

Follow the Best Strategy to be a Smarter Trader

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Abstract. Market traders often buy and sell volatile assets to maximize total returns. We have developed an optimal trading strategy model using gold and bitcoin daily price streams to meet this need. Based on a sliding window, we use the ARIMA model to predict the daily prices of gold and bitcoin, respectively. Meanwhile, the Granger causality test results showed that they were not cointegrated in the short term. We construct a multi-objective dynamic trading sub-model that considers maximum investment value, minimum trading frequency, and minimum trading risk factors based on historical and forecast sequences. Finally, we validate the optimal trading strategy model using the efficient frontier and conclude that the model is accurate.

Keywords: Optimal trading strategy; Efficient frontier; ARIMA; Multi-Objective Dynamic Trading; Sharp Ratio.

1. Introduction

Facing the complex investment market, it has always been a hot topic to explore the optimal investment plan by comprehensively weighing the risks and the benefits.

Portfolio Theory established by Nobel Prize winner Markowitz[1] is widely used in portfolio selection and asset allocation. Moreover, with rich results, Modern Portfolio Theory (MPT) has laid the foundation of research.

In terms of forecasting, Kastampa[3] and Chu [4] et al. explore bitcoin's financial asset capabilities by enabling the GARCH model, and the results show that bitcoin is a valuable tool for portfolio management, risk analysis, and market sentiment analysis.

Until now, in gold and bitcoin portfolio research, few scholars have combined historical and forecast sequences with optimal target planning to obtain optimal trading decisions. Therefore, based on the excellent results of predecessors, we started our research.

2. Model Overview

Our optimal trading strategy model, called the **Multi-Objective Dynamic Trading Model**, aims to obtain the best trading decisions of the day. The model combines time series with an optimization algorithm. It first constructs a series of submodels:

The article constructs A Sliding-window-based Price Prediction Sub-model to make short-term predictions of daily gold prices and bitcoin daily prices in terms of time series.

In terms of multi-objective optimization, we have constructed a **Multi-objective Daily Trading Strategy Sub-model**: Based on the above time series analysis results, we start from the dimension of historical information, transaction dimension, transaction frequency dimension, and transaction risk dimension, and comprehensively consider investment value and transaction frequency, transaction risk and other multiple objectives to build equations.

3. Price prediction sub-model based on sliding window

We set the historical sliding window size to 5, the future sliding window to 1, and the moving step to 1. Then, we fit the appropriate ARIMA models separately to the daily price of gold and the daily price of Bitcoin. Finally, we use the ARIMA models to make predictions on a sliding window.

3.1 ARIMA Prediction Model for Gold daily prices g_t

1. Gold daily price g_t related inspections and operations

After the stationarity test, the output P-value of the golden daily price series g_t is a non-stationary model, so the first-order difference is performed.

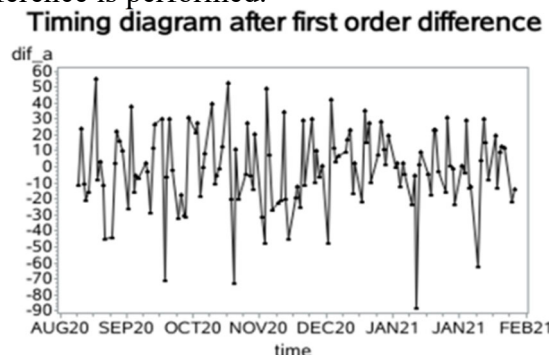


Figure 1 The daily price of gold dif g_t

Figure 2 shows the first-order difference sequence of the price of the gold day g_t . After the pure randomness test and stationarity test, the gold daily price after the difference is a stationary non-white noise sequence, and the next step is to establish an ARIMA model for the difference.

2. Determine the optimal forecasting model g_t .

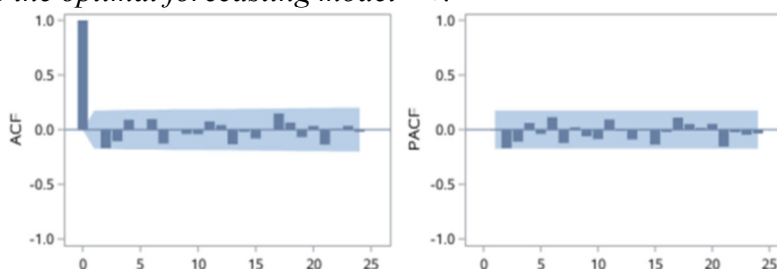


Figure 2 ACF and PADF of the dif g_t

It can be seen from the ACF and PACF diagrams that the partial autocorrelation graph of dif g_t has the first-order tailing feature, and the autocorrelation graph has the truncation feature; The BIC statistic corresponding to ARIMA (0,1,1) is only 6.357109; Therefore, we can judge that: The optimal prediction model for the price of the golden day g_t obeys ARIMA (0,1,1).

3.2 Random Walk Model for Bitcoin Daily Prices β_t

1. Stationarity and Pure Randomness Tests of β_t

The P-value of the PP test is less than 0.05, so the first-order difference series of Bitcoin daily prices are stationary. The P-value of the pure randomness test is greater than 0.05, so the first-order difference sequence dif β_t of the daily price of Bitcoin is a stationary sequence. Therefore, dif β_t can be considered as a white noise sequence.

2. Bitcoin daily price β_t random walk model

Since the first-order difference sequence dif β_t of the daily price of Bitcoin is a stationary white noise sequence, we build a random walk model β_t .

$$\begin{cases} \beta_t = \beta_{t-1} + \varepsilon_t \\ E(\varepsilon_t) = 0, Var(\varepsilon_t) = \sigma_\varepsilon^2, E(\varepsilon_t \varepsilon_s) = 0, s \neq t \\ E(\beta_t \varepsilon_t) = 0, s < t \end{cases}$$

Among them β_t is the daily price of bitcoin in the current period β_t is the daily price of bitcoin in the previous period. Traditional economists generally believe that the short-term trend of Bitcoin price is similar to the random walk model^[6]. Our results are consistent with this conclusion.

3.3 The relationship between g_t and β_t

Gold daily price g_t obeys ARIMA (0,1,1), Bitcoin daily price β_t obeys Random Walk. After performing the Granger Causality Test, we found no significant cointegration relationship.

4. Multi-objective Dynamic Trading Model

As mentioned above, the price prediction model can reasonably predict the fluctuations of gold and Bitcoin based on the sliding window.

We combine historical data and forecast data to determine constraints based on historical information dimensions, transaction dimensions, transaction frequency dimensions, and transaction risk dimensions.

This sub-model builds a multi-objective dynamic trading model with the goals of maximum investment value, minimum trading frequency $trade_times$, and minimum trading risk coefficient risk.

4.1 Multi-objective Daily Trading Strategy Sub-model

Step 1: Historical Information Dimensions

Build k_g to refer to the price fluctuation data of the last k_g days before buying or selling gold. k_β Refers to the price fluctuation data of the last k_β days before buying or selling Bitcoin and does not make any trading operations before k_g or k_β during days. The reference days k_g k_β are both more than two days.

Step 2: Transaction Dimension

In the multi-target daily trading strategy model, gold and bitcoin are allowed to buy and sell at different times, and the ratio of buying and selling gold and bitcoin is not the same.

Step 3: Transaction Frequency Dimension

- The Multi-objective Daily Trading Strategy Sub-model considers the number of transactions dimension. Construct $trade_times$ as trade frequency. The trading frequency depends on the threshold rate for reducing and increasing the rate of the position. The construction rate is the threshold value of the percentage increase or decrease. Taking gold as an example, the historical change ratio of the gold price is:

$$rate_g = \frac{\{g_t\} - \{g_{t-1}\}}{\{g_{t-1}\}} \times 100\%$$

$$\begin{cases} \text{Buy gold, if } rate_g > 0 \& rate_g > rate \\ \text{Sell gold, if } rate_g < 0 \& -rate_g > rate \end{cases}$$

- The lower the percentage change threshold $rate$, the smaller the trading frequency $trade_times$.

$$trade_times = \frac{\theta}{rate} \quad (\theta \text{ is taken as } 0.1)$$

Step 4: Transaction Risk Dimensions

The Multi-target Daily Trading Strategy Model constructs risk as a trading risk factor based on historical and future price ratios.

Step 5: Cross-validation

We use the cross-validation method to determine the optimal range of reference days.

Step 6: The multi-target planning equation of the model

$$\begin{cases} \text{Max}\{value\} \\ \text{Min}\{trade_times\} \\ \text{Min}\{risk\} \end{cases}$$

$$s.t. \begin{cases} trade_times = \frac{0.1}{rate} \\ risk = rate \times rate^{pre} \left(\frac{\omega_1 - \omega_2}{rate} \right) \\ k_\alpha \in [50, 60], k_\beta \in [35, 45] \\ rate \in [0, 1] \\ \omega_i \in [0, 1] \quad (i=1, 2, 3, 4) \\ \text{New Function value}' \end{cases}$$

k_α refers to the price fluctuation data of the last k_α days before buying or selling gold. k_β Refers to the price fluctuation data of the last k_β days before buying or selling Bitcoin. w_1 and w_3 are the ratios of buying and selling gold currently held. Then, w_2 and w_4 are the ratios of current bitcoin holdings for buying and selling. New Function value' is the optimization function of the model. It will iterate based on constraints, finally, output portfolio and investment value.

Step 7: Solve using the PSO algorithm

Aiming at the multi-target dynamic trading model of the Multi-target Daily Trading Strategy Model, this paper constructs the PSO algorithm to solve the optimal trading strategy [7]. The target value of the particle swarm algorithm locates the planning target of the Multi-target Daily Trading Strategy Model, and the particles satisfy the constraints.

PSO is initialized as a group of random particles (random solutions) and then iteratively finds the optimal solution. Each iteration updates itself by tracking two extreme values; one is the optimal solution found by the particle itself p_{best}^k , and the other is the optimal global solution p_{best} .

Define n particles p_i :

$$p_i = [rate_i, risk_i, k_{i\alpha}, k_{i\beta}, rate_i, \omega_{i1}, \omega_{i2}, \omega_{i3}, \omega_{i4}]$$

Subject to constraints:

$$\begin{cases} rate_i, risk_i \in (0, 1) \\ k_{i\alpha}, k_{i\beta} \geq 2 \\ \omega_{i1}, \omega_{i2}, \omega_{i3}, \omega_{i4} > 0 \end{cases}$$

The speed of particle iteration:

$$v(i) = c1 * r1 * (p_{ibest}^k - p_i) + c2 * r2 * (p_{best} - p_i) + w * v(i-1) * \Phi$$

The particle change is: $p_{i+1} = p_i + v(i)$

Learning Factor for Particle Iteration $c1 = c2 = 2.05$. $r1, r2$ is a random constant at (0,1). The contraction factor is $\Phi = \frac{2}{|(2-C-\sqrt{C^2-4C})|}$ ($C = c1 + c2 = 4.1$); w is the inertia weight. Indicates the ability of the particle to maintain its original state. p_{ibest}^k is the optimal solution for the i-th particle at the k-th iteration. p_{best} is the optimal solution obtained by all particles until the kth iteration. In this paper, 10 scatter points are established, and the number of iterations is set to 10,00^[10].

We also set the multi-objective weighting to a single objective. Standardizing the value, trade_times, risk. The planning objectives are as follows:

$$Max\{value' - trade_times' - risk'\}$$

We use the above PSO algorithm to output the dynamic trading strategy of the Multi-objective dynamic trading model, including investment portfolio, the time point of each transaction, etc.

4.2 Efficient frontier proving the optimal model solution

Rational investors tend to dislike risk and prefer returns. Portfolios that meet these demands are the effective frontier. Effective frontier based on the Mean-Variance Method and the effective boundary model, answering the key question of asset allocation — how to construct an optimal portfolio.

As shown, Bitcoin and gold fluctuate much more than cash and correspondingly assume higher risk, which is an important risk and return trade-off in portfolio theory.

We will calculate the mean and variance of different investment ratios based on the yield of each kind of risk asset, volatility, and covariance. Then using Monte Carlo Simulation Draw, a scatter plot of multiple Portfolio under different asset ratios of distribution and then obtain effective boundary. Last, according to the Sharp Ratio, get the least risk and the least benefit investment scheme.

Based on the large number theorem, set n days ($n \geq 30$), and the data under this time series is divided into Training Set $\{a_1, a_2, \dots, a_3\}$ and Test Set $\{b_1, b_2, \dots, b_3\}$. Based on the higher-order time-series algorithm, predicted sequences were obtained with Training Set $\{b_1^{pre}, b_2^{pre}, \dots, b_n^{pre}\}$.

The difference method is used to obtain the daily return of each investment product, $R_i = \frac{b_i^{pre} - b_i}{b_i}$ and part of the result is shown in the figure below.

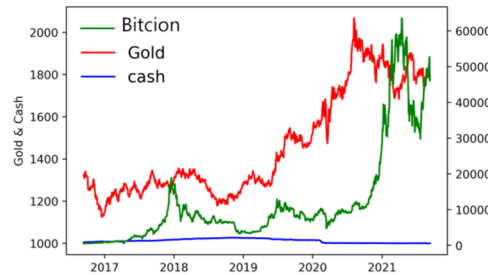


Figure 3 Markowitz Mean-Variance Method

There are three investment products: cash, gold, and Bitcoin. Its random yield is the investment ratio is $\varpi_1, \varpi_2, \varpi_3$ respectively. If the initial capital of the investor is W_0 , the returns of the three assets are $W_0\varpi_1R_1, W_0\varpi_2R_2, W_0\varpi_3R_3$ and the total return is $W_0\varpi_1R_1 + W_0\varpi_2R_2 + W_0\varpi_3R_3$. Sum the weighted returns of all stocks are summed, and we can obtain the Expected Return R_p and Expected Volatility σ_p of the Portfolio:

$$R_p = \varpi_1R_1 + \varpi_2R_2 + \varpi_3R_3$$

It is not difficult to randomly take a set of yield combinations, making it possible to find the corresponding value in the optimized trading strategy model.

The expected return of the three assets is $\varpi_1E(R_1), W_0\varpi_2R_2, W_0\varpi_3R_3$ Moreover, portfolio-expected yield is:

$$E(R_p) = \varpi_1E(R_1) + \varpi_2E(R_2) + \varpi_3E(R_3) \quad (1)$$

The variance of the portfolio yield is:

$$\sigma^2(R_i) = \sigma^2\left(\sum_{i=1}^3 \varpi_i R_i\right) = \sum_{i=1}^3 \varpi_i^2 \sigma^2(R_i) + \sum_{i \neq j} \varpi_i \varpi_j \sigma(R_i, R_j)$$

According to Markowitz Mean-Variance Method, the decision faced by investors can be expressed as:

$$\begin{aligned} \min_{\varpi_i} \sigma^2(R_p) &= \sum_{i=1}^3 \varpi_i^2 \sigma^2(R_i) + \sum_{i \neq j} \varpi_i \varpi_j \sigma(R_i, R_j) \\ s.t. \bar{R}_p &= R_f + \sum_{i=1}^3 \varpi_i [E(R_i) - R_f] \end{aligned} \quad (2)$$

Sharp Ratio Determines the Optimal Portfolio

We want to find a balance point between income and risk. The Sharp ratio variable helps make it, calculating the excess return per unit of risk.

The Sharp ratio (or Sharpe Ratio) was proposed by Nobel Prize-winner William Sharp, which aims to help investors compare returns and risks to their Portfolio. Rational investors generally fix the tolerable risk, pursue the maximum return, or chase after the minimum risk at the fixed expected return. So the Sharp ratio calculates the excess return for each total risk of one unit. The calculation formula is as follows:

$$\begin{aligned} E(R_p) &= R_F + \frac{E(R_M) - R_F}{\sigma_m} \sigma_p \\ \sigma_p &= \sqrt{\varpi_1^2 \sigma_1^2 + \varpi_2^2 \sigma_2^2 + \varpi_3^2 \sigma_3^2 + 2\varpi_1 \varpi_2 \sigma_{1,2} + 2\varpi_1 \varpi_3 \sigma_{1,3} + 2\varpi_2 \varpi_3 \sigma_{2,3}} \end{aligned}$$

R_p : desired percentage return; R_f : risk-free rate of interest; σ_p : standard deviation of the excess returns.

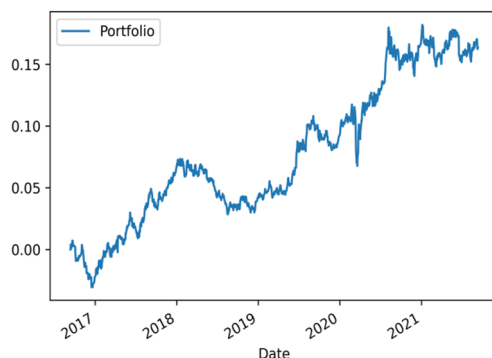


Figure 4 Fixed-weight Portfolio Cumulative Return Curve

Molecules calculated the difference by comparing an investment to a benchmark representing the entire investment category. Denominator standard deviation represents earnings volatility, corresponding to risk because greater volatility indicates high risk. Get a Sharp Ratio measuring the excess return and risk simply by dividing the mean of the excess return by its standard deviation. In addition, multiply the number of trading days to get an annualized Sharp Ratio.

At a given yield, the red star point has the optimal Sharp index, indicating the optimal Portfolio. In the pursuit of high returns, the Portfolio achieves maximum risk control, and the composition of the optimal Portfolio is $[\varpi_1, \varpi_2, \varpi_3] = [0.998, 0.002, 0]$ All cash is used for investment, gold for 62.5% of total assets, and bitcoin for 37.5%.

Blue star points have the lowest Sharp index, indicating the absolute minimum variance portfolio. With the low expected return of this Portfolio and the low expected volatility, this minimized investment risk is the most conservative Portfolio. Based on the given yield, the composition of the optimal Portfolio is $[\varpi_1, \varpi_2, \varpi_3] = [0.998, 0.002, 0]$, that is gold for 0.02% of total assets and cash for 99.8% of total assets.

Each day corresponds to a determined yield combination. The given Portfolio finds the corresponding date in the daily yield table and the daily asset holding amount solved by the Optimal Trading Strategy Model. Through testing, the investment ratio portfolio falls on the effective frontier between the red star points and the blue star points, and it proves that the Optimal Trading Strategy Model optimal portfolio model provides the optimal strategy.

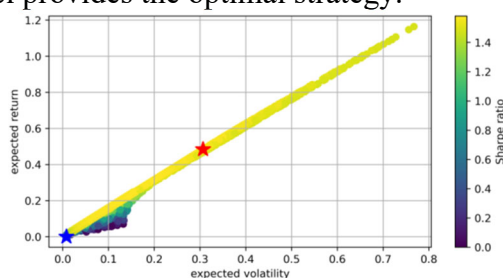


Figure 6 Efficient Frontier

5. Summary

When researching optimal investment strategies for cash, gold, and bitcoin, we forecast the daily prices of gold and bitcoin through a sliding window. Combining historical and forecast information, we consider investment problems from multiple dimensions and use multi-objective programming in mathematics to obtain optimal investment portfolios.

To fully demonstrate the accuracy of our optimal trading model, we combine multi-objective programming with an efficient frontier in the financial domain. It turns out that our optimal trading strategy model, which combines forecasting with multi-objective planning, is effective when investing.

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