Study on forest carbon sink model

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Abstract. In order to reduce the content of CO2 in the atmosphere, a carbon sink model is proposed to alleviate the greenhouse crisis. The model takes carbon accounting, biomass, soil, effective forest area, products and bioenergy as indicators to analyze forest carbon sink. On this basis, provide a reasonable forest management plan. Finally, the stability of the model is determined by sensitivity analysis. The model provides guidance for forest carbon sequestration management.

Keywords: CO_2, Carbon sink model, Forest management plan, Sensitivity analysis.

1. Introduction

Climate change poses a great threat to life on earth. In order to mitigate the adverse effects of climate change, we need to take a series of actions to reduce greenhouse gases in the atmosphere. In addition to reducing greenhouse gas emissions, we also need to increase the carbon content of carbon pools outside the atmosphere, including physical carbon storage and biosphere carbon storage [1]. This process is called carbon sequestration.

Forests play an indispensable role in mitigating global climate change and providing products and ecosystem services required for human development, with a wide range of environmental, social and economic benefits. Because forests store carbon dioxide not only in living plants, but also in products produced by trees. These wood products absorb and store carbon dioxide during their life cycle. According to the characteristics of different wood products, the product life is different, and the life of some products is even longer than the tree itself [2]. In conclusion, carbon sequestration in some forest products, combined with carbon sequestration from young forest regeneration, may allow more carbon sequestration over time compared with the carbon sequestration benefits of no deforestation at all.

Therefore, there is an urgent need for an accurate model to analyze forest carbon sink and provide reasonable forest management plan to alleviate the greenhouse crisis.

2. CO_2 FIX model

According to the division of climate, the earth's temperature zone is divided into four categories: tropical zone, subtropical zone, temperate zone and boreal zone. Among them, there are rich types of tropical vegetation, with rain forest as the main forest type [3]. The main vegetation type in subtropical zone is evergreen broad-leaved forest. Affected by the temperate monsoon, deciduous broad-leaved forest is the main forest type in the temperate zone. Due to the cold climate, the boreal zone is dominated by coniferous forest and tundra.

See Table 1 and Table 2 for data arrangement.

Table 1 Current average standing biomass [4]

Biome	Current average dry matter content
Tropical forests	241
Temperate forests	113
Boreal forests	128
Tropical savannas	59
Temperate grasslands	14
Deserts	4
Tundra	13
Wetlands	86
Croplands	4

Table 2 Conversion factors from wood to carbon tonnage of different trees

Tree species	Characteristic parameter
The oak tree	0.330
Beech	0.340
Lrb.sp(1)	0.325
Srb.sp(2)	0.205
Spruce	0.215
Fir	0.205
Douglas fir	0.235
The pine tree	0.245
Larch	0.275

The carbon accounting module uses different carbon accounting methods to determine the impacts of selected circumstances [5].

The carbon stored in the system per unit area CT_t is:

$$CT_t = Cb_t + Cs_t + Cp_t \ (MgC/ha) \tag{1}$$

Where, Cb_t is the total carbon stored in living (above plus below-ground) biomass at any time t. Cs_t is the carbon stored in soil organic matter. Cp_t is the carbon stored in wood products.

Measure and calculate the product of biomass, carbon storage in soil organic matter and forest effective area, plus the carbon sequestration of forest products to obtain the total carbon sequestration CT, the formula is:

$$CT = (Cb_t + Cs_t) * S_e + Cp_t \left(\frac{MgC}{ha}\right) \tag{2}$$

Due to the different soil characteristics and biomass types, the carbon content of forests in different temperature zones per unit area is different. With the rise of temperature, the amount of carbon sequestration increases from the subtropical zone to the tropics.

Express the biomass growth of leaves, branches and roots of plants by fractions, and the rate relative to stem biomass is as follows:

$$B_i = F_i * B_s \ (MgC/ha) \tag{3}$$

Where, B_i is the growth of biomass. F_i is a relative biomass allocation coefficient. B_s is the growth of stem biomass.

The rate and decomposition rate of microorganisms entering litter depend on the sum of temperature and summer drought value. Through the microbial decomposition process, the following flow chart is obtained (Fig. 1):

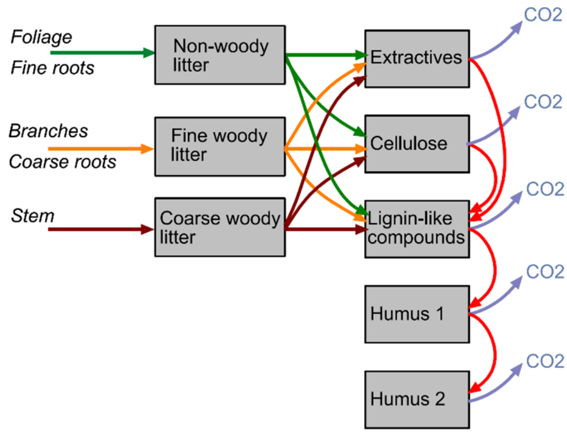

Fig. 1 Flow chart of the soil module [6]

Formula display of soil decomposition:

$$\frac{dx_i}{dt} = u_i - a_i * x_i \tag{4}$$

$$\frac{dx_j}{dt} = C_i a_i x_i + C_i a_i x_i + C_j a_i x_i - k_j x_i \tag{5}$$

Where, u_i is the input of type i garbage. x_i is the weight of organic carbon in wooden waste compartment i at time t. x_j is the weight of organic carbon in each decomposition chamber j at time t. a_i is the rate at which microorganisms invade litter i. C_{ij} is the concentration of compound j in type i litter. k_j the decomposition rate of compartment j. P_i the proportion of mass decomposed in compartment i transferred to a subsequent compartment.

If we want to find a way to manage forest, we must get information of the change of forestry area. Targeting to how to maximize carbon sequestration, we use a new definition: the effective forest area. It means the actual area of forests to absorb and stock carbon. Now we get effective forest area though two aspects: planting cycle and the growth rate, another is cutting cycle. Especially, we divide the first aspect into to two periods: growth and maturity. For the growth, the area of the tree that effectively absorb carbon dioxide changes over time. However, for the latter, the effective area will do not change. The effective forest depends on these cycles finally.

According to the annual effective forest area data of various countries, the global effective forest area data from 1999 to 2020 are calculated [7], and the following Fig. 2 is obtained.

The bioenergy module calculates the effects of using wood or wood waste for the generation of energy, and fossil fuels are represented as carbon equivalents because they are avoidable emissions and are added to the system's total inventory.

$$A = CT_t + Cbio_t \ (MgC/ha) \tag{6}$$

Where, A is total atmospheric effect. $Cbio_t$ is avoided emissions due to bioenergy use.

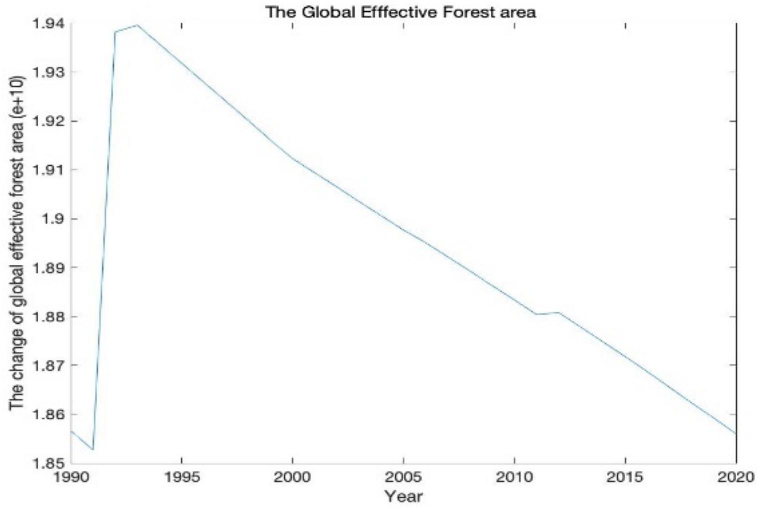

Fig. 2 Effective forest area

3. The Solution of CO2FIX model

Now, we calculate the effective area in the management of forest: Let the area of forest be S_0, and the number of trees of different ages obeys normal distribution. Then, the proportion of trees in growth is *P*:

$$P = \int_0^{t_1} \frac{1}{\sqrt{2\pi}} e^{\frac{-(S-\mu)^2}{2\sigma^2}} ds \,, 0 < \mathrm{S} < \mathrm{T}, \sigma > 0 \tag{7}$$

Where, t_1 is the growth cycle of trees. T is the oldest growth cycle of trees.

For any time t, the effective forest area is S_e:

$$S_e = S_0 \cdot \int_0^{t_1} \frac{1}{\sigma \cdot \sqrt{2\pi}} \cdot e^{\frac{-(S-\mu)^2}{2\sigma^2}} ds \tag{8}$$

Also, let the carbon dioxide absorption coefficient of forest per unit effective area is q every day. Then, in any time t that could stock the number of carbon is Q.

$$Q = \int_0^{t} S_e \cdot q \, d = S_e \cdot qt \tag{9}$$

We can find that for the model above, the effective area is constant. However, the area of the growth tree that effectively absorb carbon dioxide changes over time, this will get the same ending as enlarge the effective area. So, we let the area enlarge ΔS_e:

$$\Delta S_e = S_0 \left[\int_0^{t_1} \frac{1}{\sigma \cdot \sqrt{2\pi}} \cdot e^{\frac{-(S-\mu)^2}{2\sigma^2}} ds - \int_0^{T} \frac{1}{\sigma \cdot \sqrt{2\pi}} \cdot e^{\frac{-(S-\mu)^2}{2\sigma^2}} ds \right] \tag{10}$$

From Eq. (11), we get:

$$\Delta S_e = S_0 \int_T^{t_1} \frac{1}{\sigma \cdot \sqrt{2\pi}} \cdot e^{\frac{-(S-\mu)^2}{2\sigma^2}} ds \tag{11}$$

We can also regard ΔS_e as the area of deforested land in T equal to the effective area of planting trees in T. From what has been discussed above, we get the effective forest area is constant. So, they will use the constant rate to absorb the carbon in the air.

An example diagram of the biomass growth of leaves, branches and roots relative to the biomass growth of stems is obtained through calculation [8]. According to the data, the relationship between the natural mortality of trees and the age of trees is shown in the figure below (Fig. 3,4):

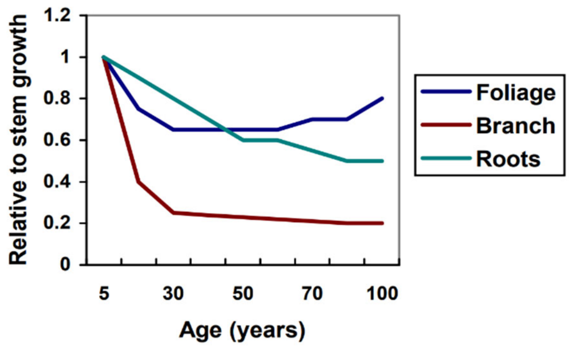

Fig. 3 Relative to stem growth

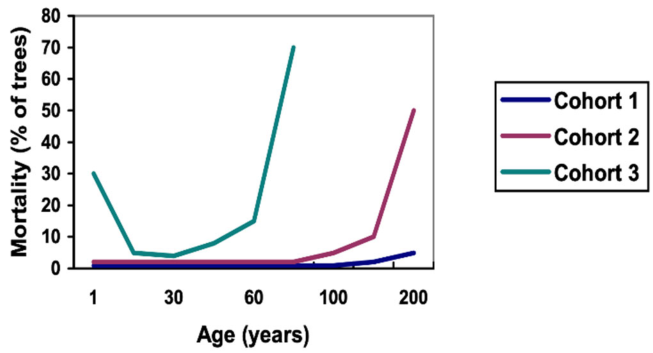

Fig. 4 Example as a function of age

It can be seen from the figure that the natural mortality of trees increases with the increase of the age of trees (Fig. 3,4).

The initial content of the soil module compartment can be determined manually or mechanically. After providing the quality information of chemical waste as a parameter, the standard value of temperature sensitivity of humus decomposition is 0.6. For conifer litter, the initial decomposition rate of soluble compounds is equal to 0.5 years and that of deciduous tree litter is equal to 0.8 years.

According to the product category, the landfill data can define a half live:

$$P_{t+1,k} = P_{tk} * \frac{1 - \ln 2}{L_k} \tag{12}$$

Where, P_{tk} is the amount of carbon in product category 'k' at time 't'. L_k is the half live for category 'k'. The following chart is obtained after data collection (Fig. 5):

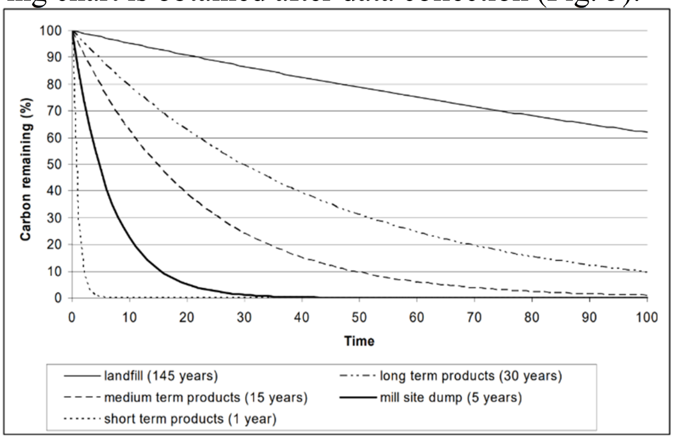

Fig. 5 Discarding curves of carbon in wood products

In this model, the carbon dioxide emission of bioenergy technology should always be stable and this value should be zero. The reason is that emissions are zero in a sustainable harvest cycle. At the same time, if it is an unsustainable harvest, net emissions will be reflected in the reduction of forest carbon stocks. Therefore, we can regard the accumulated greenhouse gases as the increased carbon emissions in forests.

The forest status of each ecological area is shown in Table 3.

Table 3 The situation of forests for each ecological zone

Ecological zone	Area (109 ha)	Carbon carrying capacity		Carbon sequestration potential (Pg C)
		Average (Mg C/ha)	Total (Pa C)	
Tropical	1.91	186±20	354.1±39.0	142.5
Subtropical	0.41	160±18	65.9±7.4	34.3
Temperate	0.63	136±13	85.4±7.9	53.7
Boreal	1.22	65±2	79.5±2.5	55.9
World	4.19	140±12	586.2±49.3	287.7

According to the model:

$$y = f(x) = \sum_{i=1}^{n} a_n x^n + b \tag{13}$$

Find the maximum value of y in the range of x∈[0,T]:

$$\max y = \sum_{i=1}^{n} a_n x^n + b \tag{14}$$

When *y* is the maximum value, the solution of *x* now is the effective forest area. At this time, the cutting area S_f in the above model can be expressed as:

$$S_f = S_O \int_0^{t_1} \frac{1}{\sigma \cdot \sqrt{2\pi}} \cdot e^{\frac{-(S-\mu)^2}{2\sigma^2}} ds - x_1 \tag{15}$$

The expression formula of planting area S_p is:

$$S_p = S_O \int_T^{t_1} \frac{1}{\sigma \cdot \sqrt{2\pi}} \cdot e^{\frac{-(S-\mu)^2}{2\sigma^2}} ds \tag{16}$$

Classified and discussed, and two situations are obtained:

$$\begin{cases} S_O \int_0^{t_1} \frac{1}{\sigma \cdot \sqrt{2\pi}} \cdot e^{\frac{-(S-\mu)^2}{2\sigma^2}} ds \le x_1 \\ S_O \int_0^{t_1} \frac{1}{\sigma \cdot \sqrt{2\pi}} \cdot e^{\frac{-(S-\mu)^2}{2\sigma^2}} ds > x_1 \end{cases} \tag{17}$$

When $S_O \int_0^{t_1} \frac{1}{\sigma \cdot \sqrt{2\pi}} \cdot e^{\frac{-(S-\mu)^2}{2\sigma^2}} ds \le x_1$, deforestation will reduce the effective area of forests and lead to the reduction of the comprehensive value of forests. Therefore, in this case, tree harvesting cannot be carried out.

This is the condition that the forest will not be cut down.

When $S_O \int_0^{t_1} \frac{1}{\sigma \cdot \sqrt{2\pi}} \cdot e^{\frac{-(S-\mu)^2}{2\sigma^2}} ds > x_1$, proper harvesting will not only not reduce the effective area of the forest, but also accelerate the speed of forest renewal, improve the carbon sequestration of the forest and its wood products [9], which is more conducive to the realization of carbon balance in the ecology.

To sum up, the model has a transition point, that is $S_O \int_0^{t_1} \frac{1}{\sigma \cdot \sqrt{2\pi}} \cdot e^{\frac{-(S-\mu)^2}{2\sigma^2}} ds = x_1$. At this transition point, the cutting area is zero.

4. Sensitivity Analysis [10,11]

4.1 Growth factor β

The growth rate of vegetation under the influence of objective factors such as climate, temperature and humidity.

According to the new scheme, calculate the planting area of each year respectively to obtain:

$$S_{p1} = S_0 \cdot \int_T^{t_1} \frac{1}{\sigma \cdot \sqrt{2\pi}} \cdot e^{\frac{-(S-\mu)^2}{2\sigma^2}} ds \tag{18}$$

$$S_{p2} = S_0 \cdot \int_T^{t_1} \frac{1}{\sigma \cdot \sqrt{2\pi}} \cdot e^{\frac{-(S-\mu)^2}{2\sigma^2}} ds\, \beta + S_{p1} \tag{19}$$

$$S_{p3} = S_0 \cdot \int_T^{t_1} \frac{1}{\sigma \cdot \sqrt{2\pi}} \cdot e^{\frac{-(S-\mu)^2}{2\sigma^2}} ds\, \beta^2 + S_{p2} + S_{p1} \tag{20}$$

According to the derivation, the expression of planting area in the nth year is：

$$S_{pn} = S_0 \cdot \int_T^{t_1} \frac{1}{\sigma \cdot \sqrt{2\pi}} \cdot e^{\frac{-(S-\mu)^2}{2\sigma^2}} ds\, \beta^{n-1} + S_{pn-1} + \cdots + S_{p2} + S_{p1} \tag{21}$$

According to the traditional scheme, the planting area of each year is calculated:

$$S_{p1}{}' = A \tag{22}$$
$$S_{p2}{}' = A\beta + S_{p1}{}' \tag{23}$$
$$S_{p3}{}' = A\beta^2 + S_{p2}{}' \tag{24}$$

According to the derivation, the expression of planting area in the nth year is：

$$S_{pn}{}' = A\beta^{n-1} + A\beta^{n-2} + \cdots + A\beta + A \tag{25}$$

4.2 Fitting factor ε

Definition: this factor is used to determine the fitting degree of the planting area of the new scheme and the old scheme.

Formula:

$$\varepsilon_i = \frac{S_{pi} - S_{pi}{}'}{S_{pi}{}'}, i = 1, 2, 3, \dots \tag{26}$$

5. Conclusion

(1) The soil and biomass modules are directly proportional to the effective area of the forest. But at the same time, if logging is not carried out for a long time, it will lead to the reduction of products and the carbon sequestration of products and bioenergy modules.

(2) In the process of tree growth, due to the increase of height and diameter, the reproduction of leaves and roots, the carbon sequestration of trees increases. However, when the trees are fully mature, the carbon sequestration tends to be stable. The mortality of trees is proportional to their age. In addition to microbial decomposition and carbon sequestration, inactive plants themselves will not carry out any biological activities, including carbon sequestration.

(3) In terms of logging, forest managers need to do two things. First of all, we should resolutely resist illegal logging, and relevant parties should also formulate rules and regulations to limit it. Secondly, according to the growth cycle of trees, determine the reasonable and effective planting cycle and cutting cycle.

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