Research on the Optimal Trading Strategy of Gold and Bitcoin
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Abstract. Market traders trade gold, and Bitcoin is aim to maximize their return. This paper utilizes
the grey prediction model to explore the optimal trading strategy and optimize fund allocation based
on dynamic programming. In addition, by comparing with other traditional trading strategies, we
discover that the grey prediction model can more accurately estimate future prices, enabling the
trader to gain steadily growing returns at a low-risk level.

Keywords: Grey Prediction Model; Trading Strategy; Fund Allocation; Dynamic Programming.

1. Introduction

Bitcoin, a kind of cryptocurrency with a limited number of 21 million, is regarded as a substitute
for gold when fighting inflation due to its decentralized mechanism and finite amount.

However, the average daily trading volume for Bitcoin in 2020 was estimated to be less than $2
billion, while gold reached more than $180 billion. Moreover, Bitcoin's average volatility over the
past 20 years was six times that of gold, resulting in a dramatically unstable market[1], which makes
it impossible to become a perfect replacement.

Considering the quantile cointegration relationship between the return of gold and Bitcoin[1], it is advisable to construct a portfolio including both gold and Bitcoin to hedge
against each other's price changes.

This paper is to establish a mathematical model to determine the optimal trading strategy and solve
the problem of fund allocation to achieve maximum total return. Based on the quantification, we
verify the effectiveness of our strategy.

2. Model Building and Analysis

All the trading data comes from London Bullion Market Association and NASDAQ, which ranges
from 2001 to 2021. Suppose we started our investment in 2016. We choose 2001 to 2012 as the back-
testing period and 2016 to 2021 as the forecasting period.

There is an essential assumption for simplification: only one trading price on the same day, which
means the opening price equals the closing price. Therefore, we only use the closing price data of
gold and Bitcoin.

2.1 Timing Strategy Model Based on Grey Prediction

The grey prediction is used to forecast an uncertain system. By accumulating the original data, we
obtain an increasing data series. Then a price prediction curve is created, by which we forecast future
prices and make deals by judging the difference between the actual and predicted price to earn profit.

Considering gold is valuable and widely used as a common currency, we assume the gold market
is efficient, and its market price can objectively reflect its actual value. Therefore, we rely on the
market price to make trading decisions: when the market price crosses the price prediction curve
upwards, gold is underestimated by the model, buy gold; on the contrary, sell gold.

Since Bitcoin is a virtual cryptocurrency, it does not have value itself. Born in 2009, the pricing
system of Bitcoin is not yet mature, so we suppose the Bitcoin market is inefficient. Therefore, we
depend on the grey prediction model to make trading decisions: when the market price is below the
price prediction curve, Bitcoin is underestimated by the whole market. buy Bitcoin; conversely, sell Bitcoin.

Furthermore, we predict future prices by constructing corresponding differential equations. The original sequence \( x^{(0)} \) is known as:
\[
x^{(0)} = \{x^{(0)}(1), x^{(0)}(2), ..., x^{(0)}(n)\}
\] (1)

After an accumulation, we get \( x^{(1)} \) as follows:
\[
x^{(1)} = \{x^{(1)}(1), x^{(1)}(2), ..., x^{(1)}(n)\}
\]
\[
= \{x^{(0)}(1), x^{(0)}(1) + x^{(0)}(2), ..., x^{(0)}(1) + x^{(0)}(2) + ... + x^{(0)}(n)\}
\] (2)

where
\[
x^{(1)}(k) = \sum_{i=1}^{k} x^{(1)}(i), k = 1,2, ..., n
\]

Average the formula (2), we get \( z^{(1)} \) as:
\[
z^{(1)} = \{z^{(1)}(1), z^{(1)}(2), ..., z^{(1)}(n)\}
\] (3)

where
\[
z^{(1)}(k) = \frac{1}{2} x^{(1)}(k) + \frac{1}{2} x^{(1)}(k - 1), k = 2,3, ..., n
\]

Establish a grey differential equation as follows:
\[
x^{(0)}(k) + a z^{(1)}(k) = b, k = 2,3, ..., n
\] (4)

Establish the corresponding Albino differential equation as follows:
\[
\frac{dx^{(1)}}{dt} + ax^{(1)}(t) = b
\]
\] (5)

Let \( u=[a, b]^T \), \( Y=[x^{(0)}(2), x^{(0)}(3), ..., x^{(0)}(n)]^T \), \( B=\begin{bmatrix} -z^{(1)}(2) & 1 \\ \vdots & \vdots \\ -z^{(1)}(n) & 1 \end{bmatrix} \), according to the least square method, by minimizing \( J(u) = (Y - Bu)^T(Y - Bu) \), we get the estimated value of \( \mu \):
\[
\hat{u} = [\hat{a}, \hat{b}]^T = (B^TB)^{-1}B^TY
\] (6)

Solve the Albino differential equation (5), we get:
\[
\hat{x}^{(1)}(k + 1) = \left( x^{(0)}(1) - \frac{\hat{b}}{\hat{a}} \right) e^{-\hat{a}k} + \frac{\hat{b}}{\hat{a}}, k = 0,1, ..., n
\] (7)

Therefore, the predicted prices can be further obtained through the model. The results of back-testing and forecasting are shown in Figure 1 and Figure 2, proving that the grey prediction model can fit the future price series.
Using the back-testing set, Figure 3 depicts the portfolio's total value in 2012. The return of gold and Bitcoin are respectively 40.13% and 3270.81%. It can be found that using our timing strategy can obtain steadily increasing income.

Using the forecasting set, Figure 4 illustrates the portfolio's total value in 2021. The return of gold and Bitcoin are respectively 81.48% and 44.17%. Besides, the maximum drawdown of gold is 0.0026, while VaR with a confidence level of 95% is 0.1434. The maximum drawdown of Bitcoin is 0.3896, while VaR with a confidence level of 95% is 457.4745, indicating that gold trading can get a higher return with lower risk, and Bitcoin trading is highly volatile.
To sum up, our model is applicable for predicting the future price of both gold and Bitcoin. Besides, using our strategies can help the trader earn excess profits.

2.2 Fund Allocation Model Based on Dynamic Programming

On the nth day of the transaction, the trader expects to maximize the value of his portfolio by buying and selling assets after the transaction. Since the closing price is unknown before the transaction is over that day, to avoid the look-ahead bias, we use closing price data from the past few trading days to predict the closing price today. Therefore, we set up the following objective function $f$:

$$
\max f = \bar{p}_n \times G_n + \bar{q}_n \times B_n - \bar{x}_n \times \bar{p}_n \times \alpha_x - \bar{y}_n \times \bar{q}_n \times \alpha_y + C_n \\
- \bar{G}_n \leq \bar{x}_n \leq \frac{C_n}{\bar{p}_n \times (1 + \alpha_x)} \\
- \bar{B}_n \leq \bar{y}_n \leq \frac{C_n}{\bar{q}_n \times (1 + \alpha_y)} \\
\bar{x}_n \times (1 + \alpha_x) + \bar{y}_n \times (1 + \alpha_y) \leq C_n \\
\bar{G}_n, \bar{B}_n, C_n \geq 0 \\
\bar{G}_0, \bar{B}_0 = 0, C_0 = 1000
$$

By adding restrictions:

$$
x_n = \bar{G}_n - \bar{G}_{n-1} \\
y_n = \bar{B}_n - \bar{B}_{n-1}
$$

At this time, objective function $f$ can be simplified to:

$$
\max f = \bar{p}_n \times (1 - \alpha_x) \times \bar{x}_n + \bar{q}_n \times (1 - \alpha_y) \times \bar{y}_n + \bar{p}_{n-1} \times \bar{G}_{n-1} + \bar{q}_{n-1} \times \bar{B}_{n-1} + C_n
$$

Let $G_{n-1}$ denote the amount of gold held on the last trading day, and $B_{n-1}$ denotes the amount of Bitcoin held on the last trading day. $G_{n-1}$ and $B_{n-1}$ are already known so that objective function $f$ can be simplified to:

$$
\max g(x_n, y_n) = \bar{p}_n \times (1 - \alpha_x) \times x_n + \bar{q}_n \times (1 - \alpha_y) \times y_n
$$

To be noted, GM(1,1) can calculate both $\bar{p}_n$ and $\bar{q}_n$, so the objective function $g$ is the function of $x_n$ and $y_n$. Therefore, we can conclude that $x_n$ and $y_n$ determine whether the trader can construct an optimal portfolio on the nth day of the transaction.

We manage the position size according to the absolute volatility of a single market to offset the impact of market volatility and position size on the value of the portfolio so that the probability of profit and loss for different transactions is the same.

Through iterative calculation, we find that the allocation of initial funds has little effect on the final value of the portfolio: when we trade gold and Bitcoin, there is no shortage of funds in their respective accounts, so the allocation will not affect the available return in the end.
Furthermore, only the extreme fund allocation weight will impact the portfolio. According to the
doctrine of risk diversification, rational investors will avoid setting extreme weight, so our strategy
is not sensitive to the initial fund allocation weight.

2.3 Comparison with Other Trading Strategies

2.3.1 Bollinger Channel and Like-Donchian Channel

Bollinger Channel [2] is a common indicator composed of a pressure line, a supporting line, and a
moving average line.

Like-Donchian Channel is comprised of an upper line, formed by the maximum closing price for
the past N days, and a lower line, formed by the minimum closing price for the past N days.

The trading strategy is formulated according to the relationship between the real price and the
upper, center, and lower lines of Bollinger Channel and Like-Donchian Channel: when the real price
is lower than the lower line of Bollinger Channel, we buy. As of the Like-Donchian Channel, buy
when the real price is higher than the upper line.

It is observed that when trading using this strategy, the fund allocation weight of gold and Bitcoin
will be too extreme to hedge the risk, resulting in tremendous loss. Therefore, it is not suitable to use
this strategy.
2.3.2 DMA Strategy

The DMA strategy judges the timing by comparing short-period moving average and long-period moving average. When the short one crosses the long one upwards, we buy.

Figure 7 demonstrates the portfolio value during the back-testing and the forecasting. Trading gold and Bitcoin using the DMA strategy both suffer a huge loss. Therefore, the DMA strategy is not a sensible trading option.

2.3.3 RSI Strategy

RSI [2] is the ratio of the sum of up points and down points in a certain period. RSI=50 is the dividing point between a strong market and a weak market. We set 10 as the threshold for buying and selling, then the timing for buying and selling is: when RSI>60, we sell; when RSI<40, we buy.

Gold's value increased steadily during the back-testing, while Bitcoin's value showed great volatility and finally suffered a loss. When it comes to forecasting, both Bitcoin and gold will suffer losses when using the RSI strategy. Therefore, the RSI strategy is not advisable for trading.
Figure 9: RSI: Back-testing and Forecasting

From the analysis above, it can be concluded that the grey prediction model can more accurately estimate the future price, achieving a greater return with less risk.

At the same time, other strategies are not universal to apply. Using them for trading may cause volatile gains or varying degrees of losses. In summary, the grey prediction model is the optimal trading model.

3. Conclusion

This paper predicts the future price based on the grey prediction model by using prior price data of gold and Bitcoin. The timing strategy is adopted to maximize the total returns according to the difference between the predicted price and the real price.

Furthermore, we also find that the allocation of initial funds has little effect on the final value of our portfolio. In contrast with other traditional trading strategies, the grey prediction model is highly accurate for forecasting future prices and is universal to apply, which implies that our timing strategy is scientific.

In the future, more advanced techniques can be used to predict price series, such as the time series analysis model, BP neural network model, and LSTM neural network model. In addition, since we only use prior closing prices during our prediction, we expect to utilize more price information, including opening prices, highest prices, etc, which is used to construct more accurate technical indicators and achieve more precise prediction results.

References

[1] [https://www.coinhills.com/]