Portfolio Risk Investment Strategy Model based on ARIMA and LSTM

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Abstract. Market trading is often fraught with uncertainty. We build a model to predict the future market movements of gold and bitcoin prices based on historical data. We also design a reasonable trading strategy that achieves the expected returns. First, we build a market forecasting model based on ARIMA and LSTM neural network to obtain a set of price change series that can reflect both the future trend and observe the recent increase to support the design of a market trading strategy. Next, we designed a portfolio investment model that combines return and risk. Combining the commission at the time of trading with the return on investment avoids the impact of considering commissions at the time of decision making. Each held product is decided separately in the investment decision, and the weighting of each investment is adjusted by planning the weighting of each investment with the expected risk. The acceptance of different risks, the R-value of ARIMA prediction results, and the stage evaluation of LSTM are integrated to adjust the weighting of risks, which finally gives three different risky portfolios of low, medium, and high.

Keywords: ARIMA, LSTM, Price Estimation, Investment Strategy, Trading Risk Minimization.

1. Introduction

As Stanley Crowe said, to make profits as big as possible and losses as small as possible, we need to do our best to build an effective trading strategy.

We need to use only the price data to forecast its subsequent trend and build a portfolio investment planning model to balance risk and return under fixed inputs and seek maximum returns under different risk levels.

2. Model Establishment and Solution

2.1 Market Price Forecast Model

2.1.1. Short-term Value Volatility Prediction Based on ARIMA Model

This section builds ARIMA models and LSTM neural networks to predict the subsequent price trends for gold and bitcoin, respectively, considering their respective characteristics. We provide a decision basis for the subsequent portfolio investment model.

ARIMA model is an improved algorithm of ARMA. If the original data does not satisfy the smoothness and white noise test after the original time series is transformed into ARMA (*p, q*) series by the difference method, the ARMA algorithm is continued to be used for forecasting.

The ARIMA model has three main parameters, *p, d, q*, where *d* denotes the number of differences of the series, *p* denotes the order of the autoregressive coefficient polynomial of the series, and *q* denotes the order of the moving average coefficient polynomial, generally abbreviated as *ARIMA* (*p, d, q*), and its structure is as follows.

$$\begin{cases} \Phi(B)\nabla^d x_t = \Theta(B)\varepsilon_t \\ E(\varepsilon_t) = 0, \mathrm{Var}(\varepsilon_t) = \sigma_\varepsilon^2, E(\varepsilon_t\varepsilon_s) = 0, s \neq t \\ Ex_s\varepsilon_t = 0, \forall s < t \end{cases} \tag{1}$$

where ε_t is the zero mean and the variance is the smooth white noise of σ_ε^2. The d-order difference is generally divided into ∇dXt = (1-B)dXt, ∇and *d* becomes the *d*-order difference operator.

B is the back-shift operator, which is defined as

$$BX_t \equiv X_{t-1},\ B^k X_t \equiv X_{t-k} \tag{2}$$

$\Phi(B)$ is the *p*th-order autoregressive coefficient polynomial and $\Theta(B)$ is the *q*th-order moving average coefficient polynomial.

For the operator polynomials $\varphi(B)$, $\theta(B)$, the following additional assumptions are usually made.

$$\begin{cases} \Phi(B) = 1 - \phi_1 B - \phi_2 B^2 - \cdots - \phi_p B^p \\ \Theta(B) = 1 - \theta_1 B - \theta_2 B^2 - \cdots - \theta_q B^q \end{cases} \tag{3}$$

i. φ(B) and Θ(B) have no common factor and $\varphi_p \neq 0$, $\Theta_q \neq 0$.

ii. The roots of $\varphi(B)=0$ are all outside the unit circle, and this condition becomes the smoothness condition of the model.

iii. The roots of $\Theta(B)=0$ are outside the cell circle, which becomes the inverse condition of the model.

Considering the trading price of the day *i* before the investment on day n as a stochastic time series *X*, the weak smoothness condition needs to be satisfied before using the ARMA model. Plot the time series of the sequence *X*.

Figure 1 will give the time series plot of bitcoin and gold. It is easy to see that both are not smooth sequences and require some stabilization.

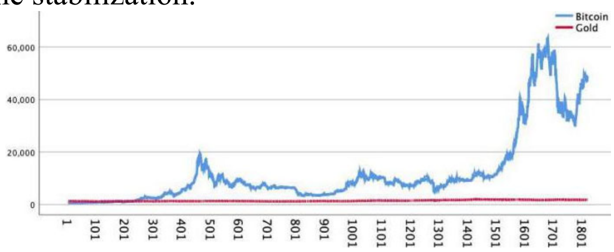

Figure 1 Time Series Plot of Bitcoin and Gold

The values in the series are logarithmicized by the following equation to achieve the initial smoothing.

$$X_t = \ln(x_t - \bar{x}) \tag{4}$$

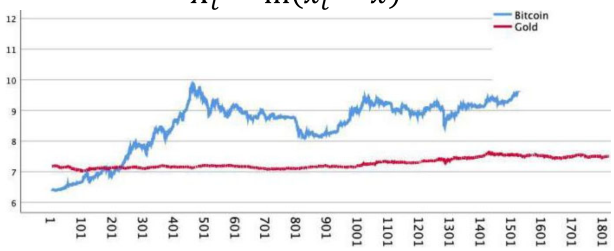

Figure 2 The Result after Smoothening Treatment

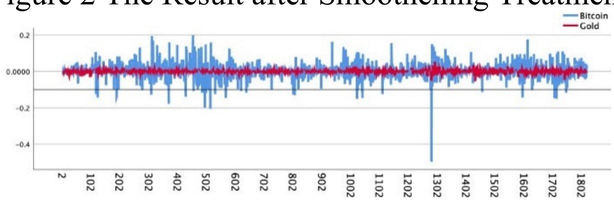

Figure 3 The Result after Smoothening Treatment

Observing the above Figure 2 and Figure 3, it can be seen that after the logarithmic processing, the maximum offset of the outlier points shows signs of substantial weakening but still fails the smoothness test.

According to the results shown above, it is easy to see that the ARIMA model has high stability for short-term time series forecasting. However, it cannot predict long-term trends, so it still needs to be combined with other models to achieve control of future trends.

Find the individual ACFs and PACFs and plot them against the number of sequences. *p, q* can be found in Figure 4 and Figure 5. So, we end up with ARIMA model parameters for Bitcoin as *ARIMA* (1, 1, 2), and ARIMA model parameters for gold as *ARIMA* (0, 1, 0).

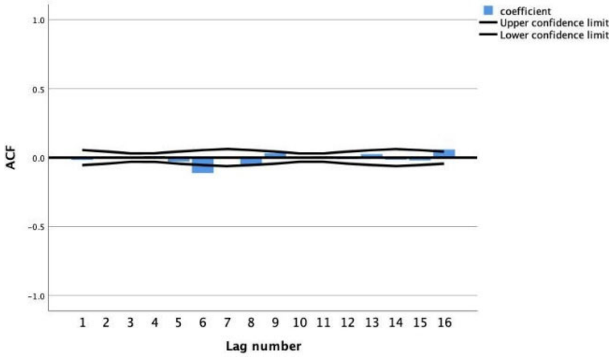

Figure 4 ACF

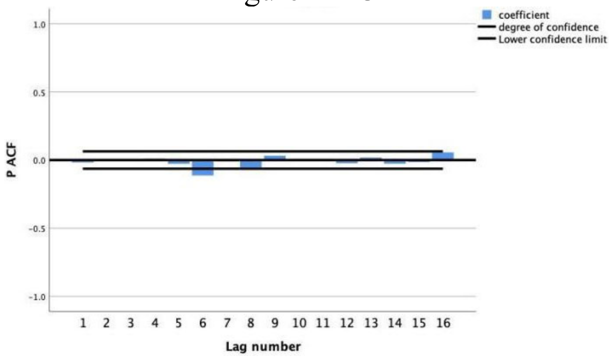

Figure 5 PACF

2.1.2. Long-term Value Trend Prediction Based on LSTM Neural Network

We construct LSTM neural networks, which combines the bidirectional propagation and memory learning. We can achieve accurate pre-training within a small amount of data by extracting features from historical data and repetitive training, accurately predicting long-term price trends within a small amount of data, and dynamically adjusting the prediction results. The results were analyzed. After analyzing the results, it is found that the LSTM neural network can accurately predict the long-term price trend network can accurately determine the bull and bear market of bitcoin and Gold bull and bear markets. It can help us to more accurately set the risk-return ratio to be considered when investing based on the market conditions at the time of risk-return ratio at the time of the investment to determine the optimal portfolio. The correlation coefficient of the predicted value can reach over 0.99. However, it also suffers from a large lag.

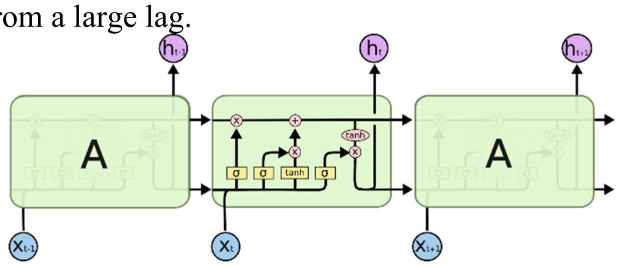

Figure 5 The principle of LSTM

Generally speaking, Recurrent Neural Network (RNN) is a neural network for processing sequential data. Compared with the general neural network, it can handle data with sequence changes.

The Long-short-term memory (LSTM) used in this model is a special kind of RNN, which is mainly used to solve the problem of gradient disappearance and gradient explosion in long sequence training. The principle of LSTM is shown in Figure 5.

We can build the LSTM neural network time series, forecasting model based on the above principles. x^t is the measured value over the five-year trading period, and y^t is the predicted value over the five-year trading period.

$$\text{Matrix}(r, N) = \begin{bmatrix} a_1 & a_2 & \cdots & a_n \\ a_2 & a_3 & \cdots & a_{n+1} \\ \vdots & \vdots & & \vdots \\ a_r & a_{r+1} & \cdots & a_{r+n} \end{bmatrix} \tag{5}$$

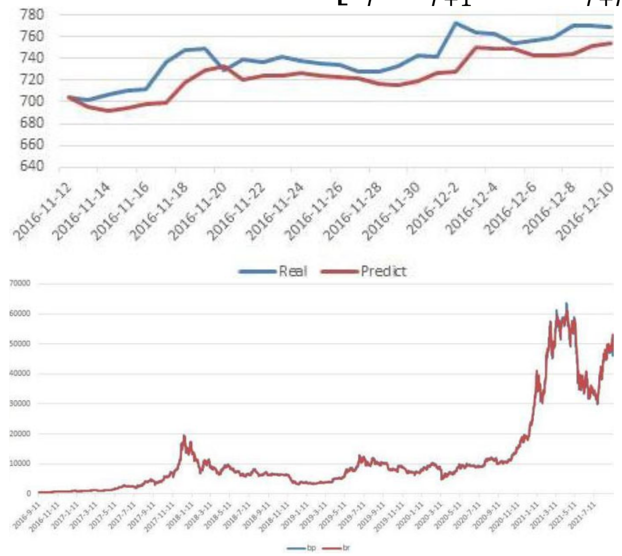

Figure 6 Partial LSTM predict bitcoin price

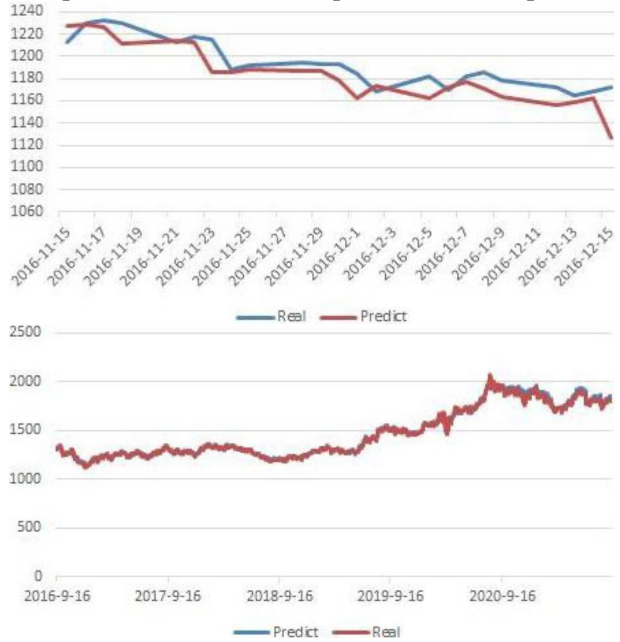

Figure 7 Partial LSTM predict the gold price

According to the results in Figure 6 and Figure 7, it is easy to see that the ARIMA model has high stability for short-term time series forecasting. However, it cannot predict long-term trends, so it still needs to be combined with other models to achieve control of future trends.

2.2 Portfolio Investment Model

2.2.1. Trading Return

We need to use the known data before the investment to predict the post-investment price trend of bitcoin and gold and develop the best investment strategy based on that, so that strategy is our objective function.

Suppose we have already given a convincing forecast of the future price trend using the ARIMA model with LSTM neural network (which will be presented separately in the subsequent sub-models). We build a continuous programming model based on time series and introduce a Sharpe coefficient for risk assessment based on this forecast. Depending on the risk investment preferences, we give different investment strategies.

Use C_n, B_n, G_n to denote the holdings of USD, Bitcoin, and gold on day n, respectively. Then, we use α to denote the commission generated by the exchange.

$$\alpha_n = [\alpha_C, \alpha_B, \alpha_G] \tag{6}$$

Where commission for cash α_C is constantly equal to 1%, the commission for bitcoin transactions α_B is constantly equal to 2%, and the commission for gold transactions α_G is 1% on trading days. No trading is allowed on non-trading days, which can be made equal to 100% in the model.

Use P_n to denote the trading price of various investment products on day n and P_{n+1} to denote the predicted price value on day n+1. Then it is possible to calculate the expected return on investment on the second day of investment R'_{n+1}, which is

$$R'_{n+1} = P'_{n+1} - P_n/P_n \tag{7}$$

It is assumed that all products can be traded normally on day n and day n+1.

Then for any product, there are three investment products to choose from on day n (here, the US dollar is considered as one of the investment products), and since there is a commission on the transaction, a new functional relationship can be constructed between the rate of return and the commission to describe the actual return on the investment. It may be expressed in terms of R:

$$R_G = (1 - \alpha_B)\,(1 - \alpha_G)\,R'_G \tag{8}$$

Take bitcoin as an example, as shown in Figure 8. G, B, C represent gold, bitcoin, and US dollar, respectively, W_i denotes the proportion of product i to the amount invested in each product, and

$$W_{ic} + W_{ib} + W_{ig} = 1 \tag{9}$$

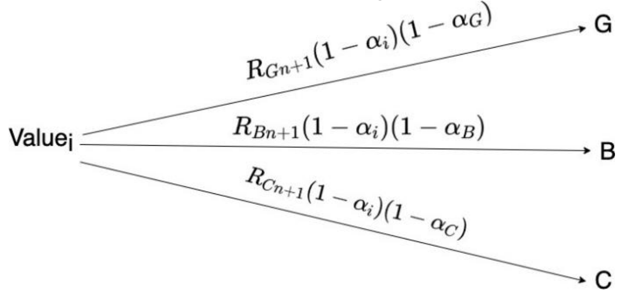

Figure 8 Three types of decisions for investment product

Define Was the weighted average operator.

$$WD = (W_C \cdot D + W_B \cdot D + W_G \cdot D) \tag{10}$$

Therefore, the total expected return at day n+1 after the investment is

$$\overline{R_B} = W(R_G + R_B + R_C) \tag{11}$$

2.2.2. Investment Risk

In addition to returns, we cannot ignore the risks involved in investing. Usually, we are used to applying the volatility of recent returns to reflect the magnitude of risk, and we can quantify this risk through the variance of investment returns soon. In fact, due to the uncertainty of stock market fluctuations, we also use the idea of grouping when we use the ARIMA model to predict the subsequent price direction.

Based on the different volatility of gold and bitcoin, we choose to use the data from fourteen days before the decision to predict the price direction of gold in the next three days and use the data from seven days before the decision to predict the price direction of bitcoin in the next two days, using the LSTM neural network to predict the approximate trend in the medium to long term future based on all past data.

Thus, for the nth day of the decision, we can use the mathematical expectation of the data from R_{n-14} to R_n to measure the expected return of investing in gold.

$$E(R) = \frac{1}{14-1}\sum_{i=0}^{13} R_{n-i} \tag{12}$$

Introducing the symbol D to denote the variance operator, the variance of the current day's ROI can be expressed as:

$$\sigma^2 = D(W_B\overline{R_B} + W_G\overline{R_G} + W_c\overline{R_c}) = \sum_{i=1}^{3}\sum_{j=1}^{3} W_iW_j\text{cov}(R_i, R_j) \quad (13)$$

In addition, the stage of price development at the time of investment also impacts the investment risk. For example, when the LSTM neural network predicts that the current time is in the appreciation period, the true risk will be small compared to the current day's ROI variance. If the risk assessment is still based on the variance of the investment return, some investment opportunities with lower returns will be missed, resulting in the investment return not being maximized.

Therefore, when an investment product is in the appreciation period, a risk factor can be designed based on the average return of the past few days to be used in the risk assessment to evaluate the actual risk more reasonably. Similarly, when an investment product is in a depreciation period, this risk factor can be used to increase the risk based on the variance calculation.

We divide the time interval by the growth trend, defining:

Bitcoin: Growth days greater than 80% and greater than 10 days in a period of appreciation, a growth rate greater than 5% in the period.

Gold: Growth of more than 80% and more than 7 days over a while, with a growth rate of more than 5% over the period.

Defining the maximum return as R_{max} and the minimum return as R_{min} during the growth period, the extreme variance of the return is R_{max}-R_{min}. The extreme variance is then compared to the average return $\bar{R}$, which describes the proportional impact of the period on the risk. It is expressed by a proportionality factor lambda expressed as:

$$\overline{O_p^2} = W\left(1 - \frac{R_{max} - R_{min}}{\bar{R}}\right) \quad (14)$$

The risk of the portfolio can still be expressed in weighted terms.

The ratio between portfolio investment risk and return can be calculated here based on the Sharp Ratio, which is used to quantify the excess return that can be generated by taking a unit of risk, and is expressed as:

$$\text{Sharp ratio} = \frac{E(R_P) - R_f}{\sigma_P} \quad (15)$$

where $E(R_p)$ is: the expected return of the portfolio, R_f: the risk-free rate of return, and σ_p: Standard deviation of modified portfolio returns.

It is easy to find that if the Sharpe Ratio is bigger than 1, the fund's return is higher than the volatility risk; if it is less than 1, it means that the fund's operational risk is greater than the return. The larger its value, the better the investment portfolio.

In each investment day, we only need to perform the above analysis for each of the three different types of investment products to get a complete investment decision plan.

2.2.3. Decision Results

At this point, we have all the data we need to know before investing. For different investors' risk-return preferences, we can find the solution that matches their preferences by restricting the Sharp Ratio.

Because the accuracy of short-term forecast data is much greater than that of long-term forecast data, we cannot rely on a limited data set to effectively predict medium- and long-term returns, but we can use the forecast results of LSTM neural network to make a general trend judgment. We try to use the best investment strategy for each day to make a portfolio in chronological order and finally get the global best result.

Based on the above considerations, with the maximum actual return of the next day as the target, we give the objective function for return planning as:

$$\max C_{\text{value}} = \overline{R_B} \cdot B_n + \overline{R_G} \cdot G_n + \overline{R_c} \cdot C_n \quad (16)$$

For any specified Sharp Ratio k, k is a constant, which can be obtained according to the deformation of its formula.

$$k^2 \sum_{i=1}^{3}\sum_{j=1}^{3} w_iw_j\text{cov}(R_i, R_j) = \left(\sum_{i=1}^{3} w_iR_i\right) \quad (17)$$

In addition, there are other constraints and their initial values. They are all given below and constitute the complete portfolio investment planning model.

$$\text{s.t.}\begin{cases} C_0 = 1000, B_0 = G_0 = 0 \\ \alpha_c = 0, \alpha_B = 2\%, \alpha_G = 1\% \\ \text{Sharp ratio } = k \end{cases} \tag{18}$$

The above model shows that with a uniquely determined Sharpe coefficient, a maximum return portfolio can be decided for each of the investment products present in hand separately. The quantiles of gold and bitcoin values can be unified by using the US dollar as the intermediary for the transformation. After the three different investment products are determined one by one, the overall investment plan for that day is also uniquely determined.

Through the mathematical expression of the model, we can see that the decision scheme of the portfolio investment is influenced by the maximum risk acceptable to the investor, the commission of the transaction, the predicted return of the various products, the actual risk, and the accuracy of the predictions.

2.2.4. Model Solution and Result Analysis

The model is essentially a linear programming model that changes dynamically over time, and its solution process is not complicated. After solving each parameter one by one concerning the model building process, the model can be gradually simplified and finally solved concerning the following process. After the solution is completed, the changes in total assets and the holdings of various assets over the entire investment period can be obtained, as shown in Figure 9 and Figure 10.

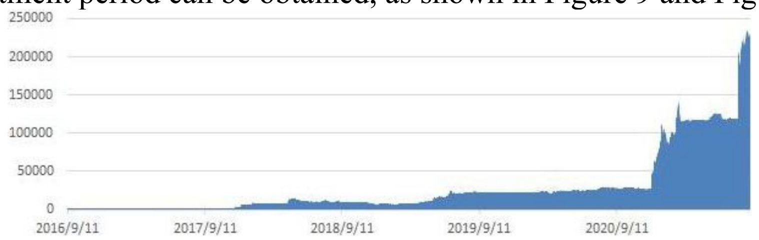

Figure 9 Total Assets

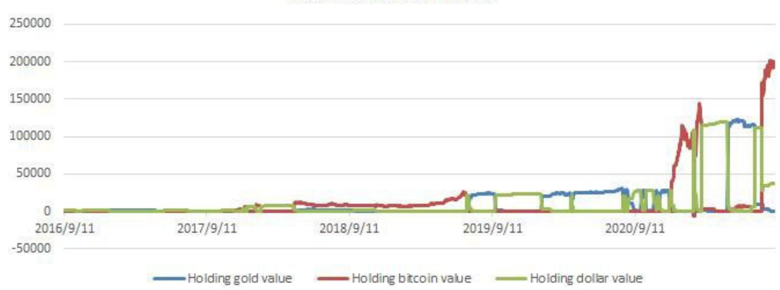

Figure 10 Assets Value Distribution

As we can see, its assets did not rise very much at the beginning of the investment and continued to do so until a year and a half later when the price of bitcoin started to rise significantly. Its investment returns are generated mainly by bitcoin, with the US dollar and gold used to hedge the risk. There is a significant rise and fall in the value of holdings.

After analyzing the results, we found that the prediction results of the LSTM neural network can accurately determine the appreciation period and depreciation period of bitcoin and gold.

3. Model Evaluation

3.1 Strength

The model introduces risk assessment indicators into the investment planning process. The single-objective planning for return is transformed into dual-objective planning for both return and risk, and a complete portfolio system is constructed.

In solving the global dynamic planning problem, the overall optimal portfolio is constructed in time order using the single-day optimal investment plan under data usage limits. The risk and return are controllable values, which reduces investment uncertainty.

The model balances the memorability of the LSTM neural network in the prediction process, the accuracy of the prediction results, and the interpretability of the ARIMA algorithm. It can analyze the price trend of investment products more effectively and conveniently and reduce the lag of prediction results.

The Poisson distribution is introduced in the forecasting process of the ARIMA algorithm, which is used to deal with the fluctuation situation generated by sudden changes in the trend of monotonic continuity. It can effectively avoid the loss caused by the deviation of the forecast trend at each node.

When the commission changes, the model can reduce the trading cost by controlling the frequency of trading changes with no change in investment return, which has strong robustness.

3.2 Weakness

The model collapses the optimal daily portfolio according to the time series to obtain the optimal global portfolio, which has limitations.

(1) The final result may not maximize the return at the end of the investment period or minimize the investment risk but is a locally optimal choice based on the forecast results after weighing the return and risk.

(2) The daily investment situation of the model is based on the forecast results of the previous days' data and does not have long-term forecasting capability. Therefore, it is only suitable for short- to medium-term investment planning.

4. Conclusion

In this section, we focus on the practical applications of our model. Our before discussion is based on ideal assumptions. In reality, we have to make some corrections and improvements.

(1) Bull and bear markets occur when the volatility of an otherwise smooth market increases and the amplitude widens because of the frenzied profit-seeking, herding effect of investors. Bull markets are good for traders to trade bitcoin and gold, and bear markets are not suitable for trading bitcoin and gold.

(2) Bold traders can buy much bitcoin early on and wait for it to appreciate, while more cautious traders can choose to trade gold futures for steady appreciation.

(3) The best option for investors is to invest all their money in bitcoin early on.

(4) The research methodology we have modeled can be applied to other areas related to futures trading markets, such as stocks.

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