

A Study of Optimal Portfolio of Gold and Bitcoin Based on Risk and Return

Yan Zhang*, Hongwei Zhang, Yu Ma

College of Mathematics, Sichuan University, Sichuan, China, 610065

*Corresponding author: vj162774@163.com

Abstract. Recently, Digital money is booming, and bitcoin shows potential in the field of investment as a representative of digital currency. According to modern portfolio theory, most of the investors are absolute risk-averse, and a diversified portfolio can effectively reduce the risk. So investors usually combine bitcoin with other assets to reduce non-systemic risks. Therefore, it is of great importance to formulate a feasible portfolio that can make steady returns for investors. For this reason, we build models to find suitable strategy to quantify the proportion of assets invested so that investors can make optimal investment decisions. We measure the return, risk, and efficiency of risk model's portfolio by sharpe ratio. And based on DEA method, the multi-stage portfolio with V-type transaction cost is evaluated by comparing the portfolio from risk model with the portfolio by applying DEA method, and finally we prove that the strategy is the optimal one. Finally, the advantages and disadvantages of this model are analyzed and summarized.

Keywords: Risk model; optimal strategy; Sharpe ratio; DEA.

1. Introduction

1.1 Background

Since Satoshi Nakamoto proposed the Bitcoin in 2009, the development and application of Bitcoin has become the forefront of financial investment. After 12 years ago, Bitcoin surpassed public expectations in 2020 and successfully attracted institutional investors [1]. Investors' positive expectations for bitcoin and the fixed total supply of bitcoin itself have pushed its prices to continue to rise. But in the meantime, because bitcoin is linked to real-world currencies and its price is highly depend on the policies and laws of various sovereign countries, the price of bitcoin fluctuates greatly with high risk and high yield coexist [2].

In addition, gold has the dual attributes of "anti-inflation" and "hedging" which can effectively reduce the volatility of the portfolio, thus increasing the Sharp ratio of the portfolio. At present, risk aversion and the expectation of interest rate factors make the gold price is volatile in a high level. Traditionally, investors would like to curb the high risk of bitcoin by increasing safe-haven assets such as gold in their portfolios. However, as bitcoin is gaining momentum, more and more people are reducing their gold holdings to invest in bitcoin, which has a serious impact on the gold market and makes the gold price unstable. Therefore, the current portfolio of gold and bitcoin is complex, and studying the proportion of the two in the portfolio can quantify the relationship between the two, clarify the risk and return level, and make investment decisions conducive to investors [3].

1.2 Literature Overview

This paper has been devoted to obtaining a model which can identify the portfolio ratio of gold and Bitcoin on a specific date to acquire the best daily trading strategy, so all of our models are based on the trading price of gold and Bitcoin on trading day.

2. Assumptions and Notations

2.1 Assumptions

To simplify the problem, we make the following basic assumptions. Although there are strong assumptions, most of our assumptions are reasonable and consistent with the underlying facts [4].

- The price of gold at the end of the day is in line with the daily closing price, and traders can sell gold at the end of the day at the closing price.
- The trade market is efficient. There is no information asymmetry and information lag.
- Traders agree to use the expected rate of return and the variance of the rate of return to represent returns and risks.
- Traders are rational people. traders want to avoid risk and want high returns. In a certain expected return level, traders are more willing to choose the portfolio of low risk.
- Markets are frictionless. Taxes and the cost of holding assets are not considered in the investment process.
- Investor-owned assets are tradable and have no constraints.
- A trade market is a perfectly competitive market. All participants in a market are price takers.
- The distribution of return on assets in the trading market is normal.

2.2 Notations

Notations are shown in Fig. 1.

variety	meaning
M	Available money on that day
M'	Increased cash on that day
B	The current quantity of Bitcoin that already has been hold
B'	Increased quantity of bitcoin
G	The current quantity of gold that already has been hold
G'	Increased quantity of gold
V _b	The current volatility of bitcoin
V _g	The current volatility of gold
k	The maximum acceptable level of risk
T	Time interval
R _y	Yesterday's investment returns
R _T	Today's instantaneous investment rate
R _n	Tomorrow's investment returns
r _p	Last strategic return
SR	Sharpe Ratio
E(R _p)	Expected annualized rate of return on investment portfolio
R _f	Annualized risk-free interest rate
σ _p	Standard deviation of annualized return on investment portfolio
X	Asset price
T	Investment day
F	Price change fitting curve
F'	Price change rate

Figure 1. Notations

3. Risk Model construction and solving

3.1 Basic model

Since most traders in real trading markets are risk averse, we take risk into account in our strategies to match real trading. The basic idea of risk algorithm is to use Markowitz portfolio theory to make the portfolio give consideration to both risk and return, so that the portfolio can reduce the non-systematic risk [5].

Considering that real markets are different from perfect competition markets, traders' investment is essentially a choice between uncertain returns and risks. Markowitz's portfolio theory describes these two key factors by means - variance. Let r_i is the expected rate of return of the risky asset i ; σ_i^2 is the risk of the asset i ; $(\sigma_{ij})_{n \times n}$ is covariance matrix; $x = (x_1, x_2, \dots, x_n)^T$ is the investment proportion vector. Through the equations [6].

$$\begin{cases} \max r(x) = \sum_{i=1}^n r_i x_i \\ s.t \sigma^2(x) = \sum_{i=1}^n \sum_{j=1}^n x_i \sigma_{ij} x_j \leq \sigma_0^2 \\ \sum_{i=1}^n x_i = 1 \\ x_i \geq 0, i = 1, 2, \dots, n \end{cases} \quad (1)$$

We solve the effective frontier of the portfolio, that is, the linear relationship between risk and return.

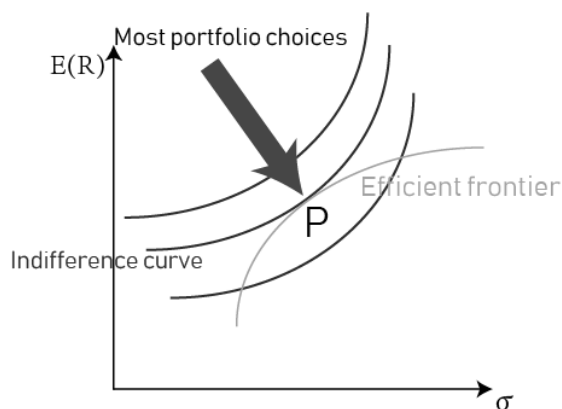


Figure 2. Efficient frontier

Efficient frontier is shown in Fig. 2. After obtaining the effective frontier, we let $n+1$ is number of observation; s_i is the price at the end of the range i ; τ is one tenth of the number of trading days in a year. we calculated the respective volatility curves of gold and bitcoin in different time periods through the formula.

$$u_i = \ln \frac{s_i}{s_{i-1}} \quad (2)$$

$$s = \sqrt{\frac{1}{n-1} \sum_{i=1}^n (u_i - \bar{u})^2} \tag{3}$$

$$\text{Volatility} \delta = \frac{s}{\sqrt{\tau}} \tag{4}$$

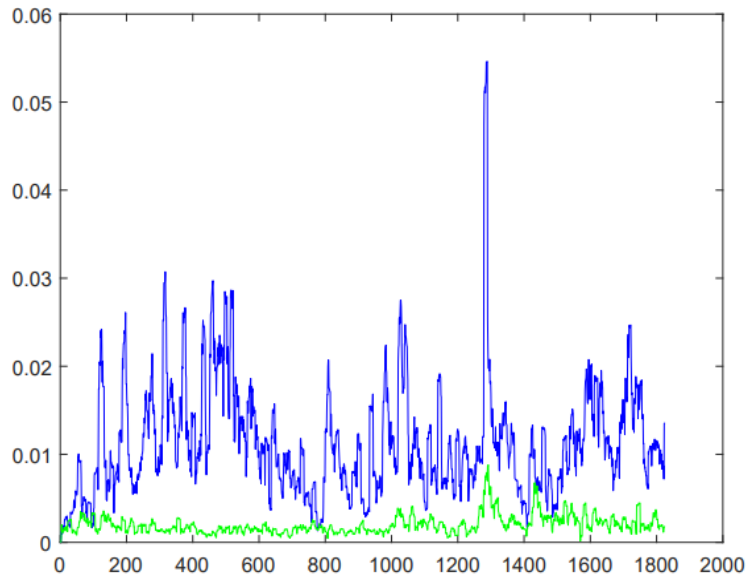


Figure 3. The volatility of the return

The volatility of the return is shown in Fig. 3.

Assuming that investors are risk-averse, we choose to use Markowitz portfolio theory to calculate the optimal portfolio when the volatility is less than 40%, and then buy on strategy.

We continue to look at the volatility of gold and bitcoin to measure risk. When volatility gets too high on one side, we think the asset is too risky and sell it in exchange for dollars. This strategy reduces the volatility of the assets we hold, allowing us to iterate again with the Markowitz portfolio. The revenue graph of the model is obtained. Investor asset change chart is shown in Fig. 4.

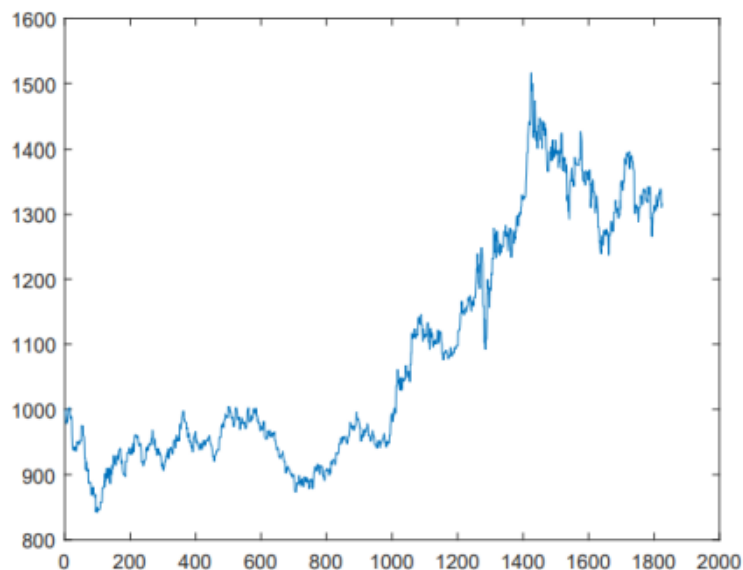


Figure 4. Investor asset change chart

3.2 Improved model

As can be seen from the asset chart above, the final return based on the basic risk model is only about 1300 dollars, which is not in line with investors' investment expectations. Therefore, we improved the model based on reality [7].

According to the market orientation of bitcoin at that time, the market price of bitcoin was rising all the way. There are four reasons for its rise:

- Bitcoin is decentralized, low transaction cost, worldwide circulation and other excellent characteristics.
- The supply of bitcoin is constant. The supply of bitcoin, which is constant at 21 million, is smaller than demand, driving up the price.
- Expectations for bitcoin are positive. Overheated expectations for bitcoin will continue to push the price higher. In addition, positive expectations for Bitcoin could lead people who already own bitcoin to hold on to them rather than sell them in the market, further reducing the supply of bitcoin in the current market and pushing up the price.
- The irrationality of investment. In reality, people's economic behaviors are mostly irrational, and it is easy to produce herd effect, that is, ordinary investors follow the trend of investing in Bitcoin, thus further raising the price of bitcoin.

The imperfect part of this model is that in the risk model, we regard the fluctuation of the return rate at a certain time as risk. In the face of large risks, our plan is to sell risky assets and wait until the risk period is over, and then optimize investment again. However, there are situations in which the market price of gold tends to rise steadily, while the price of bitcoin climbs sharply during this stage. In this case, the volatility of return rate is high, but its fluctuation is positive. Selling bitcoin in this situation would be against the right investment strategy based on the basic model [8].

So, we optimize the risk model. The idea behind this model is that as asset prices continue to rise, we gain the most from holding or buying the asset. At other times, we continue to use basic risk model to balance risks and benefits.

In the process of model calculation, we perform curve fitting on the capital price point $x = [x_1, x_2, \dots, x_n]$ of the previous 15 days when $T = 15$, and calculate the slope of each point on the fitting curve. When the slope is a strictly monotonically increasing function and the volatility is considered to be at risk, the Improved Risk Model will buy such assets.

$$F(x) = \text{polyfit}(x, T) \quad (5)$$

$$d = F'(x) \quad (6)$$

We put the price trends of the two assets into the optimization model to obtain the new asset status and the original risk model, and found that the final return has been significantly improved. Investor asset change chart is shown in Fig. 5.

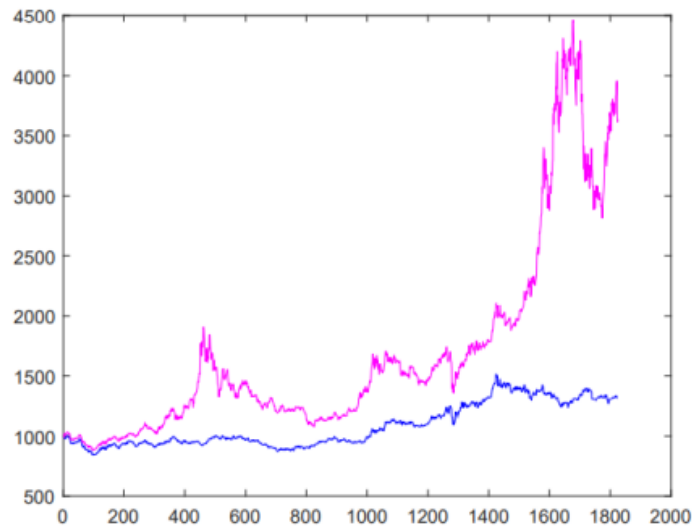


Figure 5. Investor asset change char

4. Presentation of the best strategy

We believe that a good strategy should balance risk and return to maximize the overall return. Therefore, we prove that the model we choose is the optimal strategy in the following ways.

4.1 Sharpe Ratio

Sharpe ratio provides investors with a portfolio optimization measure that considers both return and risk. It is the ratio of excess return to risk of a portfolio, which measures the risk-adjusted portfolio performance by measuring the ratio of excess return to total risk. Sharpe ratio is calculated by interest rate and variance. When the Sharpe ratio is the largest, the portfolio is the most efficient.

$$SR = \frac{E(R_p) - R_f}{\sigma_p} \tag{7}$$

The Sharpe ratio index diagram observing each decision shows that the sharpe system per decision fluctuates in the range of [0.8,2], with less than 1 under a few investment decision combinations. Therefore, in most cases, the investment decisions built by the model perform well and can maximize the returns under risk control. Sharpe ratio index diagram is shown in Fig. 6.

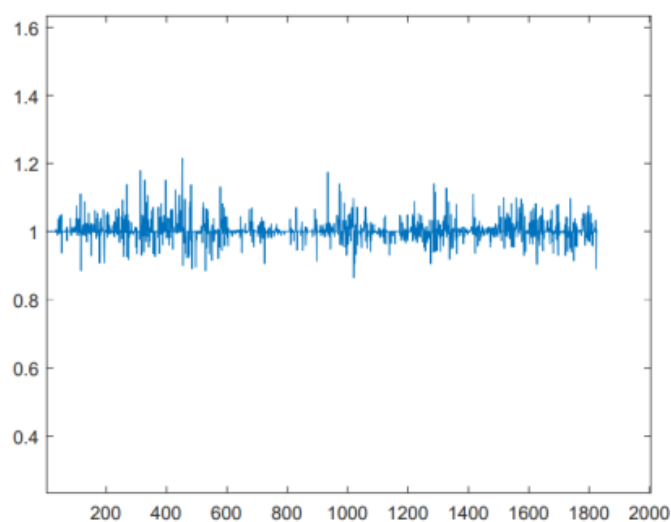


Figure 6. Sharpe ratio index diagram

4.2 DEA

DEA (data envelopment analysis) can be used to evaluate the efficiency of portfolio with multiple transaction fees. The frontier of the model linearly approximates the frontier of real portfolio. We usually take the portfolio's deviation of risk as the input index of DEA model and evaluate the performances of portfolio with measure of return [9]. Meanwhile, we solve the problem in considering of transaction costs.

Previously, we have solved the Markowitz mean variance model, the linear diagram between risk(s) and yield(r) have been calculated and the real effective frontier under the mean-variance framework can be obtained orderly.

But in reality, our portfolio is not necessarily the optimal one which is on the effective frontier. We define A (r, s) as a random portfolio, and define \widehat{PE}_r as the yield-oriented efficiency which pursue the maximum return at the same level of risks [10].

So, DEA model based on yield-oriented measure \widehat{PE}_r is.

$$\widehat{PE} = \frac{r}{r'} \quad (8)$$

In order to prove that the improved risk model provides the best strategy, we verify that the model is approach to the real frontier, that is, the portfolio obtained by the third model is the same as that obtained by DEA model.

By examining the ratio of the portfolio calculated through the third model to that calculated by DEA model for the last 20 times, the following diagram is obtained. DEA effective frontier is shown in Fig. 7.

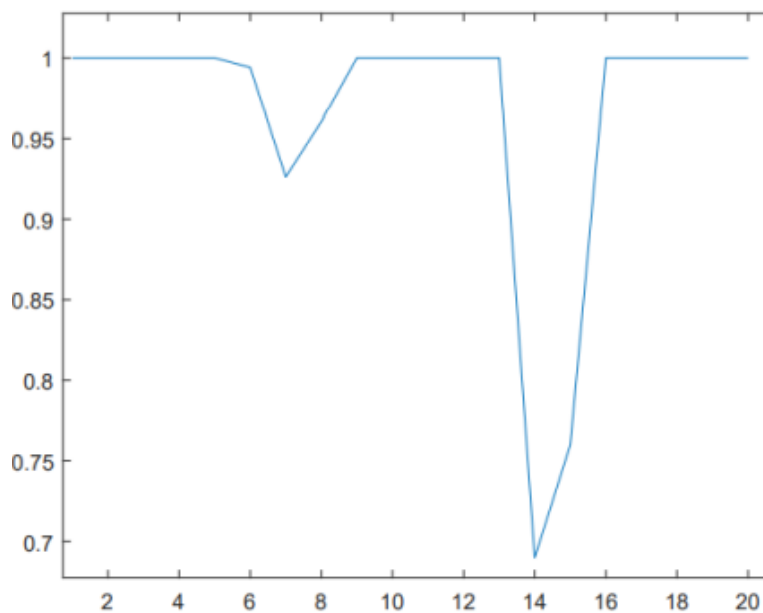


Figure 7. DEA effective frontier

The final portfolio calculated by risk model conforms to DEA model, that is, the portfolio is approaching the real frontier. Therefore, it can be concluded that the improved risk model is a optimal strategy.

5. Conclusion

The model can balance risks and benefits to maximize comprehensive bene fits. Although compared with the pure pursuit of high returns, this model is lower, but its risk is small. Therefore, it can maintain stable positive returns in the investment process. The model can adapt to investors with different risk

preferences. Investors can choose their own acceptable risk range according to their own NEN risk level. Investors only need to change a specific risk range parameter in the model to get the investment strategy suitable for their own needs. The model strategy is simple, convenient and easy to use. Therefore, the market application prospect of this model is great. The model building process has high cohesion and low coupling. It is easy to optimize for specific markets. When other factors need to be considered, such as social effects and policy impacts, we can modify them directly and then apply them to other model scenarios.

However, the model has many assumptions and some strong assumptions. Therefore, the theoretical model is difficult to reflect the risk level, so the prediction has a certain deviation.

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