose function can be expressed as $f = f_{B-S} - f_{SVR}$.

3. Commentary

3.1 Summary

Traditional option pricing models, such as the BS model, provide a mathematical description of the option pricing problem, simplify the study of the option pricing problem, lay the foundation for the development of subsequent option pricing models, and enable investors to trade and invest based on facts rather than their own preferences. However, the traditional parametric-based BS model has many shortcomings, it relies too much on the idealized assumptions of the market, and there are large deviations from the real complex and volatile financial markets, which makes the BS model much less valuable in real situations.

Machine learning is a science based on computer and artificial intelligence. Machine learning can process a large amount of data, self-learning, and self-adaptive features can improve the traditional option pricing model to a certain extent by assuming too many conditions and failing to adapt to the changing and complex market. In specific experiments10, machine learning-based option pricing methods have performed extremely well in reducing the errors generated by traditional models.Overall, parametric option pricing models are less accurate than non-parametric models, and for unified models, the accuracy of a larger training volume is higher than the accuracy of a smaller training volume.In addition, machine learning methods have superior performance in terms of time and convergence. Although there is a possibility of major abrupt changes occurring in real markets and market frictions, machine learning methods can also be generalized to pricing problems through the continuous learning method.

However, there are some drawbacks in the actual operation of machine learning. Due to the nature of machine learning methods, the learning effect of machine learning-based option pricing models is very dependent on the selection of samples, and if the sample size is insufficient and the nature of the samples themselves are not good enough (not universal, etc.), the accuracy of the pricing results will be affected. And since machine learning is based on statistical inference, the results of machine learning are theoretically subject to error. In addition, machine learning requires a large number of samples for learning implies that a large enough storage space is required, which is costly for storage space. Although the accuracy of machine learning pricing results is higher, machine learning cannot explain the theoretical implications of market prices, nor can it inform people's investment choices like traditional BS models.

3.2 Outlook

The current research on machine learning cannot explain the theoretical implications behind pricing, therefore, more in-depth research on the theory behind machine learning is needed to prove the pricing ability of machine learning. At the same time, the selection of learning samples can still be optimized, and it is hoped that the optimization of the selection of learning samples can reduce the storage cost and improve the effectiveness and efficiency of learning. In addition, machine learning is developing rapidly and new machine learning methods are emerging, so more machine learning methods can be applied to construct option pricing models in the future.

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