Research on Carbon Sequestration Problem Based on Linear Programming Model

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Abstract. To help the forest managers calculate the carbon sequestration amount, we first give the calculation method of carbon sequestration amount of trees, and then establish a parameterized forest model according to the carbon sequestration amount, volume and the products of the Chinese fir in Huitong County, Hunan province, to facilitate our simulation study. Through dynamic planning, the problem of optimal carbon sequestration can be transformed into a linear programming problem. Then we obtain the management plan that maximizes carbon sequestration amount. By studying the management plan, we find that the age of the trees harvested is concentrated in the 25-30 years old, and the optimal harvesting plan will change when the parameters of the forest change.

Keywords: Carbon sequestration; Dynamic programming; Deforestation; linear programming model.

1. Introduction

As is known to all, climate change is a big threat to us, and we should take some actions to reduce carbon emissions and absorb more carbon dioxide. Taking the Chinese fir as an example, the carbon sequestration amount of trees is related to the age of trees. With the increase of age, the carbon sequestration amount of trees first increases and then decreases. The average annual carbon sequestration amount of the Chinese fir trees during 1-7 years, 8-11 years, 12-14 years, 15-18 years and 19-20 years are 3.28，5.38，6.26，6.20 and 5.78 ($\mathrm{t \cdot hm^{-2}}$), respectively[1]. Thus, harvesting trees at different times impacts the carbon sequestration amount of a forest. In addition, the harvested trees can be made into wood products to benefit, so our harvesting plan should balance the two parts. To maximize carbon sequestration and profit more from the wood products, we should design a harvest plan.

In this paper, we design a carbon sequestration model to describe the carbon sequestration amount of the forest, and give the optimal planting scheme to maximize the carbon sequestration amount. We need to consider the carbon sequestration amount of the forest itself and the carbon sequestration amount of its products. These products will return to carbon sequestration after a certain life.

2. Model analysis

2.1 the model of the carbon sequestration amount of the forest

In recent years, different researchers have widely used methods for estimating plant carbon sequestration amount based on the relationship between biomass and stock volume. Wang Xiaoke et al. used a method for estimating plant carbon storage based on the relationship between biomass and stock, combining ecological survey data with forest census data to estimate the actual carbon storage of forest ecosystems in China. Based on this method[2], we set up the average carbon sequestration amount calculation model of different trees. The first type of tree layer is calculated for each forest ecosystem carbon storage density (P_C), and then based on the ratio of tree layer biomass and total biomass, estimate the total biomass per unit area of the forest type carbon deposits, the last times the average size of the trees of different ages. The average annual carbon storage of different trees can be obtained:

$$T_c = P_c \cdot S = V \cdot D \cdot R \cdot C_c \cdot S \tag{1}$$

In the formula, V represents the unit area accumulation of the unit forest type, D represents trunk density, C_C represents the ratio of trunk biomass to tree layer biomass, and represents the carbon content in plants, which is usually 0.45.

2.2 Solution of the harvesting management plan

Although an objective function cannot be described as a general Dynamic Programming problem, it can be characterized by its terminology.

① **Step:** Divide this problem into n stages, assuming that logging and planting are carried out once a year, then $k = 1,2,3 \dots . n$.

② **State:** x_k reference to the state vector at the beginning of year k, which is the amount of the age of the tree aged k years. A_k is the number of one-year-old trees at the beginning of the year, B_k is the number of trees that are two years old, and so on. Then the x_k is :

$$x_k = \begin{bmatrix} A_k \\ B_k \\ C_k \\ \dots \end{bmatrix} \tag{2}$$

③ **Control:** u_k Represents the control vector at the beginning of the year k, that is, the number of trees harvested for each age at the beginning of the year k. a_k represents the number of one-year-old trees decided to be harvested at the beginning of the first year, b_k represents the number of two years old trees decided to be cut down at the beginning of the first year, and so on.

④ **Equation of state transition:** $T(x_k, u_k)$ represents the $k+1$ state vector determined by the k state vector and the control vector.

⑤ **Optimal value function:** When calculating the optimal value function, it is necessary to introduce the annual carbon sequestration amount of each tree age, which is represented by the vector T_C

$$T_c = \begin{bmatrix} T_1 \\ T_2 \\ T_3 \\ \dots \end{bmatrix} \tag{3}$$

The useful life of forest products made from trees of different ages is represented by vectors S_P.

$$S_p = \begin{bmatrix} \alpha \\ \beta \\ \gamma \\ \dots \end{bmatrix} \tag{4}$$

In summary, the total carbon sequestration amount at the end of the year n, namely the optimal value function can be obtained:

$$V\left(x_1, x_2, \dots, u_1, u_2 \dots\right) = \sum_{k=1}^{n} x_k \cdot T_c - \left(\sum_{k=1}^{n-\alpha} a_k T_1 + \sum_{k=1}^{n-\beta} b_k T_2 + \dots \right) \tag{5}$$

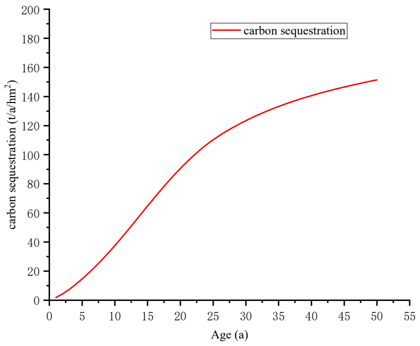
When $n - \alpha \le 0$, $\sum_{k=1}^{n-\alpha} a_k T_1 = 0$.When $n - \beta \le 0$, $\sum_{k=1}^{n-\beta} b_k T_2 = 0$ and so on.

2.3 The simulation model of the forest

The test models are all Chinese fir trees located in Huaihua, Hunan Province, China. The planting density is 2400 per hm^2. The interaction between trees and the influence of the environment is ignored, that is to say, that the trees are exactly the same on the growth curve.

According to [1], the curve of carbon sequestration amount of this kind of tree at various ages can be obtained through the fitting.

According to [3], the volume of these trees at various ages can also be obtained through data fitting.



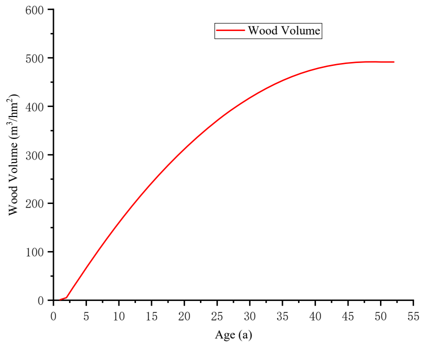


Figure.1 The Curve of carbon sequestration amount (up to one) and Tree volume (down one) over time

There are five kinds of products of the trees. We assume that all harvested trees are immediately made into wood products without loss in the same year. Their relevant data is shown in Table 1. The carbon stored in manufactured products will not be released until they reach the end of their useful life. In addition, different products have different requirements for the age of trees, and only trees that reach the required age of the products can be made into related products.

Table 1 The information about the product

Type	Price $(yuan/m^3)$	Required age (a)	Useful life (a)
building materials	584	30	100
furniture	708	10	30
paper	200	1	5
firewood	275	1	0
others	311	5	20

In this paper, we choose a Chinese fir forest size of 10,000 hectares as the research object. In year 0, the average age of trees in the Chinese fir forest was 25 and followed a normal distribution $N(25,5)$.

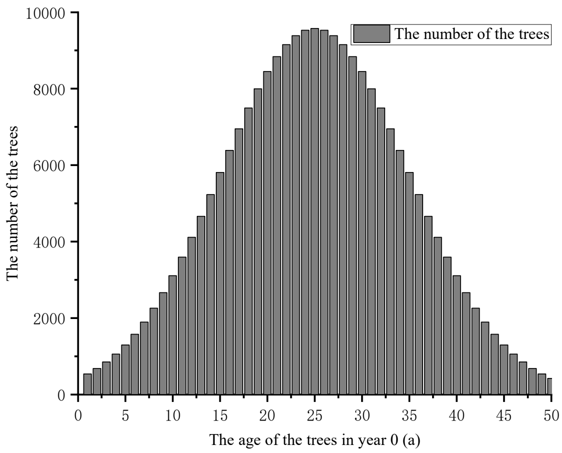


Figure 2. Age distribution of trees under initial conditions

2.4 Simulation Result

After simulation analysis, the age distribution of harvested trees is shown in the figure. In the figure, we can clearly see that the ages of harvested fir trees are concentrated between 20 and 30 years old. Therefore, accroding to the forest model, in order to maximize carbon sequestration, a rough cutting plan can be preliminarily formulated, that is, only 20-30-year-old trees are cut. If we want a detailed plan, we need to harvest the data shown in the figure year by year.

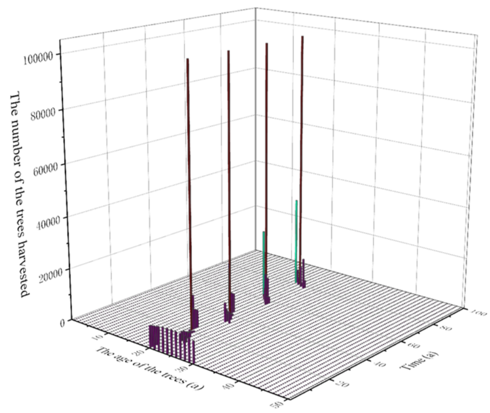


Figure 3. The number of trees harvested changes with the tree's age and time. The X-axis is the age of the trees. The Y-axis is the time, and the Z-axis is the number of trees harvested at different ages.

The number of trees in the forest can be obtained in Figure 4. As can be seen from Figure 4, trees between 0 and 25 years old have a larger age distribution, because the absorption rate of carbon dioxide during the period from 0 to 25 years old is faster than other stages. Therefore, maintaining the forest trees in this age distribution of forest carbon sequestration amount would achieve maximum amount.

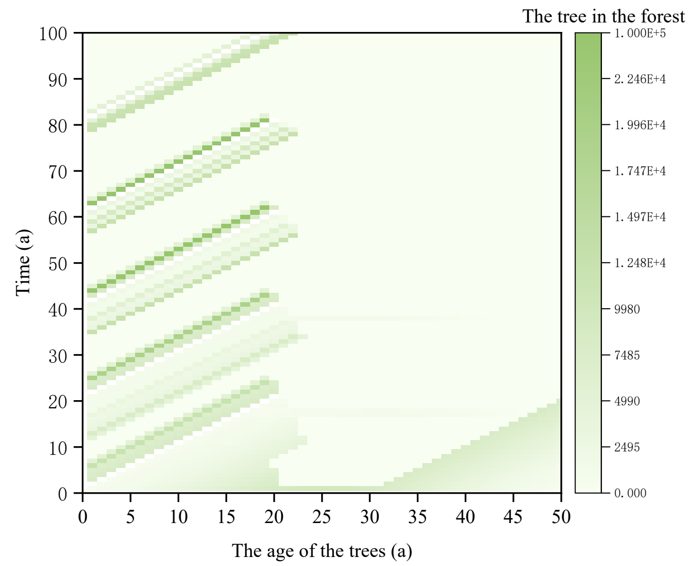


Figure 4. The number of tree living in the forest, the X axis is the age of the trees, the Y axis is the time.

Another interesting phenomenon is that all the trees harvested are used for construction material. Since building materials have the longest useful life and must meet the requirements of trees for carbon sequestration amount without considering the influence of other conditions, the products made are concentrated in building materials.

3. Conclusion

We can search for the annual carbon sequestration amount of different forests worldwide and use cluster analysis to find out how factors such as environment, policy and tree mixing affect the value of trees. And develop a prediction model with annual carbon sequestration and product values as output, which is convenient for researchers to use.

This paper found that, through the study of the management plan, the harvested trees were concentrated in the age range of 25-30 years, and with the change of forest parameters, the optimal harvesting plan also changed.

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