Approaching Portfolio Optimization through Empirical Examination

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Abstract. In this project, the study focuses on the portfolio profitability, one of the most vital quantitative-finance measurements. Out of all possible portfolios being considered, portfolio optimization is the process of selecting the best portfolio, according to some objective. It is a quantitative principle based on statistics, research methods, and advanced mathematical calculation. In this model, financial risk is calculated to usually be minimum while factors such as expected return are maximized. These factors may include physical aspects, "tangible" indicators, and financial metrics. Based on this assumption, an investor is seeking to maximize the portfolio's expected return despite a certain level of risk. Obtaining a higher expected return with these portfolios, called efficient portfolios, usually will let investors to take on more risk, so investors are sometimes forced to choose between risk and return. An asset's weight is a measure of its concentration within a particular class. In order to optimize portfolios, investors assign 'optimization weights' to each asset class and each asset within the class. By weighing the mean and variance of the whole portfolio, investors can approach the best plan with the most profitability.

Keywords: The portfolio, profitability, 'optimization weights'.

1. Problem Description

Portfolio Optimization can cultivate a mode of thinking to maximize the gain with accurate and scientific calculations in not only financial market but also real life. To approach the idea of portfolio optimization, investors can utilize the concept of marginal analysis to help them comprehend. Marginal analysis examines the benefits and costs of choosing a little more or a little less of a good. Because the thought of portfolio profitability is expedient in front of a lots of situations in people's real life, it is easy to be considerable. If you are planning a 7-day trip to Florida, you start a list of all possible activities. But due to the limitation of days and restricted vigor, you have to pick out only around eight of them. Some of them are things that you have tried, but others are not. It means that you might try ones that are completely boring, but also implies that you might experience some novel thing that can give you potential interest. With this thought, when doing the plan, you will inevitably not only consider the joy that each program can make, but also the risk that some of them maybe bring displeasure or surprising happiness. Also, because some activities cannot bring as much joy as the first try, you will not put a lot of time on incessantly doing it. Inside of this example, when you try your best to maximize your enjoyment, it is the same work that investors do when they optimize their investment portfolio. Respectively, the potential joy that you might enjoy for each program indicates expected return for each asset in stock market. The uncertainty that you might have a bad experience or surprise implies the volatility. The final joy will be considered as the return made by investors. According to this case, the essential concept of portfolio evaluation is useful in enormous usual situations.

2. Tips to Optimize Portfolio

Return maximization is the goal for every investor. To achieve it, similar to planning a trip, they have to select most suitable assets and make their ideal portfolio with taking portfolio expected return and portfolio variance into consideration. $Portfolio\ return = expected\ return \pm variance$. It is undoubtable that investors will choose the high expected return when facing to same variance.

However, when facing to different variance but same expected return, investors will try their greatest effort to achieve the highest return in the lowest level of risk.

2.1 Data Calculation

After being aware of the significance of portfolio optimization, it is important to be familiar to the way to calculate it ($Portfolio\ return = expected\ return \pm variance * importance\ of\ variance = \alpha^T \xi \pm p * \alpha^T \zeta * \alpha = \sum \alpha * Expected\ return \pm \sum\sum \alpha 1 * \alpha 2 * Cov(1,2)$). To fully explain this concept, I collect thirty-assets data from 2020 June to 2021 March. Then, I will use these thirty assets to form five different portfolios and compare them to show how the concept works.

2.1.1 For the first experiment, I pick the first 10 assets from the list, putting 10% of my money per one. After the determination of proportion of funds, the next step is to get the variance of every asset, mean of every asset, and covariance between assets.

With data:

1). For the whole portfolio mean, I transpose the proportion of funds and use the result to multiply by the overall mean of these 10 assets ($Portfolio\ Mean = \alpha^T \xi$). The mean for the whole portfolio is then 0.005287.

2). For the whole portfolio variance (risk), I make a covariance matrix for the whole portfolio. I use the transposed form of proportion of funds to times the overall covariance matrix. Finally, I use this result to times the proportion of funds again ($Portfolio\ Variance = \alpha^T \zeta * \alpha$), and the result will be the variance for the whole portfolio, which is 2.39604.

3). To see whether the portfolio can do well in the market with these indices, I set the importance of the variance to 0.5 and use the mean to minus the 0.5*variance. The profitability for the first ten assets will usually at least be 0.005287-0.5*2.39604 = -1.192733.

2.1.2 In the second try, I set the proportion of funds for the first asset as 50%, set the proportion of funds for the second asset as 10%, and set the proportion of funds for the rest as 5% (The 10 chosen assets are same)

Through the same procedures as the way of calculating the mean and variance for the first portfolio,

1). the mean is calculated to be 0.02643.

2). the variance is 4.4628.

3). The profitability for the first ten assets with 0.5 for the importance of variance is 0.02643-0.5*4.4628=-2.20497.

2.1.3 For the third experiment, I enlarge the quantity of assets from 10 to 20, setting the proportion of funds per asset as 5%, meaning that they share equal funds.

1). The mean is 0.002643.

2). The variance is 2.410.

3). The profitability for the first twenty assets with 0.5 for the importance of variance is 0.002643-0.5*2.410=-1.202357.

2.1.4 With the same assets (first 20 assets from the list), I set the proportion of funds as 0.1 for the first four assets and 0.0375 for the rest.

1). The mean is 0.005287.

2). The variance is 2.666.

3). The profitability for the first twenty assets with 0.5 for the importance of variance is 0.005287-0.5*2.666=-1.327713.

2.1.5 I then set the proportion of funds as 0.01 for the first ten assets and 0.09 fir the rest ten.

1). The mean is 0.000529.

2). The variance is 2.7354.

3). The profitability for the first twenty assets with 0.5 for the importance of variance is 0.000529-0.5*2.7354=-1.36241

Among these five portfolios, it is clear that I can get the greatest profitability when I invest in the first 20 assets and set the proportion of funds as 5% for each asset. Additionally, the importance of portfolio variance keeps being 0.5.

Table 1. Real data of mean and variance for each portfolio plan

	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1										
mean	0.005287																			
variance	2.39604																			
Proportion of funds	0.5	0.1	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05										
mean	0.026434																			
variance	4.462756																			
Proportion of funds	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05
mean	0.002643																			
variance	2.410011																			
Proportion of funds	0.1	0.1	0.1	0.1	0.0375	0.0375	0.0375	0.0375	0.0375	0.0375	0.0375	0.0375	0.0375	0.0375	0.0375	0.0375	0.0375	0.0375	0.0375	0.0375
mean	0.005287																			
variance	2.665837																			
Proportion of funds	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.09	0.09	0.09	0.09	0.09	0.09	0.09	0.09	0.09	0.09
mean	0.000529																			
variance	2.735423																			

2.2 Diversification

To maximize the return, investors that are not doing venture capital always have to weigh the expected returns against their volatilities because the relatively high expected return ensures the overall return to be positive. Thus, it is important to minimize the volatilities so that investors can stabilize their income after maximizing the expected return. In this round, diversification plays an essential role since it influences the correlation and covariance. The portfolio risk for two assets is calculated through the covariance of the portfolio and fund proportion, meaning that lessening the covariance can help decrease the risk.$(\sigma 1^2 * \alpha 1^2 + \sigma 2^2 * \alpha 2^2 + 2 * p * \alpha 1 * \alpha 2 * \sigma 1 * \sigma 2)$ ["α" is how much money the investor put on each asset. "P" is the correlation index.] is the formula of correlation if there are only two assets picked. Extremely, if the investor chooses stock 1 and stock 2 that are moving in exactly same direction and same market type, the correlation between these two will reach its biggest number, 1 (p=1). In this case, the volatility of the whole portfolio reaches its highest point. In contrast, when the investor replace stock 2 with stock 3 that is diametrically different (p=0) from the moving direction of stock 1, the risk will keep itself at the lowest point since the third term in the formula drops out. Comparing these two extreme examples, the perfectly correlated case has much more risk than completely uncorrelated case with same expected return. It is a mathematical way to reveal that the more correlations between assets a portfolio has, the more risk it has to face. According to the research done by Blume and his colleagues, 34.1% investors held only one dividend-paying stock. 50% investors held no more than two stocks. And only 10.7% investors chose to keep over 10 stocks to maintain diversification. It implied that a big number of investors did not notice the problem of low diversification, so they were more likely to lose money due to the systematic risk happening inside every stock. It was in part because some people were ignorant of the benefit brought by it, or because they thought the revenue of diversification was not significant. A research done by Statman showed the best number of stocks will be, at very least, between 30 to 40. The result surprised the team because it contradicted the past assumption that believed the best diversification was between 20.

3. Optimization Constraints

The optimization of a portfolio is usually subject to constraints, like regulatory restrictions or illiquidity. Portfolio weights can be set to target a small sub-pattern of assets in the portfolio as a result of such constraints. Despite the fact that the portfolio optimization method is affected by other factors, including taxes, transaction costs, and control costs, it may also result in a portfolio that is not sufficiently diversified.

3.1 Limitation of transaction costs

The portfolio optimization model is calculated and based on a frictionless market which has no transaction costs and taxes for each deal. As a result, there are enormous advantages to diversifying funds into lots of assets. However, in the process of adjusting the weights of the portfolio, transaction expenses are incurred. There is an incentive to re-optimize the premier portfolio frequently since it adjusts over time. But too frequent transactions would result in too-common transaction prices, so the gold standard is to find a frequency for re-optimization and trading that accurately trades off avoiding transaction fees with the out of sticking with an outdated set of portfolio proportions. In the absence of rebalancing, inventory proportions gradually deviate from some benchmark, a process known as monitoring error.

3.2 Governmental Regulation

Regulations may prohibit investors from holding several assets. Some assets may be quickly sold if portfolio optimization is unrestrained. However, quick-selling may be forbidden. According to monetary regulation, interest rates are lowered to encourage investment, and savings are encouraged by borrowing. To encourage investment, governments provide tax incentives. However, when the national inflation is high, the government will increase the interest rate. Thus, the tax cost incurred by preserving an investment is too high, it may not be practical to do a lot investment. During this process, the government imposes practical constraints.

4. Conclusion

As the investment cleaves into people's insight gradually, portfolio optimization becomes an essential and useful instrument to maximize returns. It provides enormous opportunities for investors to include a huge amount of assets in various areas. Being a method of evaluating the performance of an investment after adjusting for its risk component, portfolio optimization forces investors to calculate for the highest expected value with lowest variance. Besides, this investment strategy is also grateful among managers. Active portfolio management requires managers to track a lot of market data and stay informed on the markets. As a result, they are able to identify market opportunities earlier than others and take advantage of those opportunities for the benefit of their investors. Again, Optimal portfolios do not guarantee the best possible return from the combination. Instead, they maximize the return per unit of risk taken.

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