Portfolio Analysis Based on the U.S. Stock Market

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Abstract. As investors place more and more importance on stocks as a financial instrument, how to construct a portfolio of securities in order to avoid risks and obtain more excess profits has become an important issue in the financial world. This paper analyzes the assets of ten representative companies in different industries in the U.S. stock market. The paper uses the closing price data of these ten companies for the past 20 years to construct the optimal portfolio by means of mean-variance model and index model. The results show that in the mean-variance and index models, PG has the highest weight in the minimum variance portfolio, while AAPL in the maximum Sharpe ratio portfolio tends to have the largest weight and can be considered for inclusion in the portfolio at the time of investment. The results of this research may be useful to potential investors interested in the relevant areas of the U.S. stock market.

Keywords: Portfolio; mean-variance model; index model; U.S. stock market.

1. Introduction

The United States Department of Labor recently reported that the core inflation rate in the United States increased by 6.6 percent year on year in September 2022, a record high in 40 years, reflecting the imbalance between domestic supply and demand since the onset of the new crown epidemic and the turmoil in the world landscape, and higher commodity prices, the U.S. is facing more severe domestic price pressure. In order to curb the rising inflation rate, the Federal Reserve continues to raise the federal funds rate, which can bring down the inflation level to a certain extent, but the trend of U.S. stocks is still worried by many investors. In this situation, how to optimize the investment portfolio has become a greater concern for investors.

The advantages of diversification were first revealed by Harry Markowitz in 1952 through the introduction of mathematical analysis using rigorous mathematical formulas. He used the efficient frontier to find the minimum set of variances of a portfolio with expected returns to help investors make the best investment decisions, which marked the emergence of modern portfolio theory [1]. Investors gradually realized that diversification was more competitive compared to single security investments, and since then, portfolio analysis using scientific models has become an important topic in the field of finance.

After decades of continued development, research on portfolio optimization has made significant progress, not only in the increasing refinement of models but also in the increasing range of securities that can be combined. When the distribution under consideration is Gaussian normal, Hanoch and Levy demonstrated that the mean-variance criterion is an efficient criterion of efficiency for any individual utility function [2]. Steinbach mentioned the relationship between short-term average risk efficiency and long-term optimal and long-term capital growth [3]. Chen argued that machine learning presents important opportunities for portfolio optimization [4]. Sarwar et al. explore equity pass-through and volatility spillovers between company stocks and oil assets [5]. Wahyudi attempt to optimize asset portfolios in employer pension funds [6]. Fernando et al. argue that based on different constraints, actual generation assets can be determined based on the cost or return of each alternative technology and economic Chen attempts to reduce portfolio risk by analyzing equity and housing assets [8].

As mentioned above, portfolio theory is widely applied. However, we can find very limited analysis for specific securities, and the purpose of this paper is precisely to focus on asset allocation and better construction of asset portfolios for investors' portfolio management in the U.S. stock market.

In this regard, this paper makes the following work. First, ten representative stocks from different sectors are selected in the U.S. stock market, which are Amazon.com Inc (AMZN), Apple Inc. (AAPL), JPMorgan Chase & Co. (JPM), Microsoft Corporation (MSFT), Johnson & Johnson (JNJ), Nucor Corporation (NUE), The Coca-Cola Company (KO), The Walt Disney Company (DIS), Walmart Inc (WMT), Procter & Gamble (PG). And then the closing prices of these ten stocks over 20 years were collected, followed by cleaning the data. Then, the mean-variance model and the index model were used to construct the optimal portfolios under different constraints, i.e., the portfolio with the maximum Sharpe ratio and the portfolio with the minimum volatility.

The structure of this paper is as follows. Section 2 describes the stocks used in the study, Section 3 details the models used in this paper, Section 4 summarizes the findings, and Section 5 draws conclusions.

2. Data

The data used in the article is derived from Yahoo finance (https://finance.yahoo.com). In this paper, S&P 500 and ten representative stocks of companies in different industries in the U.S. stock markets are selected for analysis. These ten companies are AMZN, AAPL, JPM, MSFT, JNJ, NUE, KO, DIS, WMT and PG, and this article analyzes their closing prices from November 13, 2002, to November 13, 2022, and obtained a total of 2662 data. The above companies are selected because they are excellent performers in their respective industries and have been listed for more than 20 years and analyzing them allows us to draw more objective and comparable conclusions. Finally, the article converts the closing prices into returns for further calculation, and some basic information is shown in Table 1.

Table 1. Descriptive statistics of these selected assets

	SPY	AMZN	AAPL	JPM	MSFT	JNJ	NUE	KO	DIS	WMT	PG
AAR	0.0740	0.2988	0.3708	0.1304	0.1340	0.0625	0.1857	0.0628	0.1147	0.0624	0.0705
StDev	0.1481	0.3739	0.3301	0.2725	0.2228	0.1467	0.3485	0.1606	0.2479	0.1723	0.1492
Max	1.0000	0.5427	0.3623	0.2415	0.2495	0.1442	0.3419	0.1422	0.2336	0.1478	0.1314
Min	0.0000	-0.3057	-0.3289	-0.2325	-0.1656	-0.1333	-0.2476	-0.1727	-0.1891	-0.1595	-0.1184

As shown in the table above, AAPL has the highest average return and WMT has the lowest average return. Comparing the annual standard deviation reveals that AMZN has the highest annual standard deviation and JNJ has the lowest annual standard deviation. In addition, the maximum return is shown in AMZN, and the minimum return appears in AAPL.

3. Methods

3.1 Mean-Variance Analysis

The mean-variance model is a systematic solution to the dilemma of investment decisions and a tool for the conflict between high expected return and low risk [9]. The establishment of the mean-variance model provides an effective theory for diversifying risk and maximizing utility of a portfolio under risk uncertainty. Also, the mean-variance model is a simple and tractable method. A brief description of the model-related formulas will be given below.

$$\sum_{i=1}^{n} w_i = 1 \tag{1}$$

Wi represents the weighting of each asset in the portfolio.

$$Er_p = \sum_{i=1}^{n} w_i(Er_i) \tag{2}$$

Erp is the weighted average of the returns of the portfolio's securities, and Eri is the expected return for each security.

$$\sigma_p^2 = \sum_{i=1}^{n} \sum_{j=1}^{n} cov(r_i, r_j)\, w_i w_j \tag{3}$$

Cov (ri, rj) denotes the covariance of security i and security j when i ≠ j. σ_p^2 represents the calculated variance, where a larger value of variance implies greater risk. The securities with smaller variance should be selected in portfolio selection.

$$\text{Sharpe ratio} = \frac{Er_p - r_f}{\sigma_P} \tag{4}$$

The sharpe ratio is one of the three classical indicators that can consider both return and risk, and it is believed that rational investors building risky portfolios require a return on investment of at least the risk-free rate [10]. The formula Erp represents the expected annualized portfolio payoff, rf represents the annualized risk-free rate, and σ_P represents the standard deviation of the portfolio.

Although the mean-variance model can derive the optimal portfolio, it also has certain drawbacks. First of all, a large number of estimates are required to calculate the covariance, which greatly increases the computational effort. Second, the model contains only the mean and variance of the return distribution, without considering kurtosis and skewness, and the conclusions obtained are only idealized results. Third, the risk premium is a key factor in constructing the efficient frontier of a portfolio, and the mean-variance model has limited consideration of it.

3.2 Index Model

In fact, financial analysts do not like such a complex calculation process and try to find a simpler model for their analysis. To this end, in 1963 William F. Sharpe, inspired by the Markowitz model, developed a simplified index model for portfolio analysis that not only simplifies the estimation of covariances but also fully takes into account the risk premium [11]. The exponential model decomposes risk into systematic and unsystematic risk and makes it easy to analyze the ability of a portfolio to diversify unsystematic risk. Although the model simplifies the computational process, it remains valid for the portrayal of the efficient frontier.

$$r_i = \beta_i r_m + \alpha_i + e_i \tag{5}$$

ri represents the excess return of each security over the risk-free return, β_i represents the sensitivity of the security to market movement, rm represents the expected return of the market index, α_i represents the expected return of the stock when the excess market return is zero, and ei represents the residual.

$$R_p = E(r_m) \sum w_i \beta_i + \sum w_i E(\alpha_i) \tag{6}$$

Rp denotes the portfolio's expected rate of return, and wi denotes the weight of each asset.

$$\sigma_p^2 = \beta_p^2 \sigma_m^2 + \sigma^2(e_p) \tag{7}$$

σ_p^2 represents the total asset portfolio variance, where $\beta_p^2 \sigma_m^2$ implies systematic variance and $\sigma^2(e_p)$ represents non-systematic variance, which is the mean of the variance specific to the firm. From the following formula, we find that $\sigma^2(e_p)$ can be approximated to zero as n becomes infinitely large, which means that firm-specific risk can be realized to disappear completely as stock diversification increases, at which point the risk of the portfolio equals systematic risk.

$$\sigma^2(e_p) = \frac{1}{n}\bar{\sigma}^2(e) \tag{8}$$

4. Result

This paper uses the mean-variance model and the index model for portfolio analysis, respectively, and set three additional constraints, namely

1) no additional constraints.
2) wi >=0, for ∀i.
3) w1 = 0.

Next, this paper will find the allowed portfolio regions for the additional constraints described above, respectively, and the results are presented in the following tables 2-4, respectively.

Table 2. Stock weight under constraint 1

MM	SPY	AMZN	AAPL	JPM	MSFT	JNJ	NUE	KO	DIS	WMT	PG
MinVar	48.42	-2.75	-2.47	-5.68	2.15	16.32	-4.20	12.10	-7.20	21.12	22.19
MaxSharpe	-396.23	58.24	121.85	82.01	35.99	37.97	39.11	2.23	24.31	29.92	64.58
	Return	StDev	Sharpe								
MinVar	3.83	11.07	34.62								
MaxSharpe	61.87	44.48	139.09								
IM	SPY	AMZN	AAPL	JPM	MSFT	JNJ	NUE	KO	DIS	WMT	PG
MinVar	20.06	-2.06	-3.06	-5.19	1.83	28.29	-4.63	21.33	-6.95	21.50	28.89
MaxSharpe	-268.79	53.72	104.33	19.54	50.62	21.93	26.19	13.39	12.83	24.05	42.20
	Return	StDev	Sharpe								
MinVar	3.18	9.63	32.96								
MaxSharpe	52.81	39.29	134.43								

As shown in Table 2, the minimum variance portfolio has the highest weight in the mean-variance model and the lowest weight in the index model. In the maximum Sharpe ratio portfolio, the largest weight ratio is AAPL. The reason for the largest weight of PG may be that P&G has continued to grow steadily, with factories and subsidiaries worldwide and more than 300 brands of products. The share of AAPL is the biggest reason may be that its turnover has been rising rapidly for 20 years and is still on an upward trend, and the company has developed a good brand effect and has a sizeable hardcore fan base worldwide.

Under unconstrained conditions, the Sharpe ratios calculated by the mean-variance model for the minimum volatility portfolio and the maximum Sharpe ratio portfolio are greater than the Sharpe ratios calculated by the exponential model.

Table 3. Stock weight under constraint 2

MM	SPY	AMZN	AAPL	JPM	MSFT	JNJ	NUE	KO	DIS	WMT	PG
MinVar	18.77	0.00	0.00	0.00	3.08	18.65	0.00	11.88	0.00	21.90	25.72
MaxSharpe	0.00	20.20	52.06	7.98	0.00	0.00	1.38	0.00	0.00	8.56	9.82
	Return	StDev	Sharpe								
MinVar	5.92	11.29	52.45								
MaxSharpe	26.67	22.75	117.26								
IM	SPY	AMZN	AAPL	JPM	MSFT	JNJ	NUE	KO	DIS	WMT	PG
MinVar	0.00	0.00	0.00	0.00	0.00	28.00	0.00	20.77	0.00	21.99	29.24
MaxSharpe	0.00	26.92	58.13	0.00	5.40	0.00	1.17	0.00	0.00	2.59	5.78
	Return	StDev	Sharpe								
MinVar	5.29	9.95	53.21								
MaxSharpe	29.92	25.03	119.54								

In Table 3, the minimum variance portfolio also has PG as the largest weight. In the maximum Sharpe ratio portfolio, the largest weight is AAPL. In addition, NUE has the lowest weight except for the stocks with weight of 0. The reason for NUE's position in the largest Sharpe ratio is mainly due to the strong market demand for steel and steel products, while the reason for its being the smallest weight is inferred to be the volatility of commodity raw material prices.

Under the condition wi ≥ 0, for ∀i, the Sharpe ratios of the minimum variance portfolio and the maximum Sharpe ratio portfolio calculated by the exponential model are higher than those of the mean-variance model.

Table 4. Stock weight under constraint 3

MM	SPY	AMZN	AAPL	JPM	MSFT	JNJ	NUE	KO	DIS	WMT	PG
MinVar	0.00	-0.20	1.28	0.95	8.17	22.84	-0.39	15.69	-0.44	23.89	28.21
MaxSharpe	0.00	24.56	60.51	18.61	-5.94	-2.32	5.08	-12.51	-20.58	12.91	19.69
	Return	StDev	Sharpe								
MinVar	6.21	11.44	54.29								
MaxSharpe	30.06	25.17	119.45								
IM	SPY	AMZN	AAPL	JPM	MSFT	JNJ	NUE	KO	DIS	WMT	PG
MinVar	0.00	-1.23	-1.87	-3.06	4.16	31.26	-3.40	23.96	-3.95	22.94	31.17
MaxSharpe	0.00	34.94	72.62	-8.05	16.72	-9.31	7.33	-13.87	-23.32	7.96	14.98
	Return	StDev	Sharpe								
MinVar	3.91	9.71	40.26								
MaxSharpe	36.15	29.53	122.43								

Table 4 illustrates that in the minimum volatility portfolio of the mean-variance model, the largest weight is also PG, and the smallest weight is DIS. In the maximum Sharpe ratio portfolio, the largest weight is likewise AAPL, and the smallest weight is DIS.

Conditional on w1 = 0, the Sharpe ratio of the minimum volatility portfolio calculated by the mean-variance model is higher than that of the index model, but the Sharpe ratio of the maximum Sharpe ratio portfolio is lower than that of the index model.

5. Conclusion

Current research on portfolios has focused on specific sectors and their related industries, with less research on industries that span a wide range of sectors. However, in the stock market, instead, it is better to diversify risk by investing in industries with lower correlations because of their different modes of operation, sources of risk and exposure to market conditions. Based on this, this paper selects ten representative companies in ten industries: e-commerce industry, electronic high-tech industry, financial industry, computer software digital products industry, healthcare industry, steel industry, fast selling industry, entertainment and leisure industry, and retail industry for specific analysis in order to inspire and help other investors.

In this paper, two approaches, namely mean-variance model and exponential model, are used to optimize the portfolio. In this paper, three different constraints are set for the portfolio based on the reference mean-variance model and the index model, and the portfolio with the minimum variance and the maximum Sharpe ratio is calculated and selected based on this premise. Meanwhile, this paper finds a clear pattern under the three constraints of the mean-variance model and the index model, that is, under the minimum volatility portfolio, the proportion of weight of PG is always the largest and the weight of DIS is always the smallest; while under the maximum Sharpe ratio portfolio, the weight of AAPL tends to be the largest and the weight of DIS tends to be the smallest. This suggests that AAPL or PG can be included in a portfolio when investing for the need of maximum return or minimum risk, while DIS should be carefully considered. However, there are certain shortcomings in this paper, such as the portfolio composition industry is not broad enough, and the model still has a certain gap with reality, which needs to be further improved.

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