

Comparison of CAPM and Fama-French Three-Factor Model in Investment Portfolios Prediction

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Abstract. Portfolio risk assessment is important for investors in portfolio design. Based on sound forecasts of market and company prospects, an accurate investment risk assessment can provide financial benefits to investors. By comparing the CAPM and the Fama-French three-factor model, a reasonable portfolio of investment risks is constructed by selecting five companies and one fund. The firefighter's pension problem is an example of considering the proportion of the pension that is retained and the investor's lifetime to hypothesize how the portfolio will maximize benefits under different scenarios. Portfolio forecasts based on different scenarios reflect a large extent the applicability of the chosen model to the actual situation, but it is still a challenge for investors to break away from the limitations of the data to take a more expansive view of the benefits and risks.

Keywords: CAPM, Fama-French Three-Factor Model, Investment Portfolios.

1. Introduction

An investment portfolio is a collection of stocks, bonds, financial derivatives, and other securities owned by investors or financial institutions to reduce risk. Portfolios can be thought of as a collection of levels. Due to the dual objectives of safety and profitability, the first level of combination takes into account the combination of risky and risk-free assets. Combining risk-free assets is important for safety, whereas combining risky assets is necessary for profitability. Consider how to combine hazardous assets at the second level of combination. The risk-return received from any combination of two poorly correlated or negatively correlated assets will be larger than the risk return acquired from individual assets. As a result, by regularly mixing assets with low correlation, you may keep the portfolio's efficient frontier safe from danger. There are many financial products to pick from in the investment industry, each with its own set of perks and hazards. Investors may choose how they want to invest their money, whether they want to focus on a single asset, such as equities, or if they want to diversify their portfolio and increase their return potential while lowering their risk. Diversification is the term for this method. High liquidity consistent and high returns, and low investment risks are all desirable characteristics of a high-quality asset portfolio. In portfolio management, diversification is a crucial idea. Diversification strives to decrease risk by allocating investment among diverse financial instruments, sectors, and other categories. It aims to do this by investing in multiple areas that respond differently to the same event to maximize profits. While diversity is not a guarantee of avoiding losses, most investing professionals believe that it is an important aspect of reaching long-term financial goals while lowering risk.

Therefore, it is very important to obtain the weight of each part of the investment through relevant theories and calculations, reasonably and effectively diversify and reduce risks, increase returns and returns, and obtain the optimal investment portfolio. This article is for this purpose, taking four companies and a fund as the investment direction as an example, to carry out portfolio analysis and related research.

An asset portfolio is an important way to maximize risk diversification and seek greater returns. It is the focus of research by many economists, investment experts, and other professionals. Using Markowitz Portfolio Theory, construct optimum portfolios in the US stock market (MPT) [1]. The weights of various assets in the portfolio are used to compute the portfolio using the capital asset pricing model (CAPM). The stocks in each portfolio are put together according to preset criteria, and the portfolios are evaluated based on their risk and return profiles, using only equities from all asset classes as a check sample. And they reach two conclusions: the link between anticipated return and beta coefficient is less significant, and the beta coefficient alone is insufficient to explain expected return, and they give general recommendations to investors on how to choose the optimal technique for their investment portfolio. The authors used analyst expectations, fundamental data, and momentum factors to uncover supportive data sources for compound modeling in a global stock selection model for successful portfolio management, supporting the application of the APT Multifactor Model for portfolio design and risk control [2]. Between January 1997 and December 2011, three tiers of tests for stock selection and portfolio creation were designed and estimated. Strategic Asset Allocation investigates whether asset classes provide value to standard stock, bond, and cash portfolios mix mean-variance analysis with market portfolios and demonstrate how to combine mean-variance and market-neutral portfolio methodologies to arrive at optimum portfolios [3]. Using mathematical excess returns in mean-variance analysis, excess return-based mean-variance optimization to address asset allocation difficulties, maximize the Sharpe ratio of the portfolio, and finally derive real estate, commodities, and high yields for traditional portfolio additions Non-traditional assets account for only a small percentage of the market mix for maximum value. Using investment-related models, the goal is to build an ideal portfolio, evaluate BSE15 performance, rank the established optimal portfolio, and compare BSE15 securities performance using a Sharpe model [4]. During the investigation, a descriptive research design and secondary data gathering methods were employed. According to the research, 5 companies that are suitable for investment and the stock portfolio suggested by the Sharp model should have listened to more than the CAPM model. Quantile regression in financial markets is explained professionally using a quantile regression technique [5]. The overall distributional influence of variables on security returns is investigated in this study, and it is discovered that factor impacts vary substantially among return quantiles. The authors describe two broad techniques to return forecasting and portfolio creation. As a result, the novel technique gives more accurate forecasts and possibly higher-yielding portfolios than the standard conditional mean method under moderate conditions.

At the same time, the CAPM model and the Fama-French three-factor model have also attracted many discussions. CAPM and Fama French three-factor models are used to choose portfolios [6]. The study authors built portfolios by size and BV/MV from July 1926 to January 2006 using a database based on predicted returns and characteristics associated with each model. The goal of this research is to look at new options for dealing with CAPM's empirical failure. The Fama-French three-factor model is found to be superior to CAPM in explaining the expected return of an investment portfolio to a certain extent. Revisiting the CAPM and Fama-French three-factor models in stock markets looks at the applicability of two of the most extensively used asset pricing models in equity markets from 2005 to 2015 [7]. The Fama-French three-factor model was shown to be superior to the CAPM model after using two regression approaches to evaluate the two models. The goal of Optimal Portfolio Construction is to see if Sharpe's portfolio creates a single index model that offers superior investment possibilities for Dhaka Stock Exchange (DSE) investors [8]. The study in this article employs sampling approaches to choose stocks from 16 different industrial categories. The created portfolio was shown to have the optimum risk-reward combination, outperforming each individual company as well as the market index.

Finally, investment portfolios are inseparable from current events. Combined with the many events facing the world at this stage, the optimal investment portfolio in the new era is also facing unprecedented challenges and opportunities. There are also many related articles on this series of issues. The study gives a fresh round of portfolio analysis based on declining global energy

consumption as a result of the Covid-19 epidemic and the global recession, as well as low rates of SDG investment [9]. The authors look at the best portfolio investing alternatives that take the SDGs into consideration. Given the influence of carbon pricing on fossil fuel stocks, the paper proposes a model to capture the possible impact of carbon pricing on fossil fuel inventories and utilizes it to guide Bayesian portfolio design approaches [10]. The findings imply that underweighting select fossil fuel companies might help investors decrease ex post risk.

In this paper, we make an asset portfolio based on four selected companies and one fund, aiming to find the final optimal asset portfolio, clearly showing the advantages of the Fama-French three-factor model and the steps to use in the process. We mainly used the Fama-French three-factor model for portfolios. We initially selected 5 companies (including Service Corporation International (SCI), Albemarle Corporation (NYSE: ALB), Pfizer, Pioneer Natural Resources, Exxon), but due to the homogeneity of the business of Pioneer and Exxon, their correlation is relatively high, which is not conducive to the reduction of investment risk and the diversification of the investment portfolio. In the end, we chose to give up Exxon and join a fund (VOO). Next, we searched for the Fama-French 3 factor csv from Kenneth R. French's website, using the approach of the Fama-French three-factor model, and got the risk premium. By combining these with stock return data, regression analysis is performed on these factors, combined with relevant formulas, the expected Fama-French 3 factor returns are calculated according to its operation steps, and the final optimal combination is constructed. In the process of portfolio design, we divide it into two cases according to the different pension weights, and obtain two different investment ratios respectively, and obtain the optimal investment portfolio to provide guidance for investment types in similar situations.

The remainder of this paper is organized as follows: Section 2 provides a brief overview of our final selection of 4 companies and a fund along with their changing trends and market conditions over the past five years. Section 3 introduces the CAPM model and the Fama-French 3 factor model, including their basic content, application methods, and their respective advantages and disadvantages. Section 4 provides the study of the results of this investment portfolio, which is mostly based on the Fama-French three-factor model's results and analyzes the weights of each part by the situation. Our final conclusions are presented in Section 5.

2. Stocks and fund selection

Service Corporation International founded in 1962, now employs 25,000 employees and became the largest funeral service company in America. After nearly 60 years of growth, SCI has 1,302 funeral homes and cemeteries in 43 states, with a large branch network that covers 80% of households across the USA. The funeral service industry is inherently unpleasant, but the inevitability of death has kept the demand for its services constant in general. Investors should note that the funeral Services industry currently carries an Industry Rank #5, which places it in the top 2% of more than 249 Zacks industries.

According to the data from Yahoo Finance, SCI's Market Cap is not that big, only 11.412b. Because the barriers to entry in this industry are very low, there are about 20,000 funeral companies in America and SCI has only 10% of the market. But SCI's EPS (TTM) may be a big factor in investors' favor, and it's surprising that it has beaten consensus estimates in four of the past four quarters. And SCI has a solid track record. The company has been paying a dividend for a long time, and it has been quite stable which gives people confidence in the future potential. Since 2012, the dividend of SCI works out to be a compound annual growth rate of 17% a year over that time. The data shows that payments have shown some very nice upward momentum. In addition, it's encouraging to see SCI has been growing its earnings per share by 40% a year over the past five years. Earnings have been growing rapidly, and with a low payout ratio, we think that the company could turn out to be a great dividend stock.

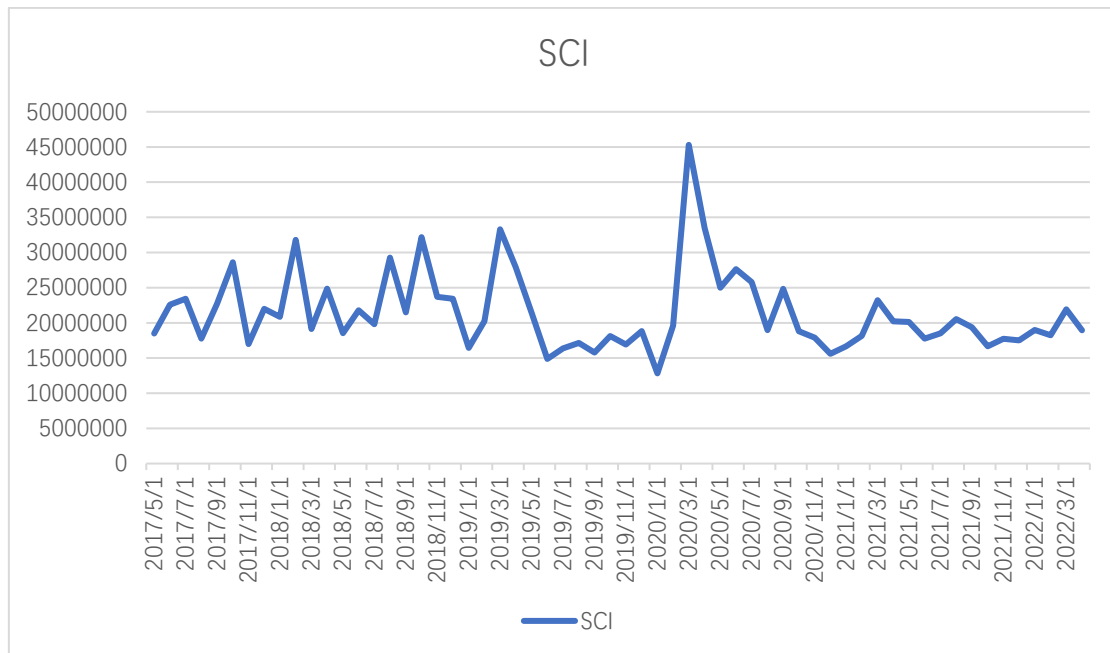


Figure 1. SCI

According to the chart of SCI, investors can clearly see that the trend of SCI is fluctuating, but the general trend is rising all the way. Because of the impact of the baby boom from 1946, America is about to enter an aging society. And with the COVID-19 outbreak in 2019, there was a very high mortality rate in the elderly, and the effects continue to nowadays. It is unfortunate and destructive for most industries, but the funeral industry has benefited. It is not negligible that SCI showed a significant downward trend at the beginning of 2020 because people are increasingly choosing more convenient and cheaper funerals and cremation. Although the profits from individual funerals have dropped, the increase in the number of funerals still allows SCI to earn a significant profit. However, as the prohibition is lifted and families have easier access to cemeteries, cremation will transform into more traditional burials, which indicates the size and expense of funerals will expand.

Mining is absolutely a vital industry that should be paid attention to when forming a suitable portfolio, in this case, ALB is a specific example. Albemarle Corporation (NYSE: ALB), headquartered in Charlotte, NC, is a global specialty chemicals company with leading positions in lithium, bromine, and refining catalysts. Recently, with the rapid boost in clean-energy technologies, ALB has realized quite profitable performance. It is stated that most of the data were taken from Yahoo Finance. The chart below roughly summarizes ALB's operating performance during recent years. Firstly, ALB holds a huge Market Cap, which is 24.711B. Secondly, the EPS (TTM) indicates that the profit after tax per unit of stock is quite promised. Lastly, the Forward Dividend & Yield greatly reach 1.58, therefore the potential is worth looking forward to.

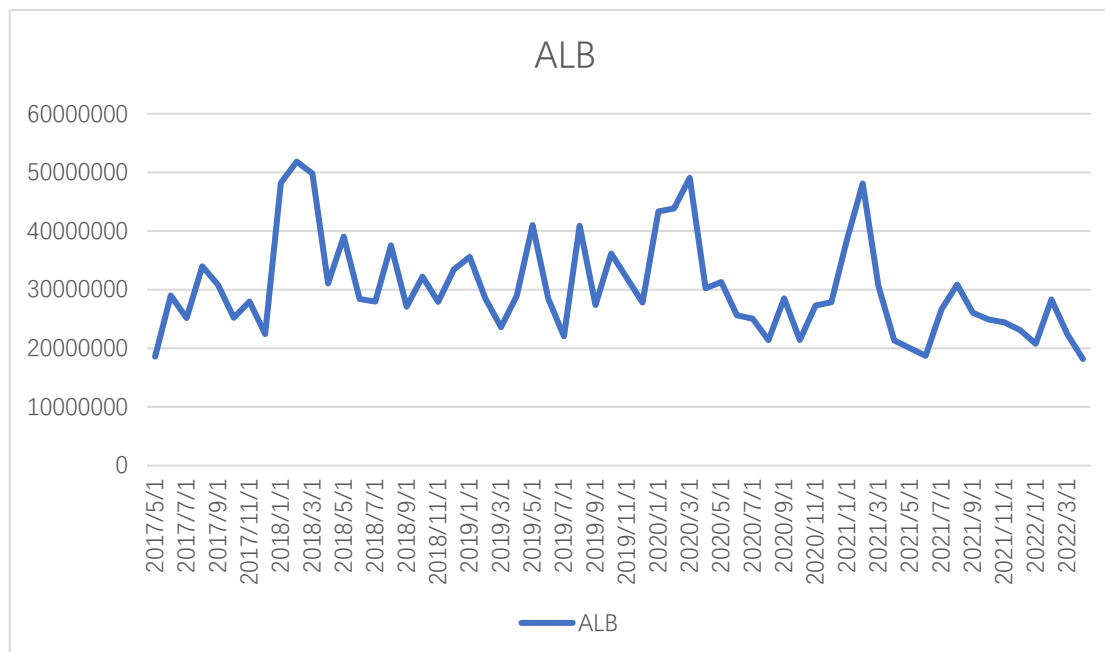


Figure 2. ABL

Indicated above is the line chart of stock price fluctuations over time. In order to facilitate subsequent calculations, the time range is limited to April 1, 2017 to January 1, 2022. From the chart, the initial point started at a quite moderate level at about 110, after some fluctuations, the stock price reached its first peak at nearly 150 at the beginning of the fourth quarter of 2017. However, it constantly experienced price drop over the next 3 years until July, 2020. Surprisingly, the line soared to the new summit of over 275 at the end of 2021.

Affected by the complex international situation, the data shown in the table may seem to be not very favorable. However, as the situation tends to a better prospect, a considerable profit is promised. Therefore it is reasonable to choose this stock.

Pioneer Natural Resources is the nation's top natural gas exploration and production corporation, committed to addressing the world's energy demands. Founded in 1997 and headquartered in Irvine, Texas, the company produces and sells oil, liquefied natural gas, and natural gas, primarily in the Permian Basin in West Texas, the Eagle Ford Shale in South Texas, and southeastern Colorado. the Raton area, and the west side of the Texas Panhandle. Due to the military conflict between Ukraine and Russia, Pioneer as a domestic energy producer is likely to gain a monopoly market position in the future, and company profits will increase significantly.

According to the data from Yahoo Finance, Pioneer has a large volume and market cap(61.72B) and is in a leading position in the same industry. It has a profit margin of 11.85%, a return on assets (Trailing Twelve Months) of 9.40%, total revenue (Trailing Twelve Months) of 17.87B, and total cash in the most recent quarter of 4.04B, and a current ratio of 1.52. At the same time, its beta value is 1.61, which is relatively high. This means that although Pioneer may fluctuate greatly with changes in policies or other factors, its investment returns are correspondingly high. And Pioneer's dividend has gone US\$2.48, which currently carries a Zacks Industry Rank 15 in the top 6% of more than 254 Zacks industries. In addition, the EPS value of Pioneer also brings a lot of information. EPS or earnings per share, as one of the important financial indicators, plays a significance role in measuring the profitability of the enterprise. In Pioneer's EPS, we can see that the value is very high and positive, from which we can infer that Pioneer has great late growth and development potential.

According to Pioneer's trend chart, in the past five years, its overall volatility has been relatively large, and it has not shown a sustained and significant upward or downward trend. In comparison, the performance of Pioneer in 2020 was slightly sluggish, but after that, it began to continue to rise as a whole, especially at the end of 2021 and the beginning of 2022, which is most obvious. As a relatively sensitive industry, Pioneer's changes in the international situation, policies and other factors are relatively significant. Although the previous period was somewhat downturn, we are still very

optimistic about the future development of this stock with the current changes and trends in the international situation.

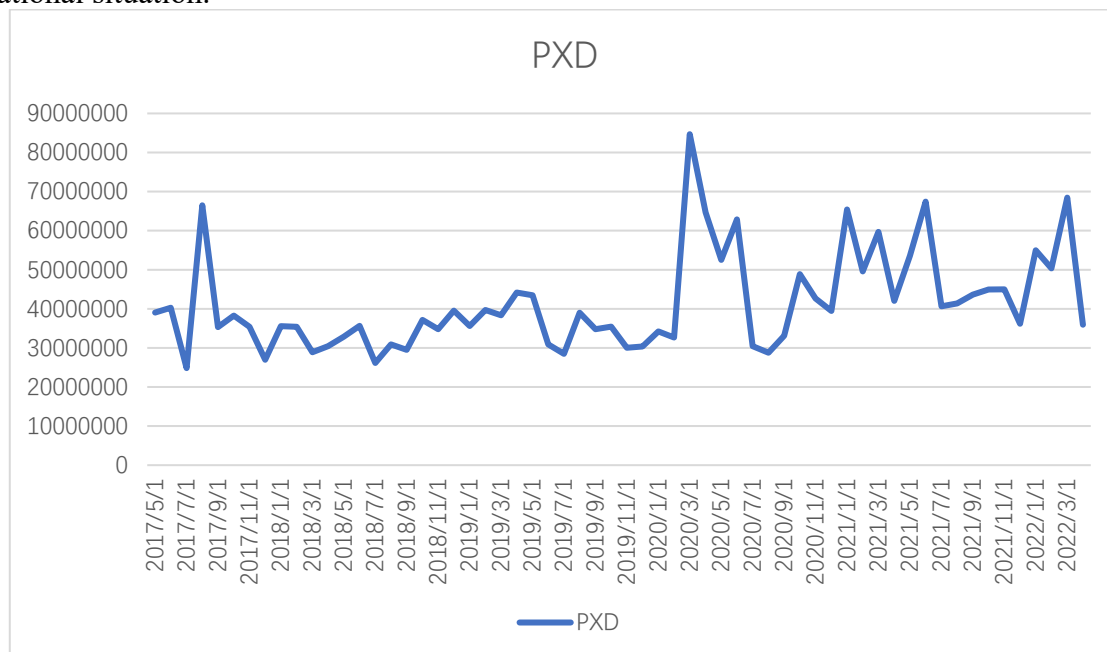


Figure 3. PXD

Pfizer, based in New York, is the world's largest pharmaceutical corporation, with the most modern production facilities and testing technologies. Founded in 1849, it was a chemical company whose main business was the production of chemical products. Pfizer's operations include six business units- Oncology, Inflammation and Immunology, Rare Diseases, Hospitals, Vaccines and Internal Medicine. Chemical medications, biological agents, vaccinations, and health pharmaceuticals are among its products' potential therapeutic and health areas.

According to the data from Yahoo Finance, Pfizer's volume and market cap(299.953B) are also very large, and It is a leading company in the field of medicine. It has a profit margin of 27.04%, a return on assets (Trailing Twelve Months) of 10.0%, total revenue (Trailing Twelve Months) of 81.29B, total cash in the most recent quarter of 31.07B, and a current ratio of 1.40. And Pfizer's dividend has gone US\$1.60. At the same time, its beta value is 0.72, which is medium level. This means that the overall change in Pfizer is not large and relatively stable. In addition, Pfizer's EPS is also positive, which means that its earnings are better, and its profitability and expected earnings are very promising.

According to Pfizer's stock price trend chart, Pfizer's previous data was relatively flat and less volatile. It finally began to climb continuously in 2021 and reached its peak in 2022. Pfizer is in the pharmaceutical industry. Due to the characteristics of the industry itself, Pfizer has been developing steadily, with few peaks, but because of its large size and strong business capabilities, it has always been in the middle and upper ranks. However, under the international situation of the globalization of the new coronavirus, Pfizer has encountered a peak of development. The global demand for drugs, vaccines and other epidemic derivatives has brought greater development opportunities to Pfizer. Therefore, we are very optimistic about this stock.

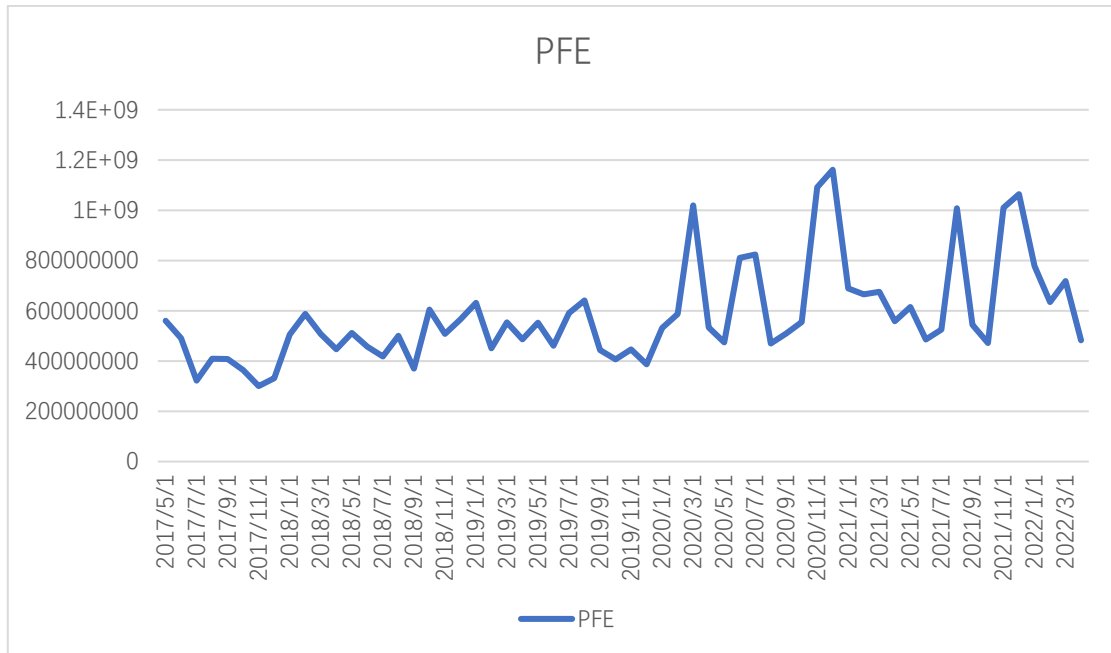


Figure 4. PEE

Exxon Mobil Corporation, the largest non-governmental oil and gas producer in the world, is committed to meeting the increasing global energy demand through advanced technology and innovation. Exxon has production facilities and sales products all over the world, and it is in the leading position in many aspects of energy and petrochemical fields. Its rigorous investment policy, its commitment to developing and applying industry-leading technologies and its pursuit of sound operation and management make it a leading position in the world. Exxon has been awarded AAA credit rating for more than 85 years in a row, and it is one of the few companies in the world that keep this record, and it ranks 10th in the Fortune 500 list in 2021.

According to the data from Yahoo finance, Exxon has an amazing market value, which is 367.518B. And its EPS (TTM) is as high as 5.39, which means that the company's profit is worth looking forward to. And its beta is 1.16 is basically synchronized with the market.

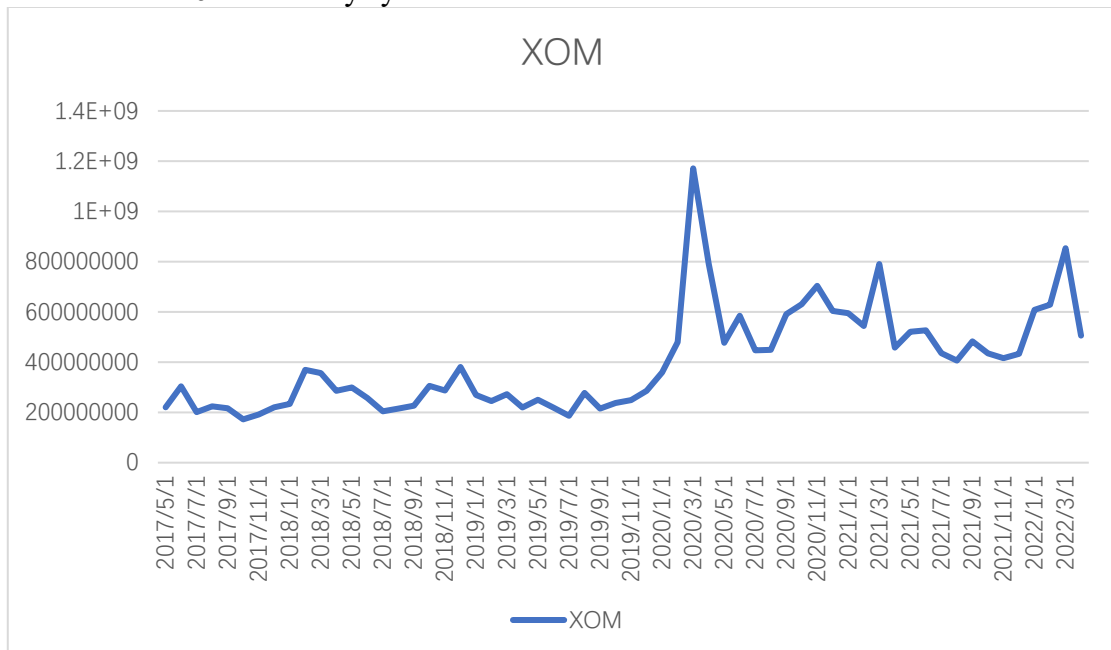


Figure 5. XOM

According to Exxon's trend chart, it is obvious that Exxon has a cliff-like downward trend in 2020. The main reason is that the COVID-19 caused the oil demand to shrink by about 20%, which had a

devastating impact on the oil business. In addition, global automobile production decreases by 60% annually, impacting the sales of chemical products and lubricants. At the same time, commercial aircraft flights were reduced by 70%, greatly reducing aviation fuel sales. However, as governments decided to coexist with the epidemic, the travel ban was gradually lifted. Coupled with the influence of Russia and Ukraine's international politics, oil prices have soared. Therefore, it can be predicted that Exxon will rebound significantly in the future.

Except for the stock companies mentioned above, in an attempt to diversify the investment, it is considered to add a well-performed fund into the optimal portfolio. The Vanguard S&P 500 Index Fund (NYSEARCA:VOO) is one of the most popular S&P 500 funds for a reason. With ultra-low fees and high trading volume, it has stood the test of time. When glancing over the summary of VOO stock, firstly the net assets equals 808.86B, and the PE ratio is 4.78, which stays in the normal range and is quite reasonable. Secondly, a 1.34% yield level assures great profitability in the future. Lastly, its beta is 1.00, indicating that this fund moves well together with the pace of the market.

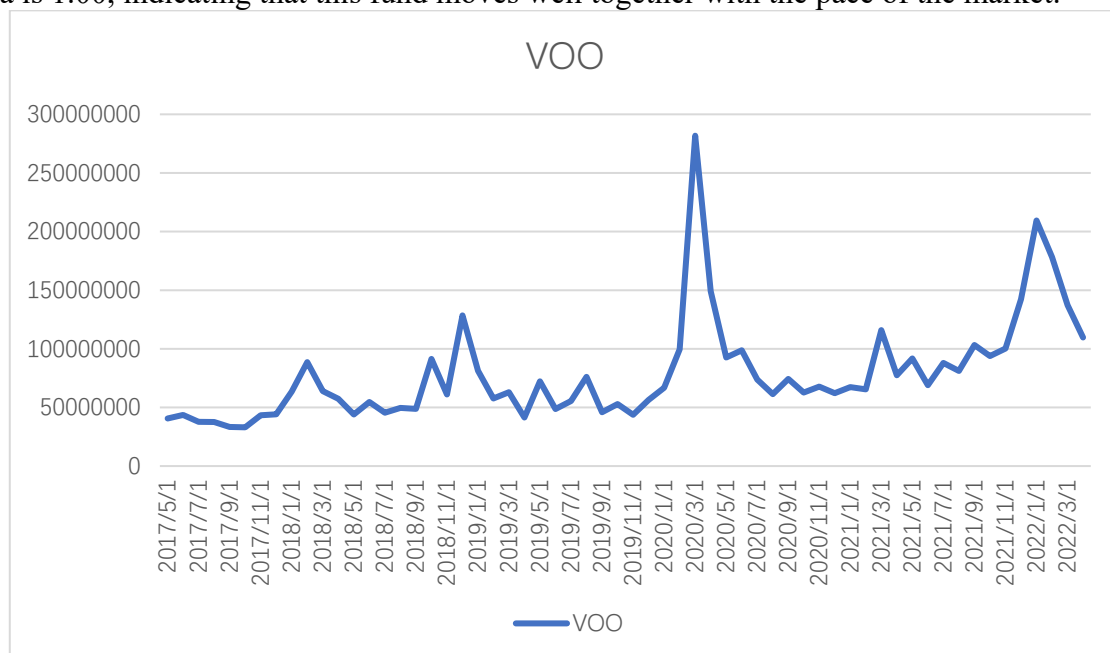


Figure 6. VOO

The line graph roughly depicted the overall trend of VOO's stock price from April 1, 2017 to January 1, 2022, it started at 23.39M point and then held a quite stable rising trend, although there were two dramatic plunges in 2019 and 2020 respectively, the fluctuations didn't make a huge impact on the whole situation. After that, the level surged to the highest 436.57 recently.

Finally, with high necessity, the market situation is accessible. From the S&P 500 index trend, we grasped some basic information. The total share price base is tremendous. Apparently, there still existed several market turmoil, it seemed unrealistic to stop the whole rising trend, especially after the midterm of 2020. The aggregate in January 1, 2022 reaches 4766.18B.

3. Assumptions of CAPM and Fama-French Three-Factor model

3.1 Assumptions of CAPM model

First, it's about CAPM, which goal is to get a new expected return and then recalculate portfolios using CAPM expected returns. The CAPM model is a qualitative jump in financial theory from normative patterns to a pattern of supply and demand for hazardous assets that generate distinctive financial asset values. The Capital Asset Pricing Model (CAPM), often known as the portfolio selection model, is one of the reference models of modern portfolio theory that has been suggested, examined, and refined by a number of famous experts in the area (Sharpe, 1964; Lintner, 1965; Mossin, 1966). Intuitively, the CAPM tells us that since everyone holds the same portfolio (the tangent

portfolio), all assets should be judged by their contribution to the risk and return of the tangent portfolio. The model knows that the only priced measure of risk is beta, and the average beta (weighted by value) in the economy is 1. Meanwhile, beta excess returns are typically found by performing a linear regression on the firm's excess returns in the market. In practice, r_M is more commonly measured using a market-weighted market index such as the S&P500. It is important to note that monthly returns avoid the noise of higher frequency returns, but allow a feasible estimate in a relatively short period of time, as beta can be dynamic, usually focusing on the previous 5 years of data. If CAPM is used as a model for analyzing and selecting the best portfolio, the same process as above is generally followed. However, since CAPM is a model that is more suitable for an ideal investment environment, it will not be the first choice for this case.

Following are several important equations about CAPM.

The first method is to compute beta using the asset's and market's historical returns. In this part, r_i represents the asset return, r_M represents the market return, σ_M^2 represents the market return variance, and $\text{cov}(r_i, r_M)$ represents the covariance between the market and asset returns. The following is the exact formula.

$$\beta_i = \frac{\text{cov}(r_i, r_M)}{\sigma_M^2}, \quad (1)$$

When it comes to the value of beta, if the asset's beta is 0, meaning it has no volatility, the expected return is equal to the risk-free rate. If the asset's beta is 1, the asset's return is the same as the market return. If an asset's beta is more than one, its cyclical return volatility is higher than the market, and it is predicted to outperform other assets. The formula for the most important CAPM is derived in order to generalize the link between an asset's expected return and its risk. In this part, r_f represents risk-free rate, and $(E(r_M) - r_f)$ is the market portfolio's projected excess return over the risk-free rate, also known as the equity risk premium. The following is the exact formula [11].

$$E(r_i) = r_f + \beta_i(E(r_M) - r_f), \quad (2)$$

$$E(r_i) = r_f + \beta_i E(r_M - r_f), \quad (3)$$

With the help of the above formula, we will get the expected return of each stock, and then use the solver tool in excel to get the optimal weight and help get the optimal investment portfolio.

The CAPM model offers several advantages, despite the fact that it is based on many assumptions. First, CAPM only examines systematic or market risk, not the unique inherent or systematic risk of a security, which eliminates the uncertainty connected with an individual security's risk, and only the general market risk becomes the major element with certainty. The model assumes that investors have a well-diversified portfolio, removing the possibility of unsystematic risk between holdings. Second, it is a superior model for calculating the cost of equity since it is extensively used in the financial sector to calculate the cost of equity and to assess the cost of financing from various sources. Finally, this is a model that is both broad and straightforward. Because this model is widely available, it may readily be used to compare inventories between nations [12].

One of the most significant models for estimating the risk and return associated with stock investment is the CAPM. Although some assumptions are made, the model's logic and ease of use make it one of the most widely recognized and reasonable methods for assisting investors in making judgments.

3.2 Assumptions of the Fama-French three-factor model

Next is the Fama-French three-factor model, which assumes that a security's returns come from two sources: company-specific events and common macroeconomic factors including GDP growth, interest rates, and market performance. Liew and Vassalou proved the function of HML and SMB which help predict the economic growth rate in the future [13]. Compared to CAPM, the three-factor model has more accurate results and provides significantly improved explanatory power due to more diverse variables from the current study [13], and is more suitable for real-world investment. The subsequent calculation process is similar to that of CAPM, except that the single factor should be

replaced by three factors in the calculation, and the final weights should also be obtained with the help of linear regression and the Sharpe ratio. The classical formula is shown below:

$$RP(t) - RF(t) = a + b[RM(t) - RF(t)] + sSMB(t) + hHML(t) \tag{4}$$

Recently, there are more surveys on the time series variation in portfolio returns, so a new formula is being tested:

$$RP(t) - RF(t) = a + b[RM(t) - RF(t)] + sSMB(t) + hHML(t) + e(t) \tag{5}$$

During the exploring process, it is suggested that the three-factor model provides a better explanation than CAPM in some specific cases. However, as Gaunt stated, the model ignores providing a convincing account for the differences in profitability between different stocks when the underlying business risk is taken into account [14]. Also, it may fail to capture the effect of conditioning information according to Ferson and Harvey's research.

Moreover, many researchers paid high attention to the time variation's effect and so they have taken the higher initiative in figuring out the result. Although some suspected the time variation in risk could probably influence the other factors, it turned out that the coefficients for the size effect and the book-to-market equity effect are all significant at the one percent level and with the expected signs [15]. In that case, it is recommended to put the time variation in beta into the formula since it makes the model robust.

Considering all the assumptions about the Fama-French three-factor model, it may be regarded as a further extension on the basis of the CAPM and solve some of its problems to a certain degree. However, this does not indicate that the model will be preferred in most of the cases, and there is no theoretical basis to prove that one of the models has an absolute advantage in the practicability of predicting results compared to others. Instead, the significance is offering a new acceptable method for investors to choose when making investment decisions.

4. Results analysis

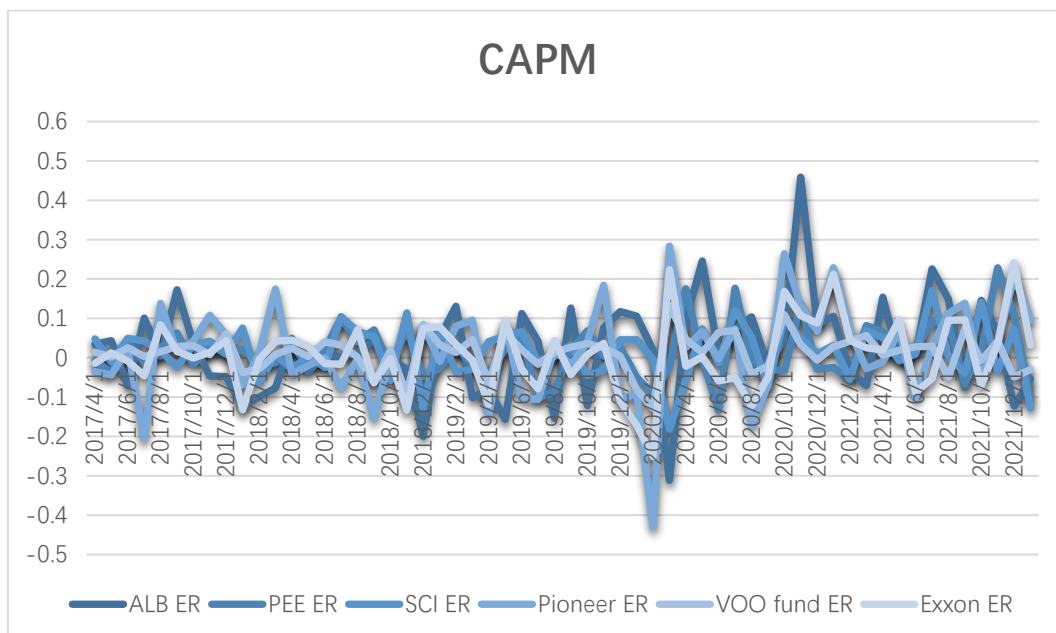


Figure 7. CAPM model

Understanding the market performance of asset classes through statistical methods of risks and returns is usually the first step to measuring the investment portfolio. Long-term investors should be as concerned with the level of wealth as they are with the returns, they receive on the wealth they invest [16]. Investors make use of the expected returns of the market and stocks to allocate between venture capital and safe investment. By calculating the arithmetic mean, geometric mean, variance, and covariance matrix of the four stocks, it is found that their risks and returns are in line with

expectations. Most of their covariance is very low, even the covariance of pioneer companies with ALB and SCI is negative.

The selected portfolio is allocated a fixed amount considering the return and standard deviation. If consumers want to maximize returns, Sharp ratio should be maximized. On the contrary, in order to reduce the risk, the variance of consumers needs to be minimized, but the return will be ignored. After the optimal risk portfolio is established, investors can allocate risk-free assets and tangent portfolios to meet their risk tolerance.

CAPM is an important model for investors to consider. The goal is to apply CAPM formula to get a new expected return and then recalculate portfolios using CAPM expected returns. When implementing the calculation in Excel it is essential to prepare market return data from S&P 500 on Yahoo Finance. After working out the market expected excess return the betas of each stock can be calculated through the regression tool. The famous relationship $E(r_i) = r_f + \beta_i E(r_m - r_f)$ helps to obtain CAPM expected returns of each stock, then the Solver tool in excel is used to calculate their respective weights. Compared to the previous arithmetic average data, CAPM emphasizes an important notion that all assets should be judged based on their contribution to the risk and return on the tangency portfolio. Because the risk premium is time-varying and the static CAPM assumes a constant risk premium, the CAPM model still has some flaws, but as a classical model, it makes analytical portfolio investing more efficient [17].

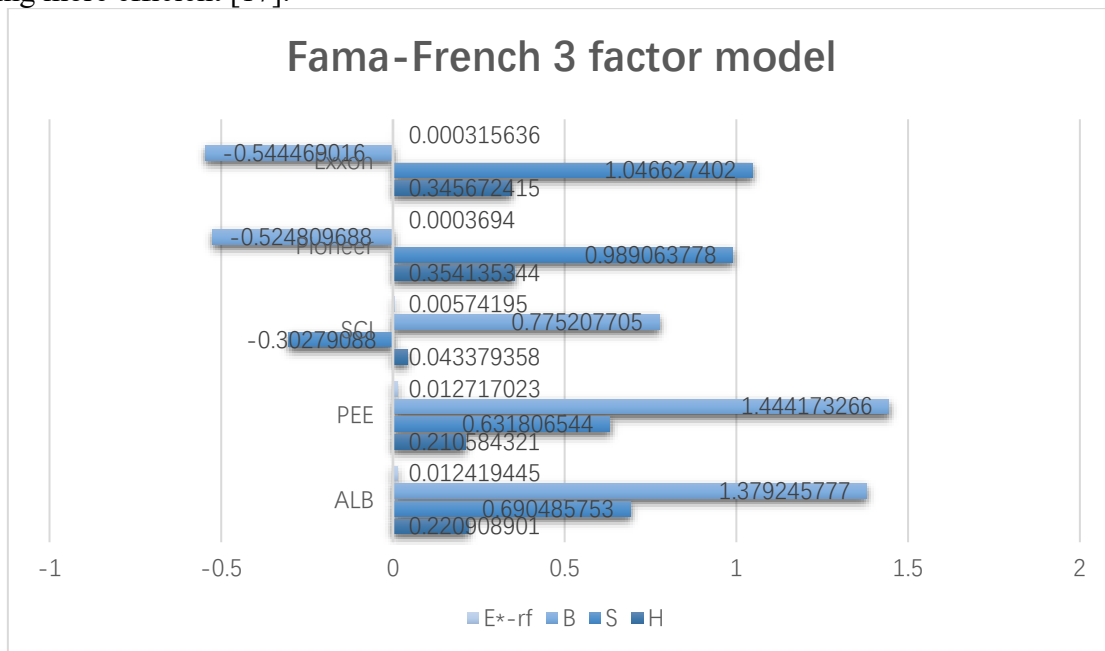


Figure 8. FF 3 Factor model

The Fama-French 3 factor model is not to be neglected by investors. The Fama-French 3 factor model is used to develop the role of expected profitability, expected investment, and book-to-market ratios as predictors of stock returns [18], which provided a wider thought of risk factors that should be considered in the expected return. Since the final investment decision is closely related to Fama-French 3 factor model result, it is necessary to take a deeper look at the calculation process. The first step is searching for Fama-French 3 factor csv from Kenneth R. French website in an attempt to get the risk premiums. After that, by combining these with the stock return data and doing regression with these factors, the betas for all factors are available. With the formula $E(r_i - r_f) = \beta_i E(r_m - r_f) + \beta_{si} E[SMB] + \beta_{hi} E[HML] + r_f$, it follows the similar pattern to calculate the Fama-French 3 factor expected return. When all the above information is ready, the final optimal portfolio can be constructed.

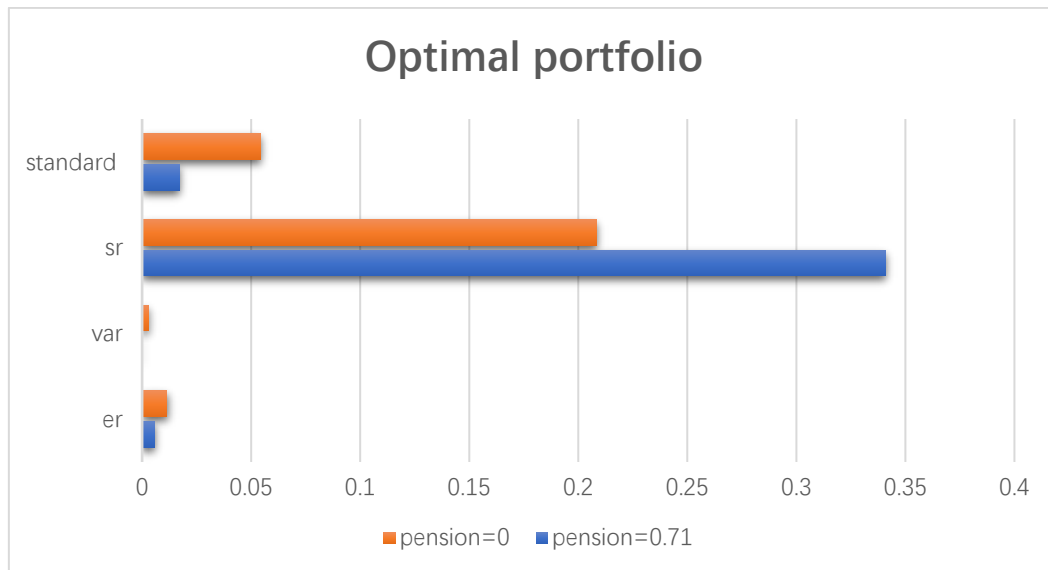


Figure 9. Optimal portfolio

In this case, it is worth noting that in order to fit the actual situation of the market, the Florida GO bond was chosen to be added into the covariance and correlation matrix as a neutral, statistic showed that Pioneer and Exxon kept a higher level of correlation at about 0.85, which could be explained because of their business homogeneity. Thus, the Exxon stock was eliminated from the portfolio to keep the diversity. In order to fully increase the rationality of the portfolio, the VOO fund was put into the analysis.

The final result was presented below. To make a further decision on how to improve our portfolio, we use the Fama-French 3 factor model again among the 4 stocks and 1 fund, through the analysis, we get the optimal stock portfolio. Considering the pension's risk premium is 0.0775, when there is no fund in the portfolio, it includes 1.64% ALB, 11.68% PEE, 8.66% SCI, 7.01% Pioneer under 71% pension weight and 12.14% ALB, 53% PEE, 14.7% SCI, 20.15% Pioneer under 0 pension weight. After putting VOO fund into the portfolio, surprisingly we get 1.087% ALB, 4.48% PEE, 4.597% SCI, 0 Pioneer, 18.8% VOO under 71% pension weight and 7.1% ALB, 15.6% PEE, 13.4% SCI, 0 Pioneer, 64% VOO under 0 pension weight. The highest Sharpe ratio is 0.435.

The VOO fund was steady as we supposed and it dramatically made the portfolio meet less risk and more profit of the market. It is reasonable to consider the portfolio as a well-performed investment choice.

5. Conclusion

While the CAPM and the Fama French three factor pricing model have been proposed and constructed quite long ago, this paper explained both of the models in detail. On the existing basis, it further explored their future development direction, the stocks and funds selected were the crucial samples for in-depth testing of the research topic.

With the help of tools such as Excel and Python, the optimal portfolio and the highest Sharpe ratio were accessible after the calculating process. Overall, the results were satisfactory, which to a large extent reflected the applicability of the selected model to the actual situation. In other words, both size effect and book-to-market equity should be considered in the expected return, Fama French three-factor pricing model is representative and efficient.

In addition, although the advantages and disadvantages of the two models have been mentioned in the paper, there is a lack of thinking to jump out of the data and then discover more possible factors which contribute to a more accurate result. This will be a major subject of portfolio research in the future.

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