Research on the Influence of Whether Rebalance Model on Quantitative Trading Investment Strategy Based on BEKK-MGARCH and Multi-Factor Model

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Abstract. The quantitative strategy of asset pricing risk management is one of the financial theory research hot spots. In this paper, we use the BEKK-GARCH model based on the relative economic theory to predict the daily volatility of gold and bitcoin; Secondly, we establish a multi-factor model with 16 factors to predict the return rate of the two assets; Finally, we establish WR model to determine the optimal position proportion of investors every day. In building the model, we use the data in the previous period to continuously slide and train our model to prevent future information leakage. In data processing, we choose the base point (0.01%) as the measurement unit, and the missing value is filled in by the average of the data of two days before and after it.

Keywords: BEKK-MGARCH; Multi-factor model; Double Ensemble; Structural Break; WR model.

1. Introduction

Asset pricing has always been the very center of modern finance theory. Traditional asset gold, and emerging currencies, bitcoin, are heavily traded. The former is an asset with low risk and low return. Even though the latter bears high risk, it provides investors with opportunities to obtain higher returns. So it is necessary for investors to timely adjust the position proportion of various assets in time to maximize the value of the assets they own.

2. Price Fluctuation Model Based on BEKK-MGARCH

We use the GARCH model to predict the variance, but because we need to get the covariance of the two sequences while getting their respective variance, we need to use the multivariate GARCH model, that is, BEKK-MGARCH [1]. Based on historical data, we can build a BEKK-MGARCH model to predict the variance and covariance of gold and bitcoin as risks. It is represented by

$$\boldsymbol{H}_t = \boldsymbol{W}'\boldsymbol{W} + \boldsymbol{A}'H_{t-1}\boldsymbol{A} + \boldsymbol{B}'\xi_{t-1}\xi'\boldsymbol{B} \tag{1}$$

To introduce some notation, suppose that W is an upper triangular matrix of parameters, and A and B are square parameter matrices of order $N(N+1)/2$. In the bivariate case $(N=2)$, A and B will be 3×3 parameter matrices. The stationarity of the BEKK-MGARCH model requires that the eigenvalues of $[A+B]$ are all less than one in absolute value. In order to gain a better understanding of how the BEKK-MGARCH model works, the elements for $N=2$ are written out below. Define

$$\boldsymbol{H_t} = \begin{bmatrix} h_{11,t} & h_{12,t} \\ h_{21,t} & h_{22,t} \end{bmatrix} \tag{2}$$

$$\boldsymbol{\xi_t} = \begin{bmatrix} u_{1,t} \\ u_{2,t} \end{bmatrix} \tag{3}$$

$$\boldsymbol{A} = \begin{bmatrix} a_{11} \\ a_{21} \end{bmatrix} \tag{4}$$

$$\boldsymbol{B} = \begin{bmatrix} b_{11} \\ b_{21} \end{bmatrix} \tag{5}$$

$\boldsymbol{H}_{\mathrm{t}}$ is a $N\times N$ conditional variance-covariance matrix, ε_t is a $N\times 1$ innovation (disturbance) vector.

By calculating the BEKK-GARCH model, we get the respective variances and covariances of bitcoin and gold. The volatility of bitcoin shows a stable trend in the long term, but it suddenly decreases after a significant sudden increase in October 2016, November 2018, March 2019, and August 2020, respectively. The same trend has occurred in gold. We speculate that it may be caused by changes in the economic environment of the whole society.

Figure 1 The data obtained from BEKK

3. Income Forecasting Model Based on Multi-Factor Model

The five-factor model [2] captures the size, value, profitability, and investment patterns in average stock returns' whose main problem is its failure to capture the low average returns on small stocks. Since this theory, academia has mined pricing factors to predict the expected return rate of stocks. Based on the multi-factor framework, our model firstly mines the factors from the price series and then forecasts the expected return rates of gold and bitcoin. In order to consider other factors, we add Asymmetry Beyond Skewness,[3] Copula,[4] Momentum,[5] Active Factor, Skewness,[6] Kurtosis,[7] Sharpe ratio, Shannon Entropy [8], and Kontoyiannis Entropy [9] into the model.

We constructed a Double Ensemble model to better use factors to estimate returns. Zhang [10] proposes Double Ensemble, an ensemble framework leveraging learning trajectory based on sample reweighting and shuffling based on feature selection to solve the problem of automatically selecting effective features.

We ensemble diversified lightGBM that are trained with different sample weights and features. Since we mine many factor data, there may be problems such as information repetition or too much noise. This weight adjustment scheme can effectively reduce the variability of weight and noise to sample the key samples of the training model and improve its information content. Thus, the accuracy of the model is improved.

Here, $X = [x_1, \cdots, x_N]T \in R, R \times F$ is a matrix where N is the number of samples, F is the number of features, and x_i is the feature vector for the i-th sample. $y = (y_1, \cdots y_N)$ is a vector of size N where y_i is the label for the i-th sample.

Each lightGBM is trained based on not only the training data (X, y) but also a set of selected features $f \subseteq [F]$ and the weights $w = (w_1, \cdots w_N)$ where w_i is the weight assigned to the i-th sample. For the first lightGBM, we use all the features and equal weights. Then We establish k lightGBMs in turn.

In the process, we sequentially construct k lightGBMs, $\mathcal{M}^1, \cdots, \mathcal{M}^k$. After constructing the k-th lightGBM, we define the current ensemble model $\bar{\mathcal{M}}^k(\cdot) = \frac{1}{k}\sum_{k=1}^{k} \mathcal{M}^k(\cdot)$ to be a simple average over the first k lightGBMs.

Then we use SR and FS. For SR, we retrieve the loss curves during the training of the previous lightGBM and the loss values of the current ensemble. We use $C \in R^{N \times T}$ to denote the loss curves where the element c_i, t is the error on the i-th sample after the t-th iteration in the training of the previous sub-model. Next, we use $L \in R^{N\times 1}$ to denote the loss values where the element li is the error of the current ensemble on the i-th sample.

$$L_f = \text{loss}\left(\bar{\mathcal{M}}\left(X_f, y\right)\right) \tag{6}$$

For robustness, we first normalize C and L via ranking. Then, we define normalized loss curves $\tilde{C} = \text{norm}\,(C)$ and normalized loss values $\widetilde{-L} =$ norm $(-L)$. We use $C_{\text{start}}, C_{\text{end}} \in R^{N\times 1}$ to denote the loss for all the samples at the start and the end of the training, respectively.

$$h = \alpha_1\left(\widetilde{-L}\right) + \alpha_2\,\text{norm}\left(\frac{C_{end}}{C_{start}}\right) \tag{7}$$

Where h, $\widetilde{-L}, C_{\text{start}}, C_{\text{end}} \in R^{N\times 1}$, the operations are element-wise.

And then divide all the samples into B bins according to the h-value. We assign the same weights to the samples in the same bin.

Because simple samples can be fitted, and noisy fitting samples may lead to overfitting, we want to be able to adjust the weights reasonably. Intuitively, we assign large weights to the samples with a descending normalized loss curve.

To avoid extreme values for the weights, we further divide the samples into B bins according to the h-values and assign the same weights to the samples in the same bin. Suppose the i-th sample is divided into the bi-th bin. The weight of this sample is assigned as follows:

$$\omega_i = \frac{1}{\gamma^k (h)_{bi} + 0.1} \tag{8}$$

Where $(h)_b$ is the average h-value for the b-th bin. Further, we use a decay factor $\gamma \in [0,1]$ to encourage the weight distribution to be more uniform in the latter sub-models of the ensemble.

Then we use FR. For FS, we directly provide the training data and the current ensemble as the input. Similar to SR, we first calculate a g-value for each feature and then divide all the features into D bins according to their g-values. To calculate the g-value for a feature, we shuffle the values of this feature, which compares the losses before and after the shuffle.

$$g_f = \frac{\text{mean}\left(L_f - L\right)}{\text{std}\left(L_f - L\right)} \tag{9}$$

For robustness against extreme g-values, we put all the features based on G values, and randomly selected features are then done from different boxes with different sampling rates and then do the same for each box in turn. At last, we concatenate and return all the randomly selected features. Therefore, we argue that shuffling is more appropriate than replacing with zeros.

We used previous data to train the yield prediction model to obtain the normal yield for gold from 2017. To better represent our forecast, we draw a comparison of the predicted returns for gold. Our forecast was obviously effective in the first two years except for some abnormal conditions on a few trading days.

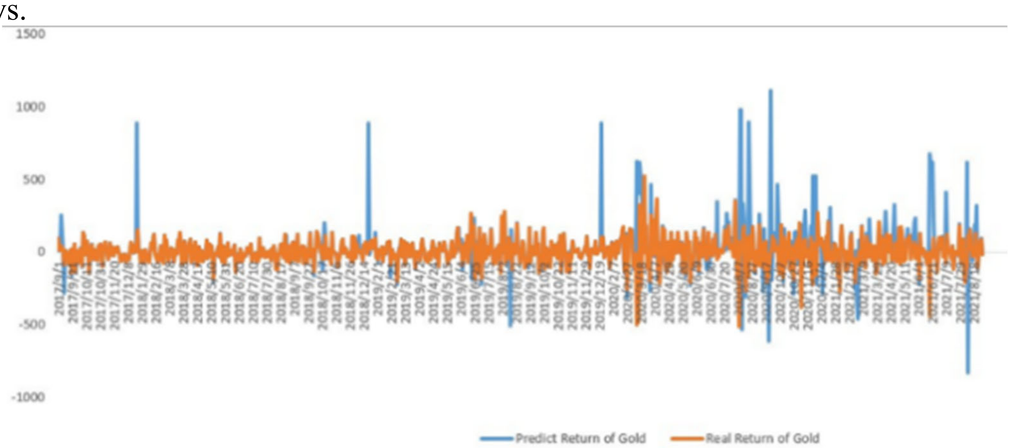

Figure 2 Comparing the predicted and real value of gold

Similarly, for the yield of bitcoin, we find that the predicted curve has a high degree of coincidence with the actual curve. There is still a discrepancy between forecast and reality, but it is within the acceptable range.

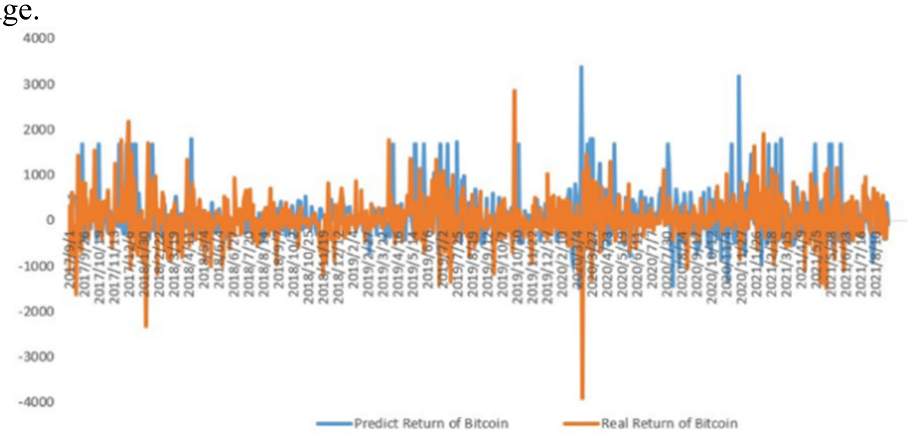

Figure 3 Comparing the accumulated predicted and actual value of bitcoin

Considering the predictive effect of bitcoin's daily yield, we also calculate the cumulative yield of bitcoin. We define the cumulative yield as the yield from buying to holding in the current period. Our prediction result is relatively ideal.

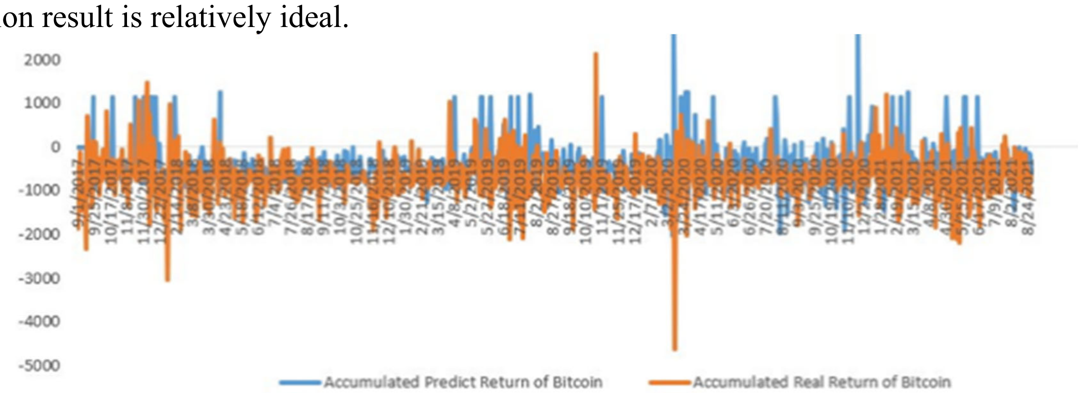

Figure 4 Comparing the accumulated predicted and actual value of gold

There are still some cases where the forecast is much higher than the real one for gold's cumulative return. However, on the whole, our prediction model still has some practical significance.

4. Whether Rebalance Model

According to Markowitz's portfolio theory [11], when investors make decisions, we consider the trading commission αgold, αbitcoin, and the current return and risk of each asset. We use the BEKK-MGARCH model to predict the risk of the two assets and the multi-factor model to predict their return. In order to take all the above factors into account, we need to build a more comprehensive model to help investors make decisions. Because the handling fee may make the cost of position adjustment more remarkable than the benefits of position adjustment, investors need to judge whether to adjust positions in advance. We believe that investors need to adjust their positions in the case of structural breaks.

The structural break is an example of a confluence of factors whose predicted outcome offers a favorable risk-adjusted return. As this break takes place, most market participants are caught off guard, and they will make costly mistakes. This error is the basis for many profitable strategies because the actors on the losing side will typically become aware of their mistakes once it is too late. Therefore, it is necessary to measure the likelihood of structural break and follow a new investment strategy. There are various methods to conduct the test, and based on that, we proposed our test model.

In order to determine whether to rebalance the portfolio, we intuitively believe that when the calculated return and variance of the portfolio change insignificantly, there is no need to rebalance positions, which means that the proportion of each asset in the current period remains the same. In order to quantify this intuition, we draw the time series graph of return bitcoin and gold. We set that when the curve changes significantly, investors should rebalance.

Referring to find the structure break of line, we first consider the slope change of the line, $|k_2 - k_1|$. We assume that when the slope change is significantly great, it shows a structural break, and then investors should rebalance. However, tests that allow for multiple variables are more convenient, and there is little information contained just in $|k_2 - k_1|$, which is easy to lead to wrong decisions. We introduce the change of return compared with the mean value of the previous period of it, $|x_2 - x_1|$, and the change of the range. $|x_2 - x_1|$ changes greatly, which means that the return of assets changes significantly. Especially when the value is positive, it indicates that investors have a great probability of obtaining greater returns in the future; When the range is extensive, the return of the asset fluctuates greatly, which will also affect the investment decision of investors. We assume k_1, k_2: the slope of return or variance graph of period $t_1, t_2; x_1, x_2$: Expectation of return or variance graph of period $t_1, t_2; Max_1, Max_2$: the maximum return or variance of period t_1, t_2; Min_1, Min_2 : minimum value of return or variance graph of period t_1, t_2. Moreover, t1 is the previous counting period, $t2$ is the current period.

Therefore, after normalization of data, the SB model is constructed to measure the degree of the structural break, we set $SB_{r.g}$ as the degree of change in the return structure of gold; $SB_{\sigma.g}$: The degree of variance structure change of gold; $SB_{r.b}$: The degree of change in the return structure of bitcoin; $SB_{\sigma.b}$: The degree of variance structure change of bitcoin

$$\begin{aligned} SB_{r.g} = |&\beta_1(k_2 - k_1) + \beta_2(\bar{x}_2 - \bar{x}_1) \\ &+\beta_3(\mathrm{Max}_2 - \mathrm{Max}_1) \\ &-\beta_4(\mathrm{Min}_2 - \mathrm{Min}_1)| \end{aligned} \tag{10}$$

We set the threshold as the value that can maximize the profit in the current period if

$$WB_t = \frac{SB_{r.g} + aSB_{\sigma.g} + SB_{r.b} + aSB_{\sigma.b}}{\alpha_{\text{gold}} + \alpha_{\text{bitcoin}}} > \text{threshold} \tag{11}$$

Then, we should rebalance the portfolio. The threshold stands for the value of maximized profit in the current period. If investors keep their portfolios unchanged, their returns will be damaged or reduced, so they should adjust their portfolios to maximize their returns.

We set ω as the proportion of assets when maximizes the objective function, k as the investment preferences of different investors, $E_{t-1}(r_t)$ as the expected return rate of the portfolio is calculated based on the multi-factor model at t time, $E_{t-1}(\sigma_t^2)$ as the variance expectation of the portfolio is calculated based on the BEKK-MGARCH model at t time, H_t as the 3×3 matrix of asset variance and covariance predicted by GARCH model.

$$\omega_t = \arg\max_{\omega} \frac{kE_{t-1}(r_t)}{E_{t-1}(\sigma_t^2)} \tag{12}$$

$$\begin{aligned} E_{t-1}(r_t) = \omega_{1,t} \times E(r_{1,t}) + \omega_{2,t} \times E(r_{2,t}) \\ +\omega_{3,t} \times E(r_{3,t}) \end{aligned} \tag{13}$$

$$\begin{aligned} &E_t(\sigma_n^2) \\ &= [\omega_{1,t}, \omega_{2,t}, \omega_{3,t}] \begin{bmatrix} \sigma_{11,t} & \sigma_{12,t} & 0 \\ \sigma_{21,t} & \sigma_{22,t} & 0 \\ 0 & 0 & \sigma_{33,t} \end{bmatrix} \begin{bmatrix} \omega_{1,t} \\ \omega_{2,t} \\ \omega_{3,t} \end{bmatrix} \\ &= \boldsymbol{\omega}^{\mathrm{T}} \boldsymbol{H}_t \boldsymbol{\omega} \end{aligned} \tag{14}$$

Where k shows the preference of investors, $E_t(r)$ is the overall income expectation of cash, gold, and bitcoin at time t, and $E_{t-1}(\sigma_t^2)$ is the overall income variance of the three assets at time $t-1$.

By substituting the data into the model, we obtain the predicted optimal asset allocation ratio in the given days, which is ω. It can be observed that the asset proportion of gold changes less frequently, while the exchange frequency of bitcoin and cash is higher, which shows that from the prediction results, investors does not need to adjust the position of gold frequently, while bitcoin and cash need to change their holding proportion frequently to maximize profit.

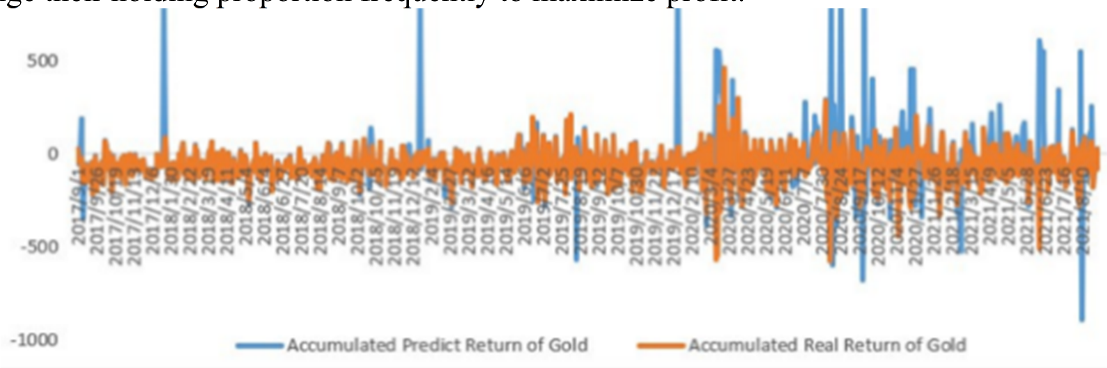

Figure 5 The data obtained from BEKK

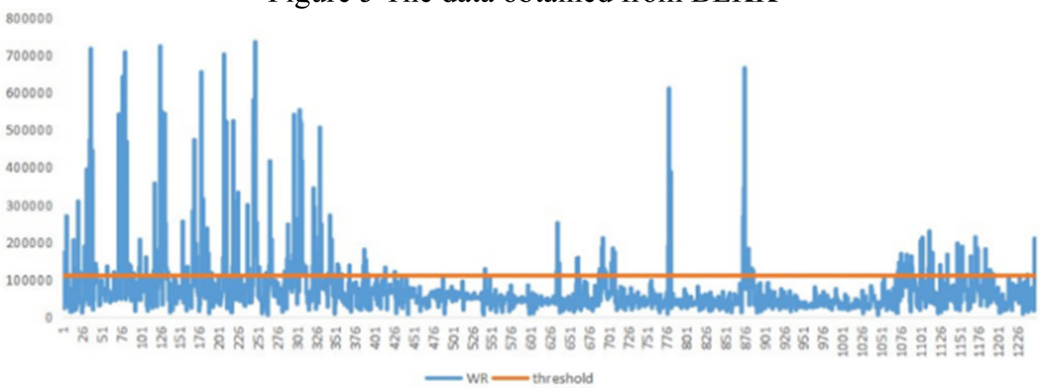

Figure 6 The value of WR threshold

The changes in the proportion of assets in the first 100 days more clearly show us this result: the best proportion of gold is close to 0 for a long time, the best proportion of bitcoin and gold changes more frequently, and their buying and selling operations are opposite at the same time point.

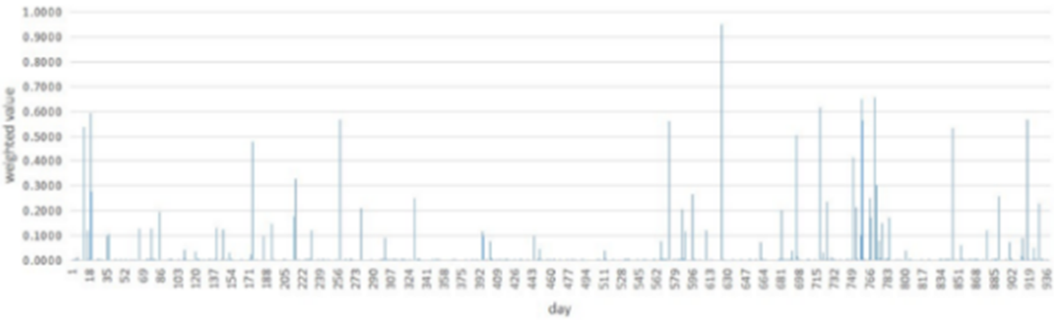

Figure 7 Calculated ω1

Figure 8 Calculated ω2

5. Conclusion

In this paper, based on the structure of the multi-factor model, we introduce Asymmetry Beyond Skewness, Copula, Momentum, Active Factor, Skewness, Kurtosis, Sharpe ratio, Shannon Entropy, Kontoyiannis Entropy, and Entropy Similarity to measure the degree of information coincidence between factors. Finally, the factors are introduced into the Double Ensemble model, and the whole model is integrated to predict the rate of return. This forecast fully considers the factors that may lead to the change of yield rate starting from the price and has strong economic significance. Then we start from the risk of the asset. Based on the BEKK-MGARCH model, we predict the variance of the current asset by using the square of the early return rate and the square of the early variance to represent the asset price volatility.

When calculating the proportion of each asset in the total capital, based on the Sharpe ratio concept and investors' preference, we construct the SB index and work out the optimal proportion in the current period. Use this ratio to calculate the maximum profit and use it as a threshold for our WR. After obtaining the expected return rate and expected volatility, we believe that investors should adjust their positions when the return rate graph changes and reflect the changes in the image by changing the slope of the curve of asset return rate or volatility, the average value, and maximum value of the index. At the same time, we also consider that the transaction fees of gold and bitcoin will also affect investors' returns and construct the WR measurement index.

Finally, we calculate the return based on the position calculated by the WR index and compare the expected return with the actual return. Our model prediction effect is good enough and can effectively help investors make the right rebalance decision.

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