

Pension Portfolio Optimization and Impact of Debt Defaults

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Abstract. Usually, the government offers two pension methods, a lumpsum buyout or an annual pension. In this paper, we provide an effective method to evaluate these two government pension methods. Initially, we constructed a pension portfolio consisting of five popular assets, such as the tech giants and popular ETFs. Then we use the Fama-French 3-factor model to calculate the expected returns of every asset. The Sharpe Ratio is the standard to make a choice that accepts the portfolio with a pension or the portfolio without a pension. Except for the Sharpe Ratios, we also took the government debt default and retirement ages into consideration. Government debt defaults contain two types, partial defaults, and full defaults. These two different kinds of defaults can have a significant impact on pension choice. With this method, we could obtain the decision boundary of the pension choice. Investors could utilize this boundary to decide whether to accept a pension or a lumpsum buyout by themselves.

Keywords: Portfolio, Pension, Pension Default, Sharpe Ratio, Fama-French 3-factor model.

1. Introduction

With considerable calculations, we picked five assets to form the optimal portfolios and recommend different investment plans under different conditions. The investors can make the final decision according to their expectations of the government's credibility and longevity.

In this paper, we used the Fama-French 3-factor model to calculate the expected return of stocks and ETFs. Considering the covariance between pension and other assets, we use the rate of Florida government bonds as the pension rate. However, the monthly expected return of pension is calculated by the annual rate of pension, not the government bond. With the expected return, we optimize the two portfolios, with pension and without a pension, by maximizing the Sharpe Ratio. Moreover, the rate of pension, which reduces due to the default, is 2.24% when the Sharpe Ratio of the portfolio with a pension is equal to that of a portfolio without a pension.

Which pension portfolio to choose may depend on the expectations of the longevity, as different stages bring different returns to the portfolio in some particular situations. Whereas, the influence of pension default cannot be neglected. Apparently, because of the economic depression and there are some natural calamities and man-made misfortunes would break the government budget for citizens' pensions. And the level of default also affects the expected returns of the pension.

The rest section of this paper is organized as follows: Section 2 describes the data of the five assets constructing the portfolio. Section 3 shows the maximum Sharpe Ratio for the portfolio and the break-even point for the pension choice. Section 4 analyzes the two factors of default that influence the pension rate and shows the decision boundary for the portfolio choice. Section 5 concludes our whole paper.

2. Data

We first selected 14 assets from big names and Forbes's top 500 small companies, calculated the correlation coefficients between them, and got rid of the assets with high coefficients. Then we

calculated the Sharpe Ratio of each portfolio formed by any five of the remaining assets. Finally, Health Care Select Sector SPDR Fund (XLV), Walmart (WMT), Apple (AAPL), Google (GOOG) and Duke (DUK), respectively from the healthcare, consumption, technology and energy sectors, were selected to form our recommended portfolio with the highest Sharpe ratio. What we used is the monthly data in the recent five years.

XLV is a Health Care Select Sector SPDR Fund which is also an ETF (Exchange-Traded Fund). In seeking to track the performance of the index, the fund employs a replication strategy. It generally invests substantially all, but at least 95%, of its total assets in the securities comprising the index. The index includes companies from the following industries: pharmaceuticals; health care equipment & supplies; health care providers & services; biotechnology; life sciences tools & services; and healthcare technology. The fund is non-diversified.

WMT refers to the stock of Walmart Inc. which comes from the Consumer Defensive sector and Discount Stores industry. Walmart Inc. engages in the operation of retail, wholesale, and other units worldwide. It was formerly known as Wal-Mart Stores, Inc. and changed its name to Walmart Inc. in February 2018. The company was founded in 1945 and is based in Bentonville, Arkansas.

AAPL refers to the stock of Apple Inc. which comes from the Technology sector and the Consumer Electronics industry. Apple Inc. designs, manufactures, and markets smartphones, personal computers, tablets, wearables, and accessories worldwide. It also sells various related services. It was incorporated in 1977 and is headquartered in Cupertino, California.

GOOG refers to the stock of Alphabet Inc. which comes from the Communication Services sector and Internet Content & Information industry. Alphabet Inc. provides various products and platforms in the United States, Europe, the Middle East, Africa, Asia-Pacific, Canada, and Latin America. It operates through Google Services, Google Cloud, and Other Bets segments. The company was founded in 1998 and is headquartered in Mountain View, California.

DUK refers to the stock of Duke Energy Corporation which comes from the Utility sector and Utilities—Regulated Electric industry. Duke Energy Corporation, together with its subsidiaries, operates as an energy company in the United States. It operates through three segments: Electric Utilities and Infrastructure, Gas Utilities and Infrastructure, and Commercial Renewables. The company was formerly known as Duke Energy Holding Corp. and changed its name to Duke Energy Corporation in April 2005. The company was founded in 1904 and is headquartered in Charlotte, North Carolina.

During the recent five years (Apr 1st, 2017 to Jan 1st, 2022), the prices of these five assets maintained an increasing trend. The Technology giant such as Apple Figure 1 (a) and Google Figure 1 (c) behave better than the other companies in expected returns. On the other hand, the volatility of these two stocks is much higher than the other three assets. Despite the lower returns, the assets of DUK Figure 1 (b), WMT Figure 1 (c) and XLV Figure 1 (e) own lower volatilities that could hedge the high risks in the AAPL and GOOG. Therefore, it is reasonable to construct a portfolio with these five assets.

The basic information, mean returns, variance, Sharpe ratio, and factors are shown in Table 1. For arithmetic mean, AAPL is the highest at a prominent 0.0328, while DUK is the lowest at 0.0090 and XLV is the second-lowest. For variance, AAPL is also the highest as 0.0073, while XLV is the lowest at 0.0019 and DUK is the second-lowest. For our Sharpe Ratio indicator, AAPL remains the highest at 0.3841 and DUK is the lowest at 0.1812.

Table 1 Arithmetic means, Variances, Sharpe Ratios, and Fama-French 3 Factors of the 5 assets.

	XLV	WMT	AAPL	GOOG	DUK
Arithmetic mean	0.0122	0.0141	0.0328	0.0216	0.0090
Variance	0.0019	0.0028	0.0073	0.0043	0.0025
Sharpe Ratio	0.2759	0.2664	0.3841	0.3279	0.1812
FF3F_Beta	0.7990	0.6796	1.3223	1.1239	0.3450
FF3F_SMB	-0.2063	-0.8073	-0.5211	-0.3782	-0.4501

FF3F HML -0.1299 -0.1312 -0.5484 -0.0259 0.1964

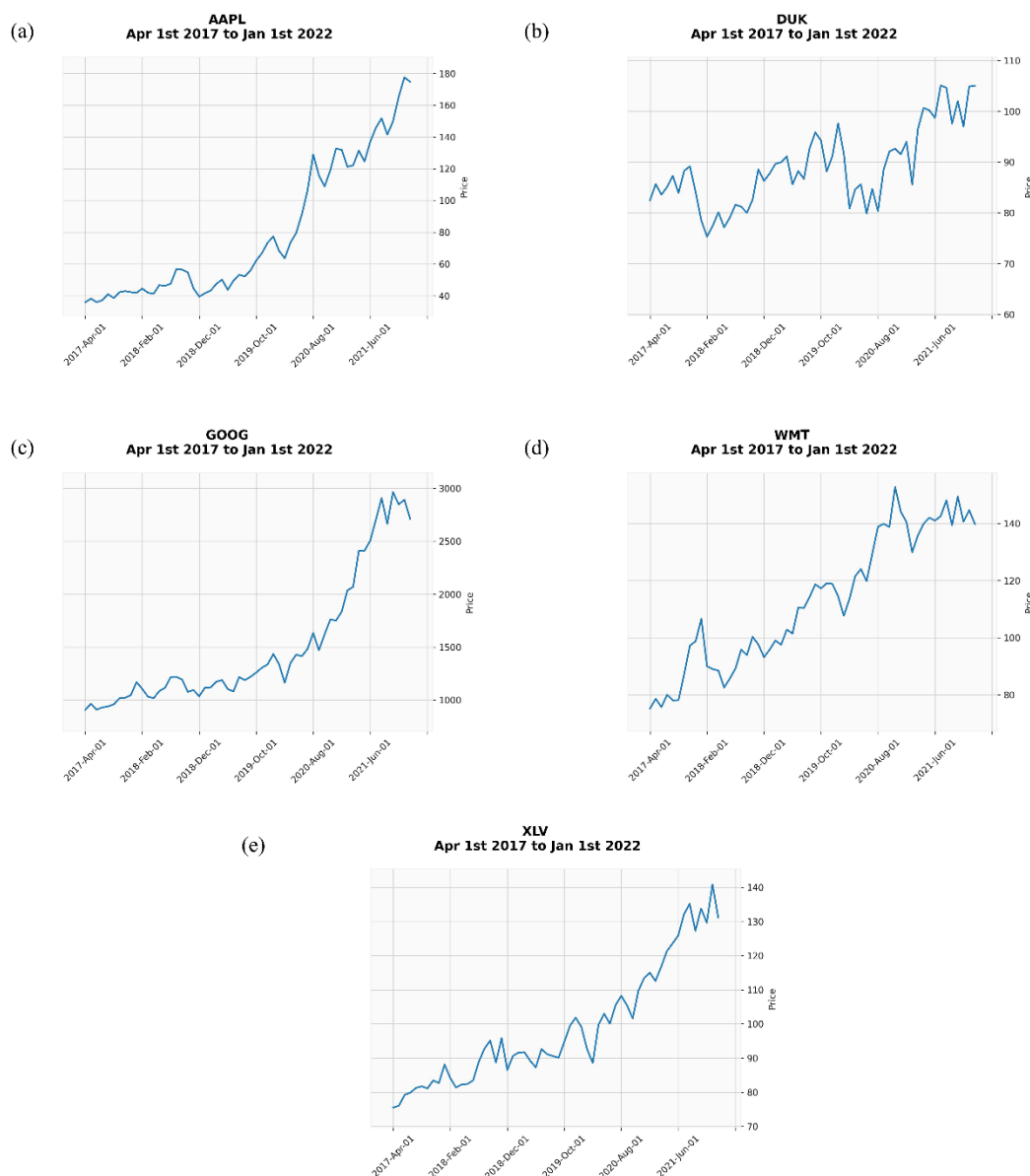


Figure 1 Prices of AAPL (a), DUK (b), GOOG (c), WMT (d), and XLV (e) from Apr 1st, 2017 to Jan 1st, 2022.

To make more accurate predictions for the future, we chose the Fama-French 3-factor (FF3F) model with more factors in consideration compared with CAPM. In the FF3F model, the beta of $MKT-R_f$ (expected market excess return) describes the market risk. SMB (Small minus Big) means size factor, and HML (High minus Low) refers to book-to-market factor.

All betas of expected market excess return are above zero as normal, and it is noteworthy that all betas of the SMB factor and most betas of the HML factor are below zero. It is because these five assets are mostly growth securities that come from large companies. They are neither value securities nor small-sized companies.

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3. Investment Portfolio

The large data prediction model for the user's electricity consumption is implemented in the Clementine software.

In this paper, Sharpe Ratio is the only standard for us to evaluate the portfolio. We could realize the maximum Sharpe Ratio by tuning the weights of different assets in our portfolio. In this section, we will calculate the portfolios with the maximum Sharpe ratio. By changing the pension rate, we could get the break-even point for the decision of whether to accept the pension.

Since Sharpe took the method initially in 1966[1], it has been accepted widely in research and companies. For almost 60 years, different researchers and traders have taken many refinements to this method, says in a bear market[2] or taking a more generalized method to calculate the Sharpe Ratio[3]. However, we used the standard method to calculate the Sharpe Ratio in this paper.

The Sharpe Ratio of our portfolio is

$$\text{Sharpe Ratio} = \frac{E(r_p) - r_f}{\sigma_p} \quad (1)$$

where r_p and σ_p are the expected return and the standard deviation of our portfolio. In our project, the Sharpe Ratio is the only standard to evaluate which portfolio should be accepted.

We usually used the arithmetic mean as the expected return, but the arithmetic mean could not reflect other factors influencing stock prices. To avoid this flaw, we calculated the $E(r)$ for every stock with the Fama-French 3-factor (FF3F) model, which integrated the influence of the market, market cap, and book-to-market values. And the equation of this model is in

$$E(r) = r_f + \beta E(r_{mkt} - r_f) + sE(SMB) + hE(HML), \quad (2)$$

where the r_f is the risk-free rate, r_{mkt} is the return of the market portfolio that we always use as the return of SPY mutual fund, SMB is the factor of the company size, and the HML is the last factor that represents the book to market value. This model was initially proposed by Fama in a series of papers[4-6] To make up for the flaw in the CAPM model, they added the other two factors, SMB and HML, to forecast the returns of assets.

The FF3F expected return was shown in Table 2, which was much different from the expected return calculated by arithmetic mean return. In addition, the monthly average return of the bond was calculated by the annual rate, not the FF3F model.

Table 2 Returns of the five assets calculated by FF3F model and arithmetic average, and returns of Florida government pension.

	XLV	AAPL	DUK	WMT	GOOG	FL GO PENSION
FF3F E(R)	0.00732	0.00886	0.00486	0.00531	0.00955	0.00624
Mean	0.01216	0.032764	0.009023	0.01411	0.02157	0.00624

With the expected returns of all assets, we could get the weight of each asset and the Sharpe Ratios of the portfolios. And the result lies in Table 3.

Table 3 The weight of each asset and the Sharpe Ratio of these two portfolios.

	XLV	AAPL	DUK	WMT	GOOG	FL GO PENSION	Sharpe Ratio
Pension	0.124	0.000	0.103	0.030	0.033	0.710	0.370
Buyout	0.458	0.074	0.194	0.030	0.244	0.000	0.163

However, in the portfolio with the pension suggested above, we ignored the default of the government pension. This led to a higher Sharpe Ratio. Figure 1 shows that the Sharpe Ratios of the two portfolios were equal when the annual interest rate of the government pension plan reduces to

2.24%. Therefore, our choice depended on the return rate of the government pension. If the rate is higher than 2.24%, we will accept the pension. Otherwise, we choose the buyout plan.

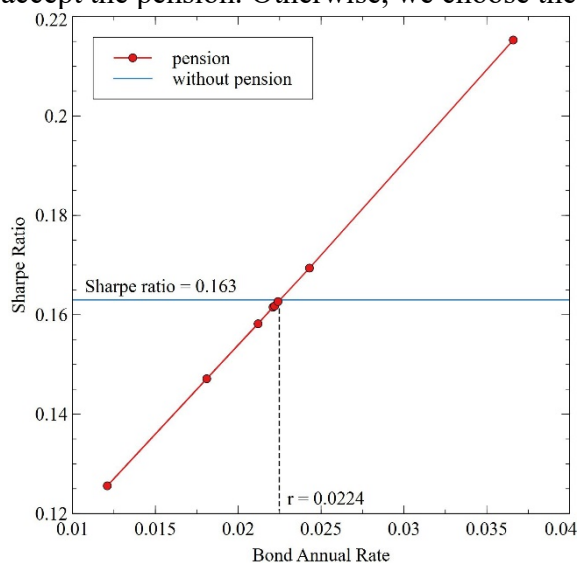


Figure 2 The red line represents the Sharpe Ratio against the pension rate.

When the rate of the pension is 2.24%, the Sharpe Ratio of the portfolio with the pension is equal to the Sharpe Ratio of the portfolio without a pension of defaults on the portfolio with a pension

4. Discussion

State pension funds are both very underfunded and highly exposed to market risks. For many state governments, therefore, pension underfunding may be an important source of public sector borrowing.

First, future benefits depend on future wages, which may be correlated with the stock market. Because of this correlation, it might make sense to discount those payments at a higher rate[7-9].

Second, states may more easily renege on future benefit payments by freezing pension plans (a so-called “soft default”), since future benefits in many instances do not enjoy the same protection as pension benefits already earned by past years of service[10].

We considered the possibility of pension default. So we calculated the IRRs where the pension is defaulted and substitute them into the portfolio to observe the effect.

Figure 3 suggests that the IRR is positively related to the growth rate, in the case that N is fixed. As the risk of the pension is given, the decrease in the expected return of the pension directly causes the fall in the individual Sharpe Ratio of the pension. Then the portfolio Sharpe Ratio declines as well.

The annual rate of return of the pension that can make the investors indifferent to choosing the portfolio with the pension or the portfolio without pension was about 2.24%. However, there is no root of g that can be found when $r = 2.24\%$, $N = [25, 26]$, $PV = 1.45$ million. In other words, when the investor is assumed to live for 25 to 26 years, and when there is only a ‘soft default’ in pension payments, the investor will always choose to accept the defined pension.

Only changes in N can significantly influence the investment decision. There are two explanations for N : 1) the remaining life of the investor, 2) the number of pension payments that he can receive. Once N declines, the pension return can fall to 2.24%, on various levels of g . A reduction in N means there are outright defaults. Outright defaults have a significant influence on the investor’s investment decision. It means the investor cannot get any future pension since the government goes into bankruptcy. For example, when the growth rate is 3%, the N that makes IRR equal to 2.24% is 14.12. It means if the investor believes the number of pension payments or the expectation of his life is more than 14.12 years, the portfolio with the pension will provide him with a higher Sharpe Ratio. On the contrary, the buyout would be acceptable.

Table 4 The number of payments against the growth rate under a constant IRR equal to 0.0224.

Number of Payment (N)	Growth Rate (g)	IRR (r)
14.12	0.03	0.0224
15.07	0.02	0.0224
16.25	0.01	0.0224
17.73	0.00	0.0224

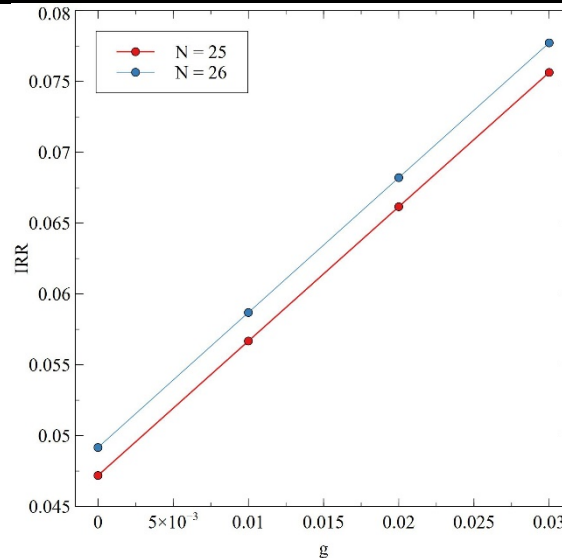


Figure 1 IRR against g under different N

5. Conclusions

In this investigation, we choose several equities that perform well in the recent five years, then we keep assets that are slightly correlated with each other and screen out the others. Finally, XLV, WMT, AAPL, DUK, and GOOG are used to form portfolios. We use the Sharpe Ratio to evaluate these two portfolios, with a pension or without a pension. The Fama-French 3-factor model calculates the more robust expected return of stocks and ETFs. This model integrates more information than the historic prices of assets, such as company sizes and book-to-market values. By maximizing the Sharpe Ratio, we obtain the weight of each asset, which could facilitate the retired to construct the most efficient portfolio. In addition, we calculated the default rate of pension which makes the Sharpe Ratio of these two portfolios equal. To compare the value of the lump sum with the value of the defined pension, we calculated their present values. The result illustrates that if the expected remaining longevity of the investor is approximately 25.89 years, at the same time there is no default in future pension payments, then the present values of accepting defined pension and taking the 1.45 million lumpsum are equal. The return rate of the lump sum is fixed, while the IRR of the pension is defined by the investor's expected longevity and possible defaults. In other words, the IRR of the pension decreases with the declines in longevity and the growth rate of the pension's cash flow. Consequently, the Sharpe Ratio of the portfolio with pension decreases with them as well. When the IRR falls to about 2.24%, the investor becomes indifferent to taking the buyout or accepting the defined pension. A smaller value of pension' IRR makes the investor prefer the 1.45 million.

To sum up, the portfolio without a pension will be attractive for the investor, when the IRR of the pension is going to be lower than 2.24%. That is to say, we advise the investor to take the lump sum payment, once he expects the remaining longevity will be lower than 14.12 years, with the cost of living increase assumed to be 3%, or he anticipates the government will be bankrupt in 14.12 years. Otherwise, we suggest the investor accept a defined pension plan.

Although we have provided the investors with the most efficient portfolio by maximizing the Sharpe Ratio, there are still several risks in the portfolio. One of the most important of them is the Fat-tail risk. Fat-tail risk: the price of every stock may behave more correlated under the fat-tail risks, such as wars, financial crises, and epidemics. Therefore, the "unrelated" stocks that should hedge the risks

will also be exposed to fat-tail events. The portfolio will change greatly if we consider these risks. Therefore, we will analyze the influence of these events and provide a more effective portfolio in the future.

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