Tests on the explanatory and predictor power improvement on Fama and French Three Factor Model by the addition of VIX index

Haoyang Chen*

Haidian foreign language shiyan school, Beijing, China

*Corresponding author: 18403018@masu.edu.cn

Abstract. The three-factor model derived by Fama and French from the CAPM is widely known for its precise attribution of the factors that affect stock returns. However, as scholars including themselves later observed, the forecasting power of their model, though somewhat better than CAPM, is still fairly weak. While the role of investor sentiments in asset pricing has already been proven, this paper proposes that the addition of the investor sentiment factor into the three-factor model will promote its ability to explain realized returns as well as forecast future stock performances. After statistical tests are employed, the result does support the proposal. However, the improvement is very limited. A hypothesis is then raised to argue the reason behind it.

Keywords: Three-factor model, VIX index, CAPM.

1. Introduction

The Capital Asset Pricing Model (CAPM) was derived in the 1960s by William Sharpe based on the efficient market theory. This model revolutionized finance by systematically relating the risk and the corresponding returns of securities that are needed to mitigate that degree of risk. However, scholars later discovered that risk premium, or the β factor in the CAPM, is not enough to explain the majority of the changes in securities' price. Researchers believed that the efficient market theory is too ideal a theory to be practically used. To reflect more precisely on the factors that influence asset price fluctuations, Eugene F. Fama and Kenneth R. French developed 5 new factors: TERM, DEF, SMB, HML, and the Rm-Rf, where the first two of which are bond market factors and the last three are stock market factors [1]. Fama and French then combine the last three factors together to form the renowned Fama and French Three-Factor Model (FF3F). Numerous empirical analyses have shown that Fama and French Three-Factor models are of great reliability in explaining the return of stocks. As Zakri Y. Bello has found, the FF3F displays remarkable improvement over CAPM in both statistical goodnesses of fit and quality of prediction [2]. Also, in multiple emerging and less developed stock markets, the FF3F also shows great reliability. As Ajlouni has found, the size and value factors in the three-factor model are recommended in the Amman stock market [3]. Given the statistical nature of the FF3F, it can be concluded that this model is fairly effective within the in-sample scale.

In terms of the out-of-sample prediction, however, the FF3F was proven to be not as effective. Although Qu has proven the superiority of FF3F over CAPM in predicting portfolio returns [4], the invalidity of FF3F still manifests. The sources of this invalidity are being disputed. According to Pettengill, Chang, and Hueng, the book-to-market ratio factor, or the HML factor, actually distorts the forecasting ability of the model [4]. Through the time-series test of 10 portfolios, they dissected the factors in the model to determine the return and risk prediction validity of each factor. The result indicates that a one-factor model formed by SMB or MKT factor is better in both realized risk and return prediction compared to a one-factor model formed by HML factor only. Furthermore, the two-factor model formed by combining SMB and MKT factors actually outperforms CAPM, the aforementioned factors models, and the original FF3F model [5]. Rahim and Hassan have also demonstrated that multi-factor models are needed to include more risk factors in the prediction. Market risk, size, and value of firms are not enough to explain, nor predict, stock returns; rather, factors like illiquidity shall also be included to further validate the model. In its essence, they argued, risk originated not from the small size of the firm, but from the illiquidity and other factors that exist as a byproduct of the small size [6].

The emergence of behavioral economics and finances aiming to explain stock returns and fluctuations by investor sentiments has also connected market behavior with the psychological conditions of the investors. The irrational nature of many investors doomed the failure of efficient market theory and one price theory in reflecting the real market and asset pricing mechanism. Brown and Cliff viewed investor sentiments as a persistent variable in the stock market as first, investors' sentiment becomes more optimistic (bullish) or pessimistic (bearish) in the long run; second, the arbitrage force will eliminate all the mispricing in the short run but is likely to become ineffective in a longer time horizon. They then concluded that the addition of a sentiment factor will be effective at describing the bearish or bullish mind of the investors. In particular, they measure the sentiment of investors by doing surveys, which have proven to be effective in 1-3 years after the survey [7]. Although their findings have accomplished great achievement in the sense that they proved the significance of including investors' sentiments in asset pricing, the method of survey is somewhat costly and may yield biased results due to multiple sampling biases. To obtain a more quantifiable measure of investor sentiments, Hu and Wang derived the notion of "hot stock" which is measured by the difference between PE value and P/M value of the company. They proved that a five-factor model with this sentiment factor added will obtain a r-square of more than 90% when conducting simple regressions, indicating remarkable power in explaining price fluctuations [8].

Dhaoui and Salah further build a revised 5-factor-model incorporating the sentiment factor using the computation method from Hu and Wang. They then tested their model with a larger sample size, gaining an averager-square of about 90%. Apart from what has been introduced, another quantifiable measure of investor sentiments is market volatility. It is reasonable to infer that investors would demand a higher rate of return under a relatively volatile market and a lower rate of return under a less volatile market condition, given the fact that volatility is directly related to uncertainty. Thus, volatility can effectively reflect the bearish or bullish sentiment of investors. Using volatility as a proxy of investor sentiments, this paper aims to incorporate it into the classic FF3F model to form a new multi-factor model. More specifically, as Whaley has described in his work, Cboe Volatility Index (VIX) is both a reflection of volatility and a good predictor of stock indexes [9-10]. Therefore, this paper will test the explanatory power as well as the predictive power if VIX is treated similar to the way with which common factors in FF3F are treated.

2. Methodology

2.1 Models

The classic FF3F incorporates three factors: HML, which measures the effect of book value and market value, SMB, which measures the effect of market size, and MKT, which measures the effect of market risk premium. The equation of FF3F is given below

$$E(r) = rf + \beta i * HML + \beta ii * SMB + \beta iii * MKT + \alpha \tag{1}$$

Where:

rf = risk-free rate in the market, often reflected by certain bond yield

βi = correlation factor of MML

HML = expected high minus low rate

βii = the correlation factor of SMB

SMB = the small minus big rate

βiii = the correlation factor for MKT

MKT = the market risk premium rate

α = the deviation of the model from the actual value.

Because the explanatory and predictive power of volatility have already been proven, the VIX index, which serves as a proxy of market volatility, will be treated the same way as other three factors. As a result, the modified equation has to form of the following

$$E(r) = rf + \beta i * HML + \beta ii * SMB + \beta iii * MKT + \beta iv * VIX + \alpha \quad (2)$$

While other notations stick to the aforementioned explanations

βiv = the correlation factor of VIX

VIX = the value of the Cboe volatility index.

2.2 Process of Computation

The objective of the paper is to test both the explanatory power and predictive power after incorporating the VIX index as a factor. However, while it is a straightforward process to test the explanatory power with realized returns and realized changes in the measures, it is obvious that no access to the future return is available to test the predictive power. To resolve the issue, this paper will use the daily data from the first trading day in 2016 to the last trading day in 2021 as sample. Then, assuming that an investor is trying to use these data at the new years day of 2022 to conclude how much of the stock price fluctuation can be explained by the model and to predict how will the stocks perform in the coming new year. Then, the data from the first trading day of 2022 to 4/22/2022 will be used as the realized "expected return" to determine how well the model predicts returns within a short period of time. It is worth mentioning the reason why daily returns are chosen rather than returns from longer time intervals like weekly or monthly. In multiple other types of research, investor sentiments are regarded as a long-term factor of influence because there is no such thing as mispricing in the short-run due to the active arbitrage force, they believed. However, from what has been observed in the past few years, manipulated investor sentiments will lead to dramatic fluctuations of the stock price. By no means should the actual value of the company deviate from its intrinsic value with that far of a distance that requires so abrupt an adjustment. As a result, it should be inferred that at least within that short period of time, the dominating force in the market isn't the arbitrage force that drives to the market price to the intrinsic value of the company, rather, individual investor sentiments and institutional investors aiming to exploit the sentiments dominate the market. Consequently, arbitrage fails to fill the gap between the actual price and intrinsic value. The VIX index that reflects the short-term sentiments of investors, thus, plays an important role in mitigating such a failure by taking short-term sentiment into account.

Then, in terms of the computation of the components in the equation, the β factor of HML, SMB, MKT, and VIX will all be computed by regressing their daily data against the corresponding daily return of the stock or portfolio returns. The slope of the line of best fit will then be taken as the value of β. The expected value, or the multiplier of the β coefficient, will be obtained by taking the simple arithmetic average of the 5 years of daily data. During the process of taking beta, the r-square obtained by regressing factors together will be the percentage of returns that can be explained by the model, or under the context of this paper, the explanatory power of models. Then, with the abovementioned processes, an expected return of the portfolio is obtained. In the course of 4 months from January fourth, 2022 to April twenty-third 2022, the portfolio is expected to have a return rate of that value. Statistical tests will be performed against the expected values and the actual values to determine whether incorporating the VIX factor improves the predictive power of the factor model.

3. Data

To test the explanatory and predictive power of both FF3F and the new multi-factor model incorporating VIX index factor, simple linear regressions are conducted using the factors against five portfolios composed of single stocks. The results are listed below.

Table 1. The results of the different portfolio model.

		Portfolio 1*	Portfolio 2	Portfolio 3	Portfolio 4	portfolio 5
FF3F	Multiple R	0.78371	0.875096	0.785413	0.708672	0.499855
	R Square	0.61420	0.765793	0.616874	0.502216	0.249855
	Adjusted Rs	0.61343	0.765326	0.616111	0.501224	0.248361
	SE	0.01151	0.008161	0.010126	0.013068	0.03128
four-factor model	Multiple R	0.783941	0.875097	0.788043	0.709376	0.499855
	R Square	0.614563	0.765796	0.621012	0.503214	0.249855
	Adjusted Rs	0.613539	0.765173	0.620004	0.501894	0.248361
	SE	0.011507	0.008164	0.010075	0.01306	0.03128

*: portfolios sorted by market size

According to these data, the explanatory power of FF3F displays great variation among different portfolios, which doesn't seem to correlate with the size of the portfolio. With the highest r-square of portfolio 4 of about 76% and the lowest r-square of portfolio 4 of about 25% r-square, the variation is more than 50%. When it comes to the adjusted four-factor model, however, the its r-square variation is very similar to that of the FF3F. Comparing horizontally the r-square of regression of factors from two models against the same portfolio, it is clear that r-square for every single regression for 4-factor model is larger than that for the FF3F. The improvement, however, is very limited. This is especially manifested by the regression against portfolio 4, for which the 4-factor modal increments the r statistic by 0.000001 and r-square by 0.000003. On the one hand, the applications of 4-factor model unanimously display improvement; on the other hand, the improvement is almost negligible. It is reasonable to conclude, therefore, that the VIX factor do increment the explanatory power of FF3F. However, to what extent can this incorporation improves such explanatory power remains to be determined.

In terms of predictive power, new methods are introduced to test it. Specifically, three statistical measures will be employed, which are Mean Average Error (MAE), Mean Average Percent Error (MAPE), and Root Mean Square Deviation (RMSD). The equations of these measures are given below

$$MAE = \frac{1}{n}\sum_{t=1}^{n} |ROt - REt| \quad (3)$$

$$MAPE = \frac{1}{N}\sum_{t=1}^{n} |\frac{ROt-REt}{ROt}| \quad (4)$$

$$RMSE = \sqrt{\frac{1}{n}\sum_{t=1}^{n}(ROt - REt)^2} \quad (5)$$

Where, under the context of this paper

n = the number of trading days of which expected returns are forecasted

ROt = the realized daily return of the portfolio at day t

REt = the expected daily return of day t

Using the expected daily return computed by both FF3F and the new 4-factor model, it is assumed that these rates of return will persist to be the expected return during the time period between the first trading day in 2022 and April 22 in 2022. Somehow, it is not as meaningful to conduct actual regressions due to the completely flat expected rates of return. Using these measures that quantify the deviation of the realized return of every trading day from the expected return is thus a viable alternative. The test statistics of the abovementioned measures are given below

Table 2. The test statistics of the abovementioned measures

		Portfolio 1	portfolio 2	portfolio 3	portfolio 4	portfolio 5
FF3F	MAE	0.014882	0.016885	0.017411	0.021857	0.032259
	MAPE	1.1274	1.084	1.0073	1.0113	0.9985

	RMSE	0.003295	0.002875	0.004979	0.003145	0.000549
	MAE	0.014822	0.016849	0.017391	0.021812	0.032246
Four-factor	MAPE	1.0632	1.0331	1.0038	1.0065	0.9877
	RMSE	0.003252	0.00283	0.004936	0.003106	0.000498

*: portfolios sorted by market size

According to the data listed above, it is very clear that for every one of the three measures, the value of the statistic for the 4-factor model's prediction is lower than the FF3F's prediction. Given the statistical meaning of these measures, this indicates that the predictions of the 4-factor model, at least in the scope of five measured portfolios, cling more closely to the actual value than the original FF3F. More specifically, the smaller MAE value for four-factor model in all five portfolios indicates that in average, the expected rate of return deviates from the realized actual rate of return by the amount of the test statistics. In terms of MAPE, this statistic measures the average deviation of the expected value from the actual value in percentage. While the value of MAE is heterogeneous for different values of data, the MAPE can be directly compared from different samples regardless of the value of the data contained in the sample. In the case of these data, because tests are conducted for a prediction made by both models against the same set of realized actual returns, the value of MAE can also be directly compared. The test of MAPE is still significant, however, in the sense that it provides a more direct and straightforward number, which is counted as a percentage, to measure the magnitude of the error of the expectation. The RMSE, which differs from the mean that MAE and MAPE measure, is the standard deviation of the difference between expected and actual value, or in the context, the deviation of the model prediction from the realized return. The smaller RMSE produced by the test, therefore, indicates that the 4-factor model performs better in the process of forecasting returns.

From the data above, it is somewhat clear that the forecasting power of the four-factor model is of greater precision than that of FF3F. However, the problem is, that the improvement in precision is very limited, just as what was observed in the case of explanatory power. It is worth to discuss of the reason behind the very limited improvements. To find the reason, the explanatory power of VIX index along is calculated.

Table 3. result of VIX regression.

		portfolio1	portfolio2	portfolio3	portfolio 4	portfolio 5
VIX regression	Multiple R	0.523207	0.594047	0.588091	0.470072	0.313435
	R Square	0.273746	0.352892	0.345851	0.220967	0.098242
	Adjusted R Square	0.273264	0.352463	0.345418	0.220451	0.097644
	SE	0.015779	0.013557	0.013223	0.016338	0.034273

According to the table above, which displays the test statistics of regressions of VIX index along against portfolio returns, VIX along can explain more than 20% of the returns on average. What could be the reason that when regressed together with other factors, the increase in explanatory power is much less? A reasonable justification is that the investor sentiments reflected by VIX index are in part also reflected by the existing factors in FF3F. The HML and SMB factors are both related to the stock price of a particular company, and MKT factor is related to the market performance. As what is previously discussed, the VIX index is measured using derivatives of listed stocks to reflect the volatility of the market. The bearish or bullish sentiment in the market will drive consumers to a certain type of stocks that they believe to be the most profitable under the corresponding situation. As a result, the price and thus the size of the company are affected, which will also yield impacts on the price and transaction volume of the derivatives. Therefore, when part of the change in VIX might be explained by the existing factors in the FF3F, it is somewhat reasonable that the VIX does not yield that much improvement to the FF3F.

4. Conclusion

In this paper, a proposal is first raised that the inclusion of an investor sentiment factor, in particular the VIX index, can effectively improve the forecasting power as well as the explanatory power of the FF3F model. The past literature has indicated the efficacy of other investor sentiment measures and used them to improve the validity of FF3F. However, few of them have employed VIX, which measures the volatility of the market, as a proxy of investor sentiment and introduced it to the factor model.

This paper treats the VIX index in the same way as what is done to SMB, HML, and MKT factors, namely, regressing them against portfolio returns to obtain the beta coefficient and using a simple arithmetic average as the multiplier. While R-square in the regression is used as an indicator of the explanatory power of the original and revised model, MAE, MAPE, and RSME are used to measure the predictive power of the models. From the result of the statistical measurement, it is observed that the inclusion of the VIX index does results in both better explanatory power and better forecasting power. However, such improvements are very limited to the extent that they are negligible in some sample portfolios. It is observed, however, that the VIX index by itself can explain a fair portion of the historical return (about 20% on average). To address that remarkable disparity, this paper purposes an assumption that it is because the price change in stocks, which can by part reflect the bearish and bullish sentiment of investors, presents already in the existing factors of the FF3F, in other words, a part of the VIX index can be explained by factors in FF3F. This assumption, however, remains to be tested quantitatively to prove its validity.

In conclusion, although the inclusion of the VIX index adds more explanatory power to the FF3F which has already been proven to be valid in explaining stock returns, the improvements are too small to be considered significant. On the other hand, although the VIX index contributes to the predictive power of the FF3F which has already proven to be an invalid predictor of stock performances, the change is also small enough that it is negligible. Therefore, the VIX index may not be the appropriate reflector of investor sentiments that should be included in the factor model. To better incorporate the psychological states of investors, other measures and approaches may need to be resorted to.

References

[1] Fama, Eugene F. and French, Kenneth R.. "Common Risk Factors in the Returns on Stocks and Bonds". The Fama Portfolio: Selected Papers of Eugene F. Fama, edited by John H. Cochrane and Tobias J. Moskowitz, Chicago: University of Chicago Press, 2021, pp. 392-449. https://doi.org/10.7208/9780226426983-018

[2] Bello, Zakri Y., A Statistical Comparison of the CAPM to the Fama-French Three Factor Model and the Carhart's Model (July 8, 2008). Global Journal of Finance and Banking Issues, Vol. 2, No. 2, 2008, Available at SSRN: https://ssrn.com/abstract=1536149

[3] Ajlouni, M. M., and M. Khasawneh. “Empirical Test of Fama and French Three-Factor Model in Amman Stock Exchange”. European Scientific Journal, ESJ, Vol. 13, no. 10, June 2017, doi:10.19044/esj.2017.v13n10p%p.

[4] Hu, Ou. "Forecasting ability of the Fama and French three-factor model-Implications for Capital Budgeting." (2003).

[5] Pettengill, Glenn, George Chang, and James Hueng. "Risk-return predictions with the Fama-French three-factor model Betas." International Journal of Economics and Finance 5.1 (2013): 34-47.

[6] Rahim, Ruzita Abdul, and Abu Hassan Shaari Mohd Nor. "A comparison between Fama and French model and liquidity-based three factor models in predicting portfolio returns." Asian Academy of Management Journal of Accounting and Finance 2.2 (2006): 43-60.

[7] Brown, Gregory W., and Michael T. Cliff. “Investor Sentiment and Asset Valuation.” The Journal of Business, vol. 78, no. 2, 2005, pp. 405–40, https://doi.org/10.1086/427633. Accessed 4 May 2022.

[8] Changsheng, Hu, and Wang Yongfeng. "Investor sentiment and assets valuation." Systems Engineering Procedia 3 (2012): 166-171.

[9] Dhaoui, Abderrazak, and Nesrine Bensalah. "Asset valuation impact of investor sentiment: A revised Fama–French five-factor model." Journal of Asset Management 18.1 (2017): 16-28.

[10] Whaley, Robert E. "Understanding the VIX." The Journal of Portfolio Management 35.3 (2009): 98-105.