Options Pricing and Trading Strategies

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Abstract. Options are tools for managing risk. As financial derivatives, options make financial markets more complete. This article introduces the concept of options, pricing methods and common trading strategies, and uses the trading of AAPL as an example to show the application strategy of options in actual investment, which is a good demonstration for others to understand options trading.

Keywords: Option Pricing, Risk Management, Option Strategy.

1. Introduction

An option is a special type of contract. The holder of the contract, that is, the buyer of the option, has the right to buy or sell the asset at an agreed price within a certain period of time or at a certain point of time specified in the contract after paying a certain amount of option premium. When the buyer of the option exercises this right, the seller of the option is obliged to cooperate with the buyer of the option to sell or buy a specific amount of the asset at the agreed price according to the contract.

From the history of options, there have been merchants using options to trade in the seventeenth century. In the seventeenth century, tulips in the Netherlands, as a symbol of status and wealth, were sought after by nobles. Rare tulips were often sold for sky-high prices. In order to snap up the market, wholesalers often sold the tulips through long-term contracts when they had not yet grown. Tulips are sold in advance, but since wholesalers are not direct growers, in order to lock in the cost of buying tulips from growers, wholesalers will sign tulip option contracts with growers in advance to lock in the highest future purchase price. When buying tulips, if the market price exceeds the contract price, the wholesaler can exercise the option to buy from the grower at the strike price. When the market price is lower than the contract price agreed at that time, the wholesaler chooses to give up the option to buy tulips directly at the market price at that time. As the earliest commodity option, tulip wholesalers successfully locked in the maximum cost of acquiring tulips through the option contract, effectively controlling the operational risks caused by price fluctuations. With the industrial revolution and the vigorous development of global transportation and trade, a variety of over-the-counter options with agricultural products as the underlying assets appeared in Europe and the United States.

With the development of trade, the use of agricultural product options has become more and more extensive, and option contracts have been gradually applied to other fields. However, the non-standardized option contracts make the transaction cost too high, which restricts the further development of options trading. April 26, 1973 CBOE (Chicago Board Options Exchange) was formally established, CBOE provides the public with standardized options contracts with stocks as the underlying asset, which marks the beginning of the era of organized and standardized modern options trading. Originally, such options traded on exchanges were also known as Exchange Traded Options. In 1982, the Chicago Board Options Exchange launched the U.S. long-term Treasury bond option futures contract, and the standardized financial futures contract was officially born. Since then, with the development of options, the options has been expanded from the previous stock to a variety of varieties, including financial options with underlying asset such as stock indices, foreign exchange, bonds and other financial products, commodity options with agricultural products, metals, fuel oil, gold, silver and other commodities and precious metals as the underlying asset, and power-based options such as carbon emission futures and water rights as the underlying asset.

2. The pricing of options

From different perspectives, options can be divided into different types, among which the most basic classification is based on the direction of buying and selling and the time of exercise. According to whether the option contract stipulates the right to buy or sell, it can be divided into call options and put options; according to the period of the buyer's exercise of power, options can be divided into American options and European options. At any point in time, the buyer of the option can exercise the right to buy or sell an option, while the buyer of the European option can only exercise the right to buy or sell on the expiration date of the option. Based on the above basic options, options have developed exotic options that are more complex than conventional options contracts, such as barrier options (meaning that the income of the option depends on whether the price of the underlying asset reaches a certain critical price within a specific time, That is, the barrier level), binary options (also known as all-or-nothing options, an option with predetermined gains and losses), etc.

As a derivative product, the value of options can be divided into two parts: intrinsic value and time value. That is, the value of the option is equal to the intrinsic value of the option plus the time value of the option. The intrinsic value of an option refers to the profit that can be obtained when the buyer of the option exercises the option. When the market price of the underlying asset is higher than the strike price, the intrinsic value of the call option is the market price of the underlying asset minus the strike price of the option. When the market price of the underlying asset is lower than the strike price, the intrinsic value of a call option is 0. The put option is just the opposite. When the market price of the standard asset is higher than the strike price, the intrinsic value of the put option is 0, and when the market price of the underlying asset is lower than the strike price, the intrinsic price of the put option is the strike price minus the market price of the underlying asset. The time value of an option refers to the premium that the buyer is willing to pay for the possibility that the option may have higher returns over time and the price of the underlying asset changes. At present, there are few ways to directly calculate the time value of an option. It is generally believed that the time value of an option is equal to the market price of the option minus the intrinsic value of the option.

At present, the two most classic and widely used methods for option pricing are Black-Scholes (B-S) model [1]and binary tree model. In 1973, Black and Scholes found after analyzing a large amount of data that the price of both financial derivatives and underlying assets obeyed Brownian motion with drift terms. They expressed the price of European options as the price of the underlying asset and the The partial differential equation of time can obtain the exact solution of the European option, where the exact solution of the European call option price is expressed as:

$$C = SN(d_1) - K\mathrm{e}^{-\mathrm{r(T-t)}}N(d_2) \tag{1}$$

where C is the price of the European call option, S is the price of the underlying asset, r is the risk-free interest rate, and K is the strike price of the contract, N(•) represents the probability function of the standard normal distribution, and d1 and d2 can be represented by the following equations:

$$d_1 = \frac{\mathrm{In}\left(\frac{\mathrm{s}}{\mathrm{k}}\right) + \left(\mathrm{r} + \frac{\sigma^2}{2}\right)(\mathrm{T-t})}{\sigma\sqrt{\mathrm{T-t}}} \tag{2}$$

$$d_2 = \frac{\mathrm{In}\left(\frac{\mathrm{s}}{\mathrm{k}}\right) + \left(\mathrm{r} - \frac{\sigma^2}{2}\right)(\mathrm{T-t})}{\sigma\sqrt{\mathrm{T-t}}} \tag{3}$$

σ is the volatility of the underlying asset.

The price of a European put option can be derived from the partial differential equation in the same way, but also from the option parity theorem from a no-arbitrage perspective.

Consider two portfolios:

Portfolio A: a European call option with strike price K and cash K*e^(-r(T-t)), at an interest rate r, cash becomes exactly K at maturity.

Portfolio B: a European put option P with strike price K plus the underlying asset at price S.

If the price of the underlying asset S is lower than K when the option expires, the option in Portfolio A will not be exercised, and the value of the portfolio is K. Portfolio B will exercise the option to sell the underlying asset at price K, and the portfolio value is also K. If the price of the underlying asset S is higher than K at expiration, the option in portfolio A will be exercised, and S will be purchased with cash K, while portfolio B will not exercise the option, the option value is 0, and the portfolio value is also S. Therefore, from the point of view of no arbitrage, combination A and combination B should have the same value, and the option parity theorem can be derived: C+ K*e^(-r(T-t))=P+S, and then the price P of the European put option can be derived:

$$P = K\mathrm{e}^{-\mathrm{rt}}N(-d_2) - SN(-d_1) \tag{4}$$

The price of the put option derived from option parity theory is consistent with the price obtained directly using the partial differential equation. Since the publication of the papers of Blake and Scholes, some scholars such as Merton, Cox, and Rubinstein have successively carried out important promotions on this theory and have been widely used[3-5].

Compared with the B-S model, which derives the price of the option through differential equations, the binary tree model discretizes the price of the underlying asset under the risk-neutral condition, and then uses the dynamic rule method to solve the price of the derivative securities [2]. The construction idea can be simplified as, if a stock and options based on the stock can be used to construct a portfolio, so that the value of the portfolio at the expiration of the option is certain, that is, the portfolio can be considered as a risk-free portfolio, and can be considered as a risk-free portfolio. To get the value of the portfolio at the beginning of the period, and the price of the stock at the beginning of the period is known, we also get the price of the option on the stock. According to this method, the pricing announcement of the binary tree single-period model of the option obtained can be expressed as:

$$C = \frac{P\,C_{\mathrm{u}} + (1-P)C_{\mathrm{d}}}{1+r} \tag{5}$$

where C is the price of the option, p is the risk-neutral probability, r is the risk-free rate, C_{u}、C_{d} are respectively the price of the option after the price of the underlying asset rises and falls after one period. Extending the single-period model to n periods, the option pricing formula of the binary tree model can be obtained:

$$C = \sum_{j=0}^{n} \frac{n!}{j! * (n-j)!} p^j (1-P)^{n-j} \, max[S_u^j \, d^{n-j} - X] \tag{6}$$

Of course, in addition to these two pricing from a theoretical point of view, there is also a Monte Carlo simulation method that simulates directly from an expectation point of view. When the option expires, the time value of the option becomes 0, so the option price is equal to the intrinsic value of the option. $C_{\mathrm{T}}\ is\ only\ related\ to\ S_{\mathrm{T}}$, so it is necessary to simulate the path of S_T, repeat it many times, and average them to get the price of option. The path of S_{T} can be simulated by the formula:

$$S_{t+\Delta t} = S_t e^{\left(\mu - \frac{\sigma^2}{2}\right)\Delta t + \sigma \varepsilon_t \sqrt{\Delta t}} \tag{7}$$

The size of Δt and the number of simulations determine the accuracy of the simulation.

Up to now, the research on various options pricing methods developed based on the above methods is still in constant discussion and development, and the current research on option pricing is still very important [3-5].

3. The application of options in actual investment

As a risk management tool, options are an important tool for investors to carry out arbitrage transactions, risk management and hedging risks. In the actual investment process, when investors have different expectations for the future, investors can construct investment portfolios by buying and selling options and underlying assets. Flexible use of portfolio strategies to maximize returns while minimizing investment risk.

1): Buying a call (put) option to get the spread income: when you are bullish (bearish) on the underlying asset, you can get the benefit of an increase in the option price by buying a call option (buying a put option), or you can hold the option to the option Expiration date to obtain the spread between the strike price and the market price.

2): Holding a long or short position in the underlying asset and the corresponding options for risk hedging: when holding a long (short) position in the underlying asset, in conjunction with holding a certain amount of put (call) options, you can choose to hedge delta [12], gamma or other risks to control the risks caused by the decline in the price of the underlying assets.

3): Bull market spread portfolio: When we are optimistic about the underlying asset, but we think that the increase is not large, in order to control the risk, we can use a long call option with a low strike price and a call option with a high strike price with the same expiration date. A short call to construct a bull spread can also be constructed using a long put with a low strike price and a short put with a high strike price. Although this combination locks in the upper limit of the investment's gains, it simultaneously establishes the lower limit of the losses and reduces the investment cost.

4): Bear market spread portfolio: The bear market spread portfolio is just the opposite of the bull market spread portfolio. When you are pessimistic about the underlying asset, you can construct a bear market spread portfolio for investment. It can be composed of a long call option with a high agreement price and an agreement The lower price consists of a short call option with the same expiry date, and it can also consist of a put option with a high strike price and a short put option with a lower strike price.

5): Butterfly spread combination: when you judge that the underlying asset will not fluctuate too much when it expires, you can construct a butterfly spread combination to make a profit. The specific method can be through a short call option with a moderate agreement price and an agreement price High, a long call with a low agreed price, or a short put option with a moderate agreed price and a long put option with a low agreed price and a high agreed price.

6): Straddle and wide-straddle spread portfolio: Contrary to the butterfly spread portfolio, the straddle portfolio is suitable for situations where the investment target will fluctuate violently, but the direction of the fluctuation cannot be determined. The construction method is to buy one put and one call option with the same agreed price and expiration time, and the wide straddle is to buy a call option with a higher strike price and a put option with a lower strike price.

7): Other investment portfolios: The above investment portfolios mainly have the same maturity time, according to different maturity times, the same agreement price and the difference period combination, according to the different maturity time and different agreement prices and the diagonal combination, etc.

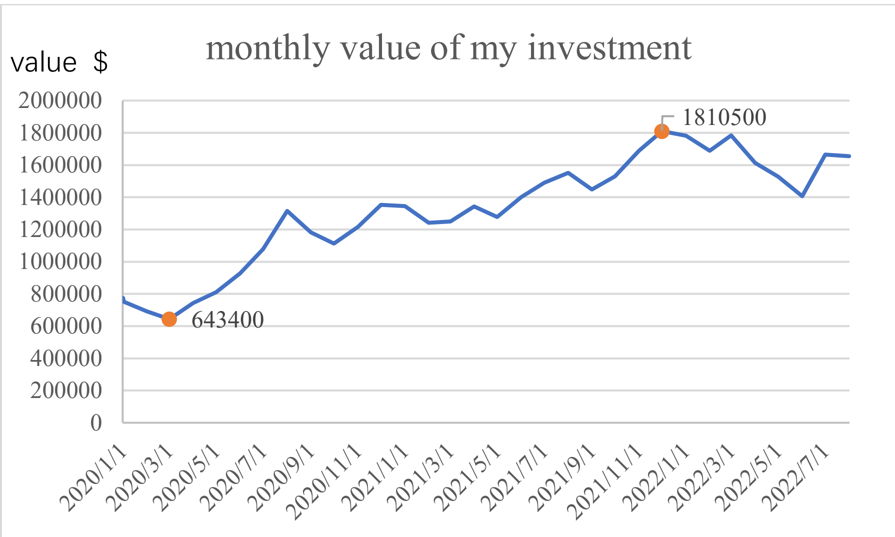

Figure 1. Monthly value of my investment

To demonstrate the role of futures investment strategies. Let's take an example of investing in Apple as a simple demonstration. Suppose today is August 1, 2022. On January 2, 2020, I purchased

10,000 shares of Applet computer (AAPL). The closing price of AAPL's stock on January 2, 2020 was $75.09 per share, so the size of my initial investment $750,900.

As of August 1, 2022, the closing price of Apple's stock is 161.51$ per share, and during the investment period, Apple has paid a total of 10 dividends, and the cumulative dividend income is 3.93$ per share, regardless of compound interest, at the end of maturity The value of my investment portfolio is $1,654,400, the yield is 120.32%, the absolute return is $903,500, and the annual yield is 35.77%.

In terms of return rate, the month with the maximum loss was from January 2020 to February 2021, with a loss ratio of 11.68%.In absolute terms, the month with the maximum loss was from March 2022 to April 2022, with a loss of 16,9600$.

In terms of rate of return, the month with the maximum return was from July 2020 to August 2020, with a return of 21.44%. The month with the maximum absolute return is from June 2022 to July 2022, and the return reached 257900$.

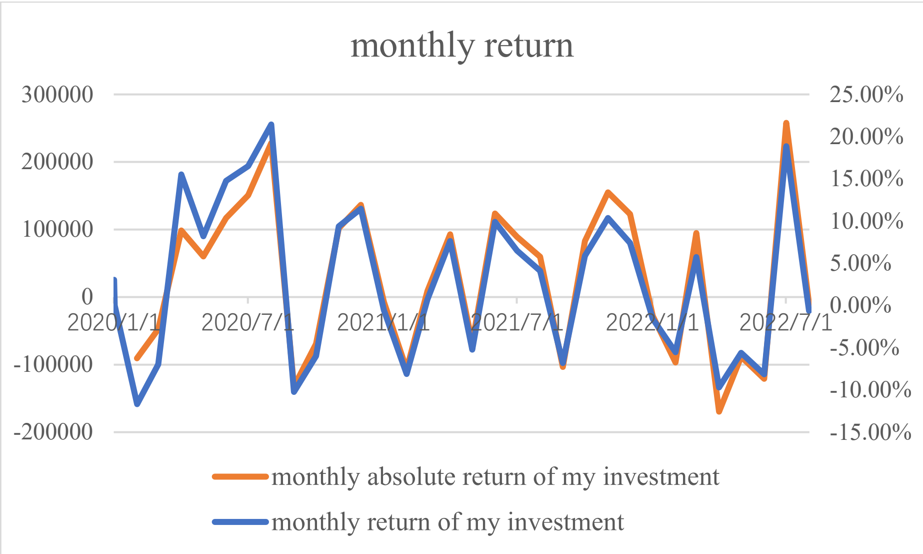

Figure 2. Monthly return of my investment

For better risk management, I want to protect 80% of my gains until June 21st 2024. My current profit is 903500$. So the tolerable loss is 180700$, that is, the stock price can fall by 18.07$ per share, that is, to buy a put option with a strike price of 147.37$. there are only put options with strike prices of 145$ and 150$ in the exchange Options, the prices at the close of trading on August 1st were: 16.80$ and 18.95$. Through the linear interpolation method, the price of the put option with the strike price of 147.37$ can be calculated to be about 17.82$, that is, I can spend 178,200 to ensure that on June 21st 2024, my investment return can keep at least 80% of the current return.

Of course, in order to reduce costs, we are willing to sell options to finance the purchase of the above options, that is, to give up some opportunities to increase investment income to pay for the funds needed to buy put options. Call options with strike prices of 190$ and 195$ are 19.15$ and 17.35$ respectively. Using interpolation, we can see that the strike price of a call option with a price of 17.82$ per share is 193.69$.

I can also directly give up a certain upward opportunity to get a premium. On August 1st, 2022, the closing price of Apple stock is 161.51$, I sell a call maturing on June 21st 2024 with strike at 20% above the closing price on On August 1st 2022, the strike price of this option is 193.81$, and the price of this option can be roughly calculated to be 17.77$. In the case of ignoring the subsequent dividends, when the option expires, the investment portfolio I hold will be The value map can be represented in Figure 3.

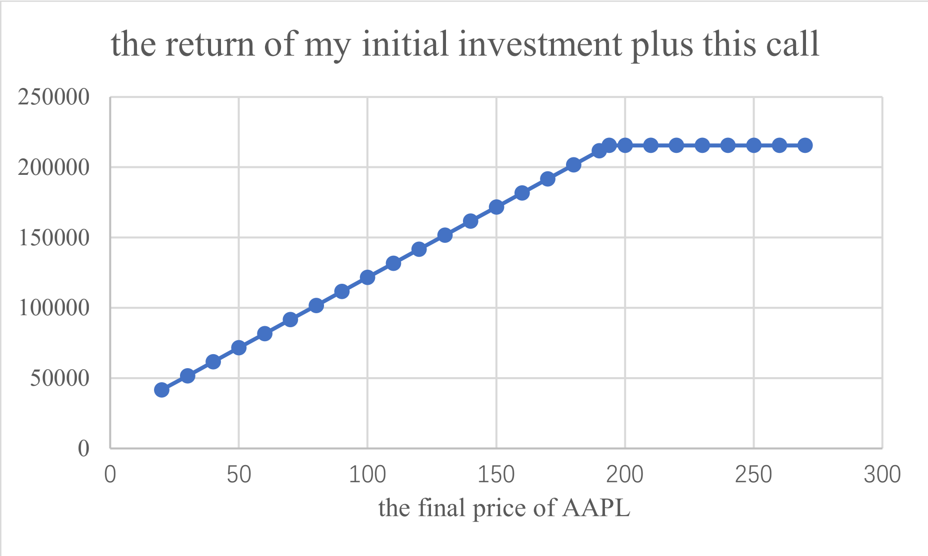

Figure 3. The return of my initial investment plus this call

4. Conclusion

As a financial derivative product, options make the financial market more complete and products more abundant. For investors, the emergence of options allows investors to effectively hedge against risks that are unfavorable to themselves, and retain risks that are beneficial to themselves to prevent losses from price fluctuations; at the same time, it enables investors to lock in costs; further relying on the arbitrage strategy of options, As long as investors make correct judgments about the future, no matter which direction changes can make profits; at the same time, investors can also obtain high returns through the high leverage feature of options.

However, as the most flexible financial derivatives, options must be effectively regulated, otherwise for speculative traders, options trading can bring unlimited losses.

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