An option pricing model focus on spread based on the stock price of Apple and Tesla

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Abstract. Option on a spread is typically valued by the difference between two stock prices. However, there may be several limitations for this form if the two prices differ enormously. We thus establish the option based on the difference of profits as opposed to pure stock prices and the option value is largely determined by the growth rate or potential profits. In the valuation process, we use the geometric Brownian motion model to estimate the stock price in 15 days and compute the rate of return as well as the investment profits. Consequently, the value of the option could be calculated if the strike price is given. To gain a more general conclusion we simulate the valuation process 1000 times and test the sensitivity of the option based on several input variables. Our work is to provide a new form of option on a spread that could be utilized in more general combinations of stocks.

Keywords: Option on a spread; Rate of return, Profits.

1. Introduction

Spread options provide a valuable tool for traders to simultaneously take positions in two or more assets and take profits of their price spread. Consider for instance the following example from the finished product oil and crude oil market. In January, the $2.69 / BBL spread between crude oil and gasoline and heating oil was an ideal profit, so the refiner buying crude oil and selling gasoline and heating oil, locking in a gain of $3.80 / BBL. At the expiration date, the crude oil go up to $1 a barrel from when he bought, gasoline prices rose to 54.29 cents a gallon, heating oil prices rose to 49.50 cents a gallon, and the refiner closed his position in the futures market. If the refiner had not hedged, his gains would have been limited to $3.13.

From this example, we can know that the application of the spread options has a positive effect on mitigating risks and hedging. Thus, the application of the spread option is very broad. Options on the spread between crude oil and refined products are traded on the NYMEX. Electricity spark spread options between the price of power and the fuel needed for its generation are regularly traded over the counter. It is easy to notice that the assets in these options are correlated. In this paper, we want to research the pricing and sensitivity of options on the spread on two correlated assets (AAPL&TESLA) and hope to make some improvements in the pricing formula. By doing these works, we hope our model can have a better effect on hedging between two assets and may can generalize it to spread options on the other two correlated assets.

By reading the previous research, we recognize that several studies have focussed on the valuation of spread options. A straightforward way of calculating the value of the option is to apply a Monte-Carlo simulation, assuming certain correlated stochastic dynamics of the assets. In 1993 Ravindran proposed to use Gauss-Legendre quadrature to approximate the value of the spread option [1]. The equation is unbiased and gives very accurate results. The option spread model of the Kirk-formula is based on the assumption of correlated log-normal dynamics of underlying and it can give an accurate result with high correlation [2]. The Carmona-Durrelman procedure allows deriving a family of upper and lower bounds for the spread option price [3]. This model believes that we can get an accurate price by calculating the supremum of the lower bounds. Researchers also use the lower bounds to calculate results [4].

The article written by Baeva derived a closed-form analytical expression for the price of a spread option on two log-normal underlying asset processes [5]. The solution was compared with widespread approximations and direct Monte-Carlo simulations [6-7].

Though these models can get more accurate results, we notice these regular models may meet some problems. A common option on a spread focuses on the spread between two stock prices and investors could gain profits if they predict the situation appropriately. This form of option may not be relatively practical, however, if two stocks vary enormously in their price, for instance, one is about $100 while the other is only $3. In this case, the value of the option is mostly determined by the first stock. Thus, the positive effect of the spread option like hedging is little.

In our design, we use the rate of return to compute profits, making the comparison of two stocks differing in the magnitude of their price more realistic. In this paper, we use AAPL and TSLA as an example because they have a big stock price gap and meanwhile we think they have a positive correlation. We hope we can build a better model to calculate the spread option on two correlated assets and vary enormously in their price.

The reminder of the article is organized as follows. Section 2 describes the different firms' information. Section 3 presents the definition of option on the spread. Section 4 introduces the valuation and result analysis of the spread option. In the Section 5, we conduct the sensitive analysis. For Section 6, we give some suggestions for the investors. Section 7 is the extension and Section 8 summarize the conclusion.

2. Firm Description

2.1 Apple lnc.

Apple Inc. (AAPL) is an American high-tech company. It was founded by Steve Jobs, Steve Gary Wozniak, and Ronald Gerald Wayne on April 1, 1976, and named Apple Computer Inc. (Apple Computer Inc.), and changed its name to Apple on January 9, 2007. Headquartered in Cupertino, California. The main business of APPL is computer hardware, computer software, consumer electronics, and information display system. We can know that AAPL is a technology innovative company and focus on computer science. After we know about the basic information of the APPL we search the stock price data of the AAPL and use it to estimate the operating conditions of AAPL. We plotted the monthly closing price of Apple over the past three years and get the graph below. From this chart, we can know that the overall trend of stock prices of AAPL is upward. The stock price upward from 41.61 in 2019.1.1 to 148.97 in 2021.9.1. Though the stock price has some fluctuations in 2020 maybe because of the NAVID-19, we still can suggest that the operating conditions of AAPL are good.

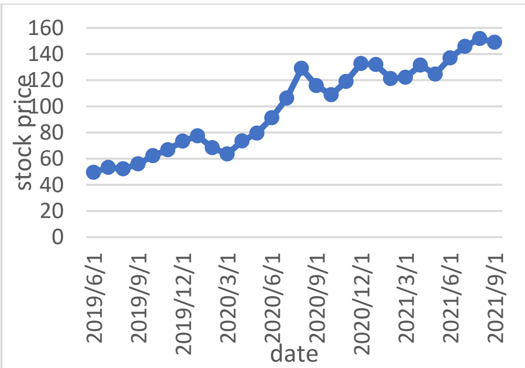

Figure 1 Stock price of Apple lnc.

We also compared Apple's sales figures for the first figure of 2019 and the first quarter of 2020. We can see that the operating income and net profit have increased.

Table 1. Description of Apple lnc.

	2020	2019
Net sales	$111,439.00	$91,819.00
Cost of sales	$44,328.00	$35,217.00
Net profit	$67,111.00	$56,602.00
Operating income	$28,755.00	$22,236.00

2.2 Tesla

Tesla is an American electric vehicle and energy company that manufactures and sells electric vehicles, solar panels, and energy storage equipment. Headquartered in Palo Alto [1], it was co-founded by Martin Eberhard and Mark Happening on July 1, 2003. After we know about the basic information of the TSLA we search the stock price data of the TSLA and use it to estimate the operating conditions of TSLA. We plotted the monthly closing price of Tesla over the past three years and get the graph below. From this chart, we can know that the overall trend of stock prices of AAPL is upward from 2019 to 2020 The stock price from 41.61 in 2019.1.1 to 148.97 in 2021.9.1. But the stock prices it go down slightly and fluctuated up and down in 2021.

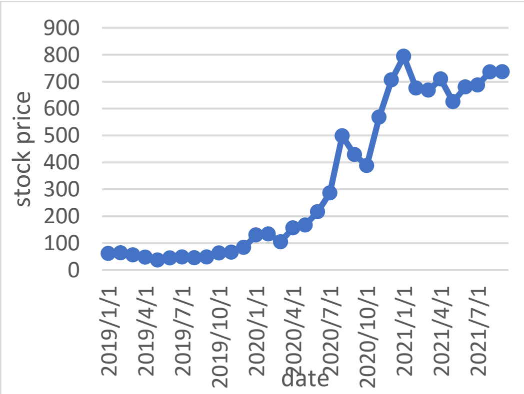

Figure 2 Stock price of Tesla lnc.

We also compared tesla's sales figures for the first figure of 2019 and the first quarter of 2020. We can see that the operating income and net profit have increased

Table 2. Description of Tesla lnc.

	2020	2019
Net sales	8771	6303
Cost of sales	6708	5112
Net profit	2063	1191
Operating income	86000	26000

3. Introduction of option on the spread

3.1 Definition of Option on the Spread

The classic option is defined on one underlying asset, while the spread option can be regarded as a simple extension of the classic option, defined on two underlying assets [8]. And these two underlying assets, could be any two indices. Option on the spread derives its value from the spread, between the prices of two assets [9-10]. For example, you can buy call options on oil and sell calls on steel. Then you get the difference as the value on the expiration date.

The idea of spread options is simple, but it can be extended in a variety of ways that are useful in solving many practical problems. In currency and fixed income markets, a spread option can be an option based on two interest rates or the difference between two yields. In commodity markets, spread options can be based on the difference between the prices of inputs and outputs of the same product at different locations (positional spread), at different times (calendar spread), or in a production process (process spread), and the difference between the prices of different grades of the same product (quality spread).

In our design, we choose two options that are defined on different assets and the value of our option is the spread between profits of two stocks. The return is based on the geometric Brownian motion and the parameters of the model are based on the previous data.

3.2 Introduction to Spread Option Model

To calculate the future value of an asset (ST), we use the geometric Brownian motion model. The formula of this model is

$$S_T = S_0 * e^{\left(\alpha-\frac{\sigma^2}{2}\right)*\mathrm{T}+z*\sigma*\sqrt{\mathrm{T}}} \tag{1}$$

In this formula, S0 is the value of an asset now. α is equal to the risk-free rate minus the dividend of the stock

$$\alpha = r_f - \delta \tag{2}$$

σ is the standard deviation of the stock's return, T is the amount of the trading days, z is a random number that conforms to a normal distribution

This is the normal formula of the GEM model. We should notice that when using this stochastic process, the expected return α and standard deviation σ are assumed constant at each moment in time, and the asset value changes continuously. We choose the recent 15 days data hoping to make it more accurate Compared to the normal formula, we make some changes in our model.

$$S_{A_{15}} = S_{A_0} * e^{\left(\alpha_A-\frac{\sigma_A^2}{2}\right)*\frac{15}{252}+z_1*\sigma_A*\sqrt{\frac{15}{252}}} \tag{3}$$

$$S_{T_{15}} = S_{T_0} * e^{\left(\alpha_T-\frac{\sigma_T^2}{2}\right)*\frac{15}{252}+z_2*\sigma_T*\sqrt{\frac{15}{252}}} \tag{4}$$

We considered the correlation of the two options, hoping to further refine this role. We use the correlation to calculate the z1 and z2 because we think it can reflect the relationship between the two stocks and we can get a better result.

$$z_2 = z_1 * \rho + \sqrt{1-\rho^2} * z \tag{5}$$

ρ is the correlation between the two stocks.

3.3 Meaning of the Model

We use this model due to the following reasons.

(1) The expectation of geometric Brownian motion is independent of the price of a random process (stock price), which is consistent with our expectation of real market

(2) Geometric Brownian processes only consider positive prices, like real stock prices

(3) Geometric Brownian motion takes on the same roughness as the price trajectory we observe in the stock market.

(4) The calculation of the geometric Brownian motion process is relatively simple.

4. Valuation And Result Analysis

4.1 Valuation

In this part, we design the option on a spread based on the difference in profits of a potential investment, as opposed to stock price, of Apple and Tesla.

4.1.1 Daily Rate of Return

We select stock prices of Apple and Tesla from December 31st, 2019 to December 30th, 2020. Then the daily rate of return is calculated:

$$r_{A_i} = \frac{s_{A_{i+1}}}{s_{A_i}} - 1 \tag{6}$$

$$r_{T_i} = \frac{s_{T_{i+1}}}{s_{T_i}} - 1 \tag{7}$$

In the formula, r_{T_i} represents the ith daily rate of return of Tesla while s_{T_i} represents the ith stock price of Tesla.

4.1.2 Stock Price Estimation

Based on the daily rate of return, standard deviation, which represents the volatility of an investment, and α, which represents the average rate of return, could be computed.

$$\sigma_A = \sqrt{\sum_{i=1}^{252} \frac{\left(r_{A_i} - \overline{r_A}\right)^2}{252-1}} \tag{8}$$

$$\sigma_T = \sqrt{\sum_{i=1}^{252} \frac{\left(r_{T_i} - \overline{r_T}\right)^2}{252-1}} \tag{9}$$

$$\alpha_A = r_f - \delta_A \tag{10}$$

$$\alpha_T = r_f - \delta_T \tag{11}$$

Among the formula, r_f is the risk-free rate of return and δ is the dividend rate of each company.

Based on the calculated data, the estimated stock price in 15 days could be computed in which we choose August 26th, 2021 as day zero.

$$S_{A_{15}} = S_{A_0} * e^{\left(\alpha_A - \frac{\sigma_A^2}{2}\right) * \frac{15}{252} + z_1 * \sigma_A * \sqrt{\frac{15}{252}}} \tag{12}$$

$$S_{T_{15}} = S_{T_0} * e^{\left(\alpha_T - \frac{\sigma_T^2}{2}\right) * \frac{15}{252} + z_2 * \sigma_T * \sqrt{\frac{15}{252}}} \tag{13}$$

Considering the price of the stock may be volatile, z_1 and z_2 are two dependent random variables that are normally distributed to quantify the fluctuation of price.

We use ρ to quantify the statistical correlation between the rate of return of two stocks. Then z_2 and z_1 are correlated by the formula:

$$z_2 = z_1 * \rho + \sqrt{1 - \rho^2} * z \tag{14}$$

where z is a normally distributed random variable.

4.1.3 Profits

The expected returns of two stocks are computed in a way similar to the daily rate of return:

$$R_A = \frac{S_{A_{15}}}{S_{A_0}} - 1 \tag{15}$$

$$R_T = \frac{S_{T_{15}}}{S_{T_0}} - 1 \tag{16}$$

We assume an investment of c dollars each, then the profits are calculated:

$$P_A = c * R_A \tag{17}$$

$$P_T = c * R_T \tag{18}$$

4.1.4 Valuation of the Option

The option between two profits could be valued by the formula:

$$v = max(P_A - P_T - X, 0) \tag{19}$$

where X is the strike price of the option.

4.2 Simulation

The average value of the option is calculated through the simulation process.

In the beginning, we just set several primary conditions for the option and then simulate 1000 times.

$$c = 10000 \tag{20}$$

$$X = 10 \tag{21}$$

In the simulation process, only the two random variables z_1 and z_2 are changing while other parameters remain constant.

4.3 Result Analysis

The simulation process shows that the average value of the option on day 15 is 717.75 dollars. Considering the time value, the value on day 15 could be discounted to gain the value on day zero, or in other words, the day investor buys the option.

$$V_{15} = 717.75 \tag{22}$$

$$V_0 = \frac{V_{15}}{(1+r_f)^{\frac{15}{252}}} = 716.90 \tag{23}$$

The distribution of the value in 1000 times simulation is shown in the following graph:

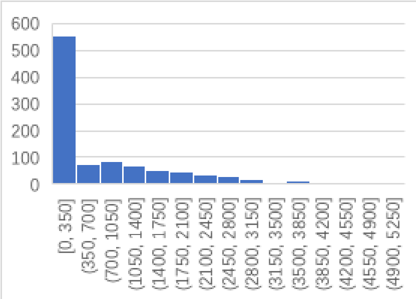

Figure 3 Distribution of the average option value in day zero.

5. Sensitivity Analysis

The purpose of sensitivity analysis is to demonstrate how rapidly the value of the option changes to variations in the parameters. In this part, we select strike price, volatility, investment, and correlation as the input value of sensitivity analysis and then aim to find the potential changes between the input value and the value of the option.

5.1 Strike Price

There is an obvious downwards linear correlation between the strike price and the value of the option.

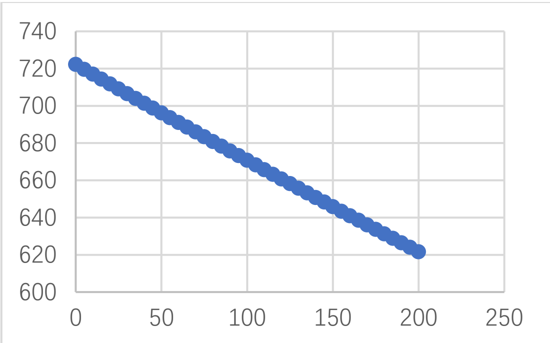

Figure 4 Sensitivity analysis: strike price

5.2 Volatility

Risks of certain investments in the stock market could be partly reflected by volatility, or more specifically, standard deviation.

5.2.1 Apple

It could be found that when the volatility of Apple is approximately 0.36, the value of Apple reaches the lowest point.

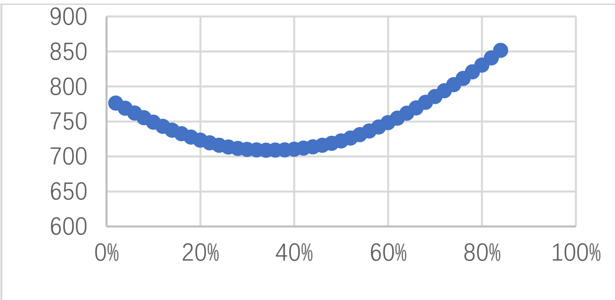

Figure 5 Sensitivity analysis: volatility of Apple

5.2.2 Tesla

It could be found that when the volatility of Tesla is approximately 0.26, the value of Apple reaches the lowest point.

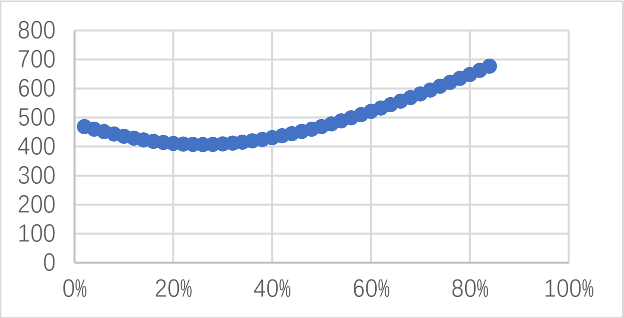

Figure 6 Sensitivity analysis: volatility of Tesla

5.3 Potential Investment

There is an upwards linear correlation between potential investment and the value of an option.

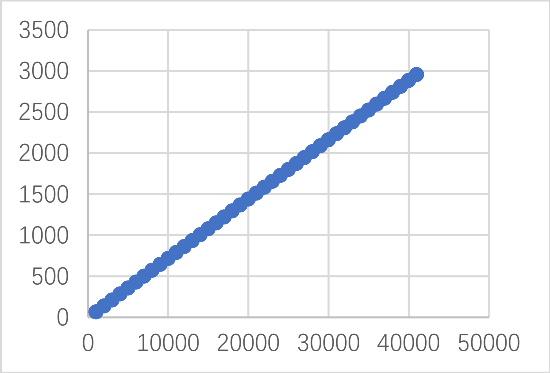

Figure 7 Sensitivity analysis: investment

5.4 Correlation

There is a downwards linear correlation between potential investment and the value of the option.

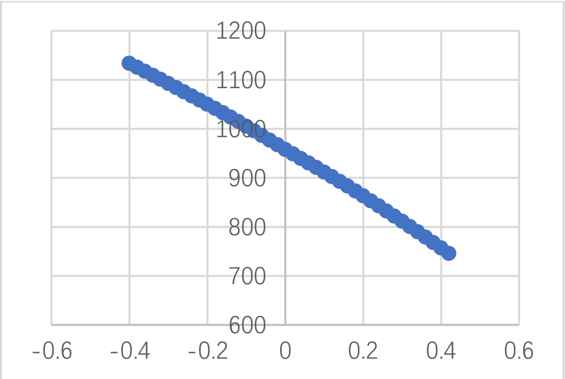

Figure 8 Sensitivity analysis: correlation

This is relatively intuitive since when the correlation of the two stocks rises, its difference would decrease.

6. Suggestions To Investors

6.1 Firm Selection

This option on a spread is a difference between the profits of two stocks, having little connection with the absolute prices of a certain stock. In this case, it is recommended that the buyer of the option need to consider a combination of companies that differed in their growth rate while the seller needs to select a combination of companies similar in their growth rate.

6.2 Industry Selection

Some industries, like high tech or luxury industry, may be more volatile than others. Thus investors may pay more attention to the strongly changing or fluctuating fields since options regarding companies in that field are more likely to earn profits.

Additionally, the option could also include companies in different industries, like one in the high tech industry while the other in a manufacturing one. The difference between profits of the two companies may be relatively stronger, leading to a higher probability of gaining.

6.3 Time Selection

There are periods when the capital market fluctuating violently and companies tend to behave in a diversity of styles. Some stocks may surge at a high speed and others would drastically decline or just remain unchanged. Investing at that time is likely to gain profits between two styles of stocks.

6.4 Model Selection

Geometric Brownian motion (GBM) is a random process in continuous time in which the logarithm of the random variable follows Brownian motion. Thus, we think that the Geometric Brownian motion takes on the same roughness as the price trajectory we observe in the stock market.

7. Extention

In our design of the option, we focus on the expected rate of return of different stocks, and the difference between profits, which are computed based on the expected rate of return, could be utilized for the option on a spread.

7.1 Result of a single calculation and simulation

Based on the formula and previous data of Apple and Tesla, we computed the following indexes to value the option value.

Table 3. results of calculation

Indexes	
Risk-free rate of return(%)	0.07%
The dividend rate of Apple(%)	0.58%
Alpha of Apple(%)	**-0.51%**
The dividend rate of Tesla(%)	0%
Alpha of Tesla(%)	**0.07%**
Volatility of Apple	0.47
Volatility of Tesla	0.89
The stock price of Apple in day 0($)	148.36
The stock price of Tesla in day 0($)	711.20
The estimated stock price of Apple in day 15($)	**179.64**
The estimated stock price of Tesla in day 15($)	**815.68**
Expected rate of return of Apple(%)	21.09%
Expected rate of return of Tesla(%)	14.69%

Investment($)	10000
Profits of Apple($)	**2108.64**
Profits of Apple($)	**1469.06**
Strike price($)	10
Value of the option in day 15	**629.58**

To gain an average value of the option, we use a simulation process for 1000 times with the normally distributed random variable z_1.

Table 4. results of simulation

Indexes	
The average value of the option in day 15($)	717.75
The average value of the option in day 0($)	716.90
Probability of loss(%)	**47.4%**

7.2 Results of sensitivity analysis

We test what the change would be for the option value with the input value of strike price, volatility, investment, and correlation. Results reflect that there is a downward linear trend for strike price and correlation while there is an upward linear trend for investment. The result for the volatility is relatively more complex since the relationship is nonlinear and the lowest point could be found.

8. Conclusion

In our design, we select two high-tech companies, Apple and Tesla, for the option. In the reality, there are a vast number of listed companies, and some combinations of the companies may be more appropriate than our selection. Some future investors, additionally, could also select more than two companies and value the design with advanced mathematical methods like Cholesky decomposition. The design of the option can make some difference in the options market while it also has several drawbacks or some unsolved problems which demand further research in the future.

This type of innovation may provide several advantages to the options market. Investors can have more choices and the design of the option on a spread may be comparably more flexible since the limitation of stock selection may be reduced. We can also predict that the design may offer some inspiration for the option on a spread. According to the option comparing two profits of stocks, it is also probable that other indexes can be used to formulate a new category of option. Consequently, the option on a spread may have rich forms instead of just paying attention to the prices.

At the beginning, we estimate prices of stocks on day 15 based on the previous data in 2020 while it is likely that in the year 2021 the expected standard deviation of the daily rate of return may be distinctive from the year 2020. Thus, the prediction may not accurately reflect the value of the option and the condition in the expiration day would be quite different.

The volatility of the stock price is quantified by the normally distributed random variables z_1 and z_2 but the stock market is far more complicated. The value could not be precisely calculated just by some random variables, leading to a similar error in decision making to the former drawback. The valuation of the option requires multiple computations, increasing hugely in complexity. Therefore, it would be challenging for investors to understand the principle of the option fully.

In the simulation process, we gain the average value of the option on day zero is approximately $700 and this is too large compared to the strike price. In this case, the constant variables, like the strike price X and assumed investment c, are expected for further modification. The magnitude of the strike price may be close to that of the investment. A more precise setting of the constant variables is required in order to revise the design.

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