

The Diversification of Portfolios under Different Conventional Scenarios in the Shock of COVID-19

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Abstract. Asset portfolio theory has traditionally been widely used in the financial field. Several studies have been made to demonstrate the superiority of this theory and many people have done many efforts to develop it. However, in the last two years, various industries have been hit by the COVID-19, and this paper is designed to investigate whether the asset portfolio still has a good risk diversification ability under different constraints in this context. In this paper, a portfolio is constructed using stocks of 10 companies from 5 industries and the SPX index, and the Markowitz Model and Index Model are used to calculate the returns and standard deviations of the portfolio under the four constraints. As a result, the limitation of short selling to stocks will increase the risk and reduce the profitability of the portfolio somehow but a well-diversified portfolio can reach the most requirements in the market, and the inclusion of the broad index into the portfolio has positive effect, which means a better diversification and higher profitability. Consequently, it can be concluded that the portfolio still has good risk diversification even under the shock of the COVID-19. This research could facilitate the study of portfolio theory under several different scenarios as well as contribute to investors' better understanding of the risk diversification.

Keywords: Risk diversification; portfolio theory; Markowitz Model; Index Model; Sharpe ratio.

1. Introduction

Risk is a common phenomenon in social and economic life. In the field of financial investment, the discussion of risk and return has always been a hot and difficult issue. Investments are exposed to systematic and unsystematic risks. Systematic risk is unavoidable. So, if investors want to hedge risks, they need to focus on under the same return, how to minimize unsystematic risk through a reasonable mix of securities investments. So that they can achieve risk diversification in pursuit of the return maximization and risk minimization.

Before the 1950s, although the idea of using portfolios to achieve lower risk and higher return has existed in academia, it mostly focused only on the simple relationship between the investment timing of individual investors and stock prices, or only on a generalized discussion without precise quantitative analysis of the diversification on the risk and return of investments.

In the 1930s, before the emergence of portfolio theory, investors were already using portfolios in practice, but different investors had different understandings of portfolios, and the relevant academic theories had different interpretations and explanations of portfolios.

The total risk of the investment of more than one risky investment is not simply related to the risk of each individual investment separately. At the same time, the risk of the investment of securities in several separate fields would be lower than the risk of the investment focusing on securities in only one field [1].

It has been proposed a theory of ordinal choice under uncertainty, while also noting the phenomena of the tendency to favor higher returns and lower risk" Although this theory did not consider the portfolio [2-3].

The risk of an investment could be eliminated by investing in multiple securities and assumed there is always a portfolio that satisfied both return maximization and risk minimization [4].

An article named "Portfolio Selection" was published in the "Journal of Finance". The portfolio theory assumes that investors are risk averse, which means that they prefer a portfolio with lower risk for the same level of return. At this point, with the x-axis as the risk of the portfolio, i.e., the variance, and the y-axis as the return of the portfolio, the investor can label all possible portfolios on the axes.

Finally, an upward sloping curve can be drawn to connect all the most efficient portfolios, called the "efficient frontier", while the portfolio below the curve is not the most efficient because it does not achieve the maximum return for a given level of risk. Consequently, an investor can construct a portfolio consisting of multiple assets that can achieve higher returns without increasing risk. Alternatively, with a certain rate of return on investments, an investor can construct a portfolio to achieve the lowest risk [5]. After that, this portfolio is developed that there is sufficient assumption that the only feasible asset portfolio is the minimized variance under a given expectation [6]; Later paper did not make such an assumption and suggested that the "frontier" algorithm is suitable for any covariance matrix, which extended the scope of its application [7].

It has been proposed the "Safety-First Portfolio Theory", which takes the mean and variance of a portfolio to select. However, it differs from the previous theory that (1) it requires passive investment mentioned in previous thought [5], while this thought is applicable to any security. (2) it allows investors to select a "worthwhile" portfolio from the efficient frontier mentioned in previous thought [5], while this thought recommends a specific portfolio [8].

Investors were seeking an efficient combination of mean-variance of monetary assets. His purpose is to illustrate the feasibility of holding money. Based on this it has been proposed the famous "Tobin's Separation Theorem" which constructs a theory of portfolio is consist of n kinds of risky assets and one risk-free asset-cash. Since all assets are monetary assets and therefore the risk is market risk [9].

The capital asset pricing model (CAPM) and the Sharpe ratio was developed based on portfolio theory. The Capital Asset Pricing Model (CAPM) describes the relationship between the systematic risk of an asset and its expected return. The Sharpe ratio measures the performance of an investment, after being adjusted for risk, relative to a risk-free asset [10].

Most articles discuss diversification in a general sense, but do not clearly argue why diversification is worthwhile. On this basis he argues for the benefits of diversification on the assumption that risks are independent. Unfortunately, the assumption of risk independence does not always hold in practice, and covariance plays a role [11].

Thus, the birth of modern portfolio theory provided the basis for subsequent related academic developments, and the related theories of portfolio and risk and return assessment became increasingly complete and sophisticated.

Diversification is critical to the achievement of risk management and investment strategy objectives. However, in the last two years, various industries have been affected by the COVID-19. In this situation, whether investors or financial practitioners can still use portfolio theory to maintain returns with a risk diversification has not been studied using quantitative methods. Based on this, I conducted a specific quantitative analysis of the last 10 years of data up to 2021 to verify whether the ability to diversify risk using a portfolio is affected by the COVID-19. It is found that most constraints have little influence on the risk and profitability of a well-diversified portfolio. If there is no limitation of short selling, it is a good choice to short sell some stocks in the portfolio, because it can reduce the correlation of the whole portfolio. In addition, we can introduce a broad index into portfolio, which can also reduce the risk and strengthen the maximum profitability. In conclusion, the portfolio can still achieve risk diversification under the shock of the COVID-19. The composition of this paper is mainly divided into: the first part is the background of this paper's research and previous literature review on related issues, the second part is the introduction of the selected sectors and companies in the portfolio, the third part gives the model chosen to be used in this paper and the related data calculation results and related analysis, and the fourth part is concludes the feasibility of portfolio theory on risk diversification under COVID-19.

2. The initial data of individual stocks and SPX

2.1 The introduction of stocks and their stocks prices

To build a portfolio with well diversification, I select 10 stocks from 5 industry field healthcare, technology, energy, consumer defensive and consumer cyclical. They are Pfizer Inc (PFZ),

Qualcomm Incorporated (QCOM), Akamai Technologies Inc (AKAM), Microsoft Corporation (MSFT), Chevron Corporation (CVX), Exxon Mobil Corporation (XOM), Imperial Oil Limited (IMO), The Coca-Cola Company (KO), PepsiCo, Inc (PEP) and McDonald’s Corporation (MCD).

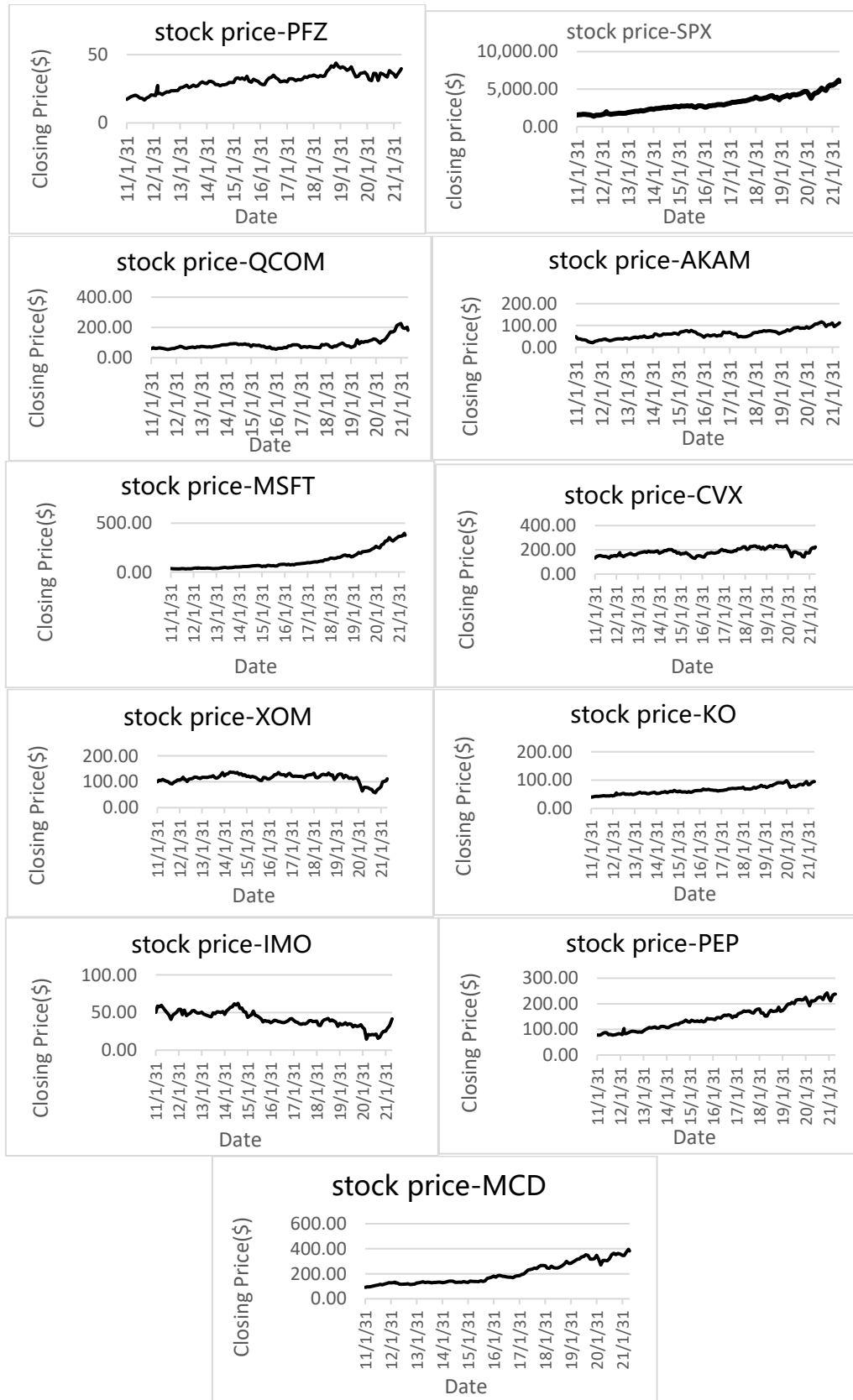


Fig. 1 Stock prices for individual stocks and SPX
 Source: Investing Finance [12]

(1) Healthcare

I select Pfizer as they represent of healthcare field. For more than 170 years, PFZ has been a science-based, creative, patient-first biopharmaceutical company based in New York, USA.

(2) Technology

QCOM is a wireless communications technology research and development company which was founded in 1985. AKAM is a CDN service provider. AKAM provides comprehensive cloud security services that offer acceleration of HD streaming media worldwide. MSFT is an American multinational computer technology company whose best-selling products are the Microsoft Windows operating system and Microsoft Office software.

(3) Energy

CVX is one of the world's largest multinational energy companies with operations in more than 180 countries. XOM is the largest publicly traded U.S. oil company by market capitalization. IMO is one of the largest integrated oil companies in Canada. Its operations cover all segments of the oil and gas resources chain, including exploration, production, and marketing.

(4) Consumer Defensive

With a 48 percent global market share and two of the top three beverages, KO is the world's largest beverage manufacturer. PEP is a significant consumer goods company in the world.

(5) Consumer Cyclical

MCD is a major global restaurant chain. Burgers, French fries, fried chicken, drinks, ice cream, salads, fruit, and other fast-food products are the main goods sold.

2.2 The correlations of individual stocks and SPX

Table 1. Correlations of the returns of 10 stocks and SPX index

correlations	SPX	PFZ	QCOM	AKAM	MSFT	CVX	XOM	IMO	KO	PEP	MCD
SPX	1.00	0.55	0.52	0.34	0.58	0.70	0.69	0.67	0.51	0.53	0.52
PFZ	0.55	1.00	0.29	0.18	0.13	0.43	0.37	0.35	0.35	0.48	0.38
QCOM	0.52	0.29	1.00	0.19	0.40	0.24	0.27	0.31	0.20	0.35	0.22
AKAM	0.34	0.18	0.19	1.00	0.24	0.17	0.13	0.21	0.09	0.13	0.03
MSFT	0.58	0.13	0.40	0.24	1.00	0.34	0.30	0.32	0.26	0.28	0.36
CVX	0.70	0.43	0.24	0.17	0.34	1.00	0.86	0.78	0.38	0.34	0.42
XOM	0.69	0.37	0.27	0.13	0.30	0.86	1.00	0.78	0.36	0.31	0.35
IMO	0.67	0.35	0.31	0.21	0.32	0.78	0.78	1.00	0.35	0.27	0.37
KO	0.51	0.35	0.20	0.09	0.26	0.38	0.36	0.35	1.00	0.69	0.61
PEP	0.53	0.48	0.35	0.13	0.28	0.34	0.31	0.27	0.69	1.00	0.53
MCD	0.52	0.38	0.22	0.03	0.36	0.42	0.35	0.37	0.61	0.53	1.00

Firstly, we can see that except AKAM, the absolute value of correlation coefficients(r) between SPX and all other each nine stocks greater or equal to 0.5 and less than 0.8, and the correlation between SPX and AKAM is not very low. It is because that on the one hand, the broad market index is calculated by the weighted average of individual stocks, so the relationship between the broad market and individual stocks is proportional, that is, when the broad market rises, individual stocks will also rise, and when the broad market falls, individual stocks fall; on the other hand, the broad market index will influence individual stocks performance, usually when the broad market index falls, investors will think that the current stock performance is not so good and will reduce trading, resulting in a decline in stock prices.

Then, as for the correlations between individual stocks, most correlation coefficient is less than 0.5, which represents the low correlation.

But there are small number of the absolute value of correlation coefficients greater or equal to 0.5 and less than 0.8 and even a correlation coefficient greater than 0.8. The reason is that these stocks are in a same industry field. It will not bring too much impact to the diversification ability of this

portfolio. In addition, for the whole portfolio, there are 5 industry fields to diversify the risk. So, from an overall perspective, we can assume that this portfolio is well-diversified and bears less risk while achieving the same return.

3. Two models and five scenarios

The data of above 10 stocks and SPX index is used to form a portfolio using Markowitz Model and Index Model.

$$\text{Portfolio} = W_{SPX} \times \text{stock}_{SPX} + W_{PFZ} \times \text{stock}_{PFZ} + W_{QCOM} \times \text{stock}_{QCOM} + W_{AKAM} \times \text{stock}_{AKAM} + W_{MSFT} \times \text{stock}_{MSFT} + W_{CVX} \times \text{stock}_{CVX} + W_{XOM} \times \text{stock}_{XOM} + W_{IMO} \times \text{stock}_{IMO} + W_{KO} \times \text{stock}_{KO} + W_{PEP} \times \text{stock}_{PEP} + W_{MCD} \times \text{stock}_{MCD} \quad (1)$$

3.1 Markowitz Mean-Variance Model

How to quantify portfolio risk and return, and how to balance these two indications for asset allocation, is a pressing issue for investors. Consequently, Markowitz theory was created by Markowitz in the 1950s and early 1960s. The main problem of asset allocation is how to diversify investments to minimize risk and maximize return. The Markowitz mean-variance model is used to find out the optimal asset allocation ratio and it is the first to introduce mathematical and statistical methods into portfolio theory.

Assuming that there are n kinds of risky assets in the market, the returns of the assets are r_1, r_2, \dots, r_n , and the investor's allocation to each risky asset is w_1, w_2, \dots, w_n , the return of the portfolio is $r_p = \sum_{i=1}^n w_i r_i$. ($\sum_{i=1}^n w_i = 1$)

Thereby, the expected rate of return of the portfolio is:

$$E(r_p) = \sum_{i=1}^n w_i E(r_i) \quad (2)$$

The variance of return of the portfolio is:

$$\text{Var}(r_p) = \sum_{i=1}^n w_i^2 \text{Var}(r_i) + \sum_{i \neq j} w_i w_j \text{Cov}(r_i, r_j) \quad (3)$$

The upper half of the curve is called the Efficient Frontier and the points inside the curve are called the feasible set.

3.2 Index model

Sharpe proposed the index model, which provides insight into portfolio diversification. Assume we select a portfolio of n securities that is equally weighted.

If M denotes market index, then, its excess return is $R_M = r_m - r_f$, and standard deviation σ_M . The factor β_i can be estimated using linear regression between observations of R_i and R_M . α_i is non-market risk premium. The excess rate of return on each security is given by

$$R_i = \alpha_i + \beta_i R_M + \epsilon_i \quad (4)$$

Similarly, we can write the excess return on the portfolio of stocks as

$$R_p = \alpha_p + \beta_p R_M + \epsilon_p \quad (5)$$

As for variance, firstly, the total risk consists of systematic risk and firm-specific risk, so the variance of return on each security is

$$\sigma_i^2 = \beta_i^2 \sigma_M^2 + \sigma^2(\epsilon_i) \quad (6)$$

So, the variance of the portfolio is

$$\sigma_p^2 = \beta_p^2 \sigma_M^2 + \sigma^2(e_p) \tag{7}$$

As the number of stocks in this portfolio grows, the percentage of portfolio risk attributed to non-market factors shrinks. Investors will be unconcerned about this component of the risk because it is diversified elsewhere. Market risk, on the other hand, persists regardless of the number of enterprises in the portfolio. As a result, systemic risk is considered non-diversifiable.

3.3 The portfolio under 5 scenarios

To examine the performance of the portfolio, this paper applies several 5 scenarios:

(1) A "free" issue with no further optimization restrictions to show how the area of allowed portfolios in general, and the efficient frontier, look when no constraints are applied.

(2) This extra optimization restriction is intended to mimic FINRA Regulation T, which permits broker-dealers to allow their customers to have positions that are backed in whole or in part by the customer's account equity (50 percent or more):

$$\sum_{i=1}^{11} |w_i| \leq 2, \tag{8}$$

(3) This additional optimization constraint is intended to emulate certain arbitrary "box" weight constraints that the client may provide:

$$|w_i| \leq 1, \text{ for } \forall I, \tag{9}$$

(4) This extra optimization constraint is intended to emulate typical limitations in the US mutual fund industry: a US open-ended mutual fund is not permitted to have any short positions; for further information, see Section 12 (a) (3) of the Investment Company Act of 1940:

$$w_i \geq 0, \text{ for } \forall I, \tag{10}$$

(5) Finally, we'd like to test if include the broad index in our portfolio has a beneficial or negative impact, thus we'll explore one extra optimization constraint:

$$w_1 = 0. \tag{11}$$

4. Result

This section shows the portfolio performance under different scenarios.

4.1 The results and analysis of the minimum variance portfolio using 2 models

Table 2. The weights of each stock and SPX in minimum variance portfolio in 5 scenarios using Markowitz Model

	SPX	Pfizer	QCOM	AK AM	MS FT	CVX	XOM	IMO	KO	PEP	MCD
Constriant1	0.44	0.07	-0.05	0.04	0.06	-0.10	0.14	-0.13	0.10	0.21	0.19
Constriant2	0.44	0.07	-0.05	0.04	0.06	-0.10	0.14	-0.13	0.10	0.21	0.19
Constriant3	0.44	0.07	-0.05	0.04	0.06	-0.10	0.14	-0.13	0.10	0.21	0.19
Constriant4	0.22	0.07	0.00	0.04	0.08	0.00	0.00	0.00	0.12	0.26	0.18
Constriant5	0.00	0.14	-0.02	0.07	0.15	-0.08	0.22	-0.11	0.15	0.25	0.23

Table 3. The return, standard deviation, and Sharpe ratio of minimum variance portfolio in 5 scenarios using Markowitz Model

	Return	Standard deviation	Sharpe ratio
Constriant1	13.463%	10.798%	1.247
Constriant2	13.463%	10.798%	1.247
Constriant3	13.463%	10.798%	1.247
Constriant4	12.970%	11.408%	1.137
Constriant5	12.807%	11.122%	1.151

The data of minimum variance portfolio is same under constraint 1, constraint 2 and constraint 3. This means that the portfolio itself satisfies constraint 2 and constraint 3, so these two restrictions do not reduce the portfolio's profitability risk, or we can say this portfolio have reach the stringent requirements for the limitation of risk.

The standard deviation under constraint 4 is the biggest. Meanwhile, It is clear that some weights of individual stocks under other constraints are negative, which means there are short position of stocks in the portfolio. According to Table 1, the correlation coefficient among these stocks is always positive, so to diversify the risk, investors need to short sell some stocks. From the above we conclude that the limitation of short selling to stocks reduces the portfolio's ability to diversify risk.

Through the comparation of the minimum variance portfolio under constraint 5 and constraint 1, we can know that the standard deviation is greater than under constraint 1 in minimum variance portfolio, which means more risk. So even the correlation coefficients between SPX and other individual stocks is not very low or near to 0, the introduction of the broad market index leads to the better diversification ability of overall portfolio but rather than increases the risk.

Table 4. The weights of each stock and SPX in minimum variance portfolio in 5 scenarios using Index Model

	SPX	Pfizer	QCOM	AKAM	MSFT	CVX	XOM	IMO	KO	PEP	MCD
Constriant1	0.32	0.09	-0.03	0.01	0.04	-0.07	-0.06	-0.10	0.25	0.30	0.23
Constriant2	0.32	0.09	-0.03	0.01	0.04	-0.07	-0.06	-0.10	0.25	0.30	0.23
Constriant3	0.32	0.09	-0.03	0.01	0.04	-0.07	-0.06	-0.10	0.25	0.30	0.23
Constriant4	0.00	0.09	0.00	0.01	0.03	0.00	0.00	0.00	0.26	0.32	0.25
Constriant5	0.0	0.13	-0.02	0.02	0.08	-0.03	-0.02	-0.08	0.29	0.35	0.28

Table 5. The return, standard deviation, and Sharpe ratio of minimum variance portfolio in 5 scenarios using Index Model

	Return	Standard deviation	Sharpe ratio
Constriant1	14.016%	9.666%	1.450
Constriant2	14.016%	9.666%	1.450
Constriant3	14.016%	9.666%	1.450
Constriant4	11.683%	10.260%	1.139
Constriant5	13.080%	9.827%	1.331

The result under 5 scenarios using Index model is same as using Markowitz Model. It is that constraint 2 and constraint 3 do not reduce the portfolio's profitability risk, constraint 4 impairs the portfolio's ability to diversify risk and constraint 5 brings more risk to the minimum variance portfolio.

4.2 The results and analysis of the maximum Sharpe ratio portfolio using 2 models

The above discussion of minimum variance portfolios focuses on the risk of the portfolio, in addition It is necessary to consider the return of the portfolio, so we use the Sharpe ratio to consider the return and risk together

Table 6. The weights of each stock and SPX in maximum Sharpe ratio portfolio in 5 scenarios using Markowitz Model

	SPX	Pfizer	QCOM	AKAM	MSFT	CVX	XOM	IMO	KO	PEP	MCD
Constriant1	0.79	-0.01	-0.08	0.00	0.34	0.01	-0.22	-0.16	-0.19	0.19	0.33
Constriant2	0.70	0.00	-0.06	0.00	0.34	0.00	-0.17	-0.16	-0.09	0.15	0.30
Constriant3	0.79	-0.01	-0.08	0.00	0.34	0.01	-0.22	-0.16	-0.19	0.19	0.33
Constriant4	0.00	0.03	0.00	0.02	0.47	0.00	0.00	0.00	0.00	0.17	0.28
Constriant5	0.00	0.11	-0.03	0.04	0.54	0.05	-0.11	-0.14	-0.12	0.26	0.40

Table 7. The return, standard deviation, and Sharpe ratio of maximum Sharpe ratio portfolio in 5 scenarios using Markowitz Model

	Return	Standard deviation	Sharpe ratio
Constriant1	21.883%	13.767%	1.590
Constriant2	21.046%	13.284%	1.584
Constriant3	21.882%	13.767%	1.590
Constriant4	18.455%	13.472%	1.370
Constriant5	21.613%	14.449%	1.496

According to Table 7, Both return and standard deviation under constraint 2 are smaller than under constraint 1, and the Sharpe ratio is roughly same as under constraint 1. From this we conclude that this constraint has weak restriction on the portfolio.

The data of minimum variance portfolio is same under constraint 3 and constraint 1. This means that the portfolio itself satisfies constraint 3, so this restriction have no impact on the portfolio's profitability.

The Sharpe ratio under constraint 4 is the smallest. This means that investors have smallest return for each unit of risk taken under constraint 4. Meanwhile, It is clear that some weights of individual stocks is negative, which means there is short position of stocks in the portfolio. From the above we can conclude that the limitation of short selling to stocks reduces the profitability of the portfolio.

Through the observation and comparison of the minimum variance portfolio under constraint 5 and constraint 1, we can find that the return under constraint 5 is less than under constraints 1 and the standard deviation is greater than under constraint 1 in minimum variance portfolio, which means less profit and more risk. Consequently, we can conclude that the inclusion of the broad index (SPX) into our portfolio has positive effect on whole portfolio, that is higher Sharpe ratio, which means a higher maximum profitability.

Table 8. The weights of each stock and SPX in maximum Sharpe ratio portfolio in 5 scenarios using Index Model

	SPX	Pfizer	QCOM	AKAM	MSFT	CVX	XOM	IMO	KO	PEP	MCD
Constriant1	0.97	-0.03	-0.01	0.00	0.32	-0.21	-0.32	-0.21	0.04	0.18	0.26
Constriant2	0.60	0.00	0.00	0.01	0.31	-0.12	-0.20	-0.16	0.08	0.21	0.28
Constriant3	0.97	-0.03	-0.01	0.00	0.32	-0.21	-0.32	-0.21	0.04	0.18	0.26
Constriant4	0.00	0.00	0.00	0.01	0.41	0.00	0.00	0.00	0.03	0.20	0.32
Constriant5	0.00	0.06	0.03	0.03	0.46	-0.11	-0.22	-0.15	0.16	0.32	0.40

Table 9. The return, standard deviation, and Sharpe ratio of maximum Sharpe ratio portfolio in 5 scenarios using Index Model

	Return	Standard deviation	Sharpe ratio
Constriant1	23.735%	12.580%	1.887
Constriant2	20.981%	11.404%	1.840
Constriant3	23.734%	12.580%	1.887
Constriant4	17.668%	12.527%	1.410
Constriant5	21.782%	12.682%	1.718

According to Table 8 and Table 9, the result in 5 scenarios obtained by Index model is similar with using Markowitz Model. But most returns using Index Model are greater than using Markowitz Model and standard deviations using Index Model are all smaller than using Markowitz Model, which lead to the greater Sharpe ratio in Index Model. It means that the slope capital allocation line is steeper, so investors can have larger effectiveness.

5. Conclusion

To verify that even under the impact of the COVID-19, the portfolio theory can still achieve a good risk diversification in the long run overall, the authors selected stocks from 5 sectors and the SPX index for the last 10 years including 2020 and 2021 to construct a new portfolio and used the Markowitz Model and the Index Model based on the previous related theories in 5 scenarios respectively. Their returns and standard deviations are calculated. It is found that the portfolio is well-diversified because the correlation coefficients between the portfolios are mostly small. Further, the five restrictions are discussed.

In terms of the minimum risk in the five scenarios, the conclusions using Markowitz Model and Index Model are the same. There is no additional limiting effect on the portfolio for constraint that allows broker-dealers to allow their customers to have positions that are backed 50% or more by the customer's account equity, with a constraint designed to emulate some arbitrary "box" weight requirements, which may be provided by the client compared to no constraint, so it can be concluded that a well-diversified portfolio already satisfies these limits. For constraint 4, the prohibition on holding short positions increases the minimum risk of the portfolio, so it can be concluded that to reduce the risk of a portfolio, one can choose to hold short positions in some of the stocks. For constraint that allows the weight of SPX equal to 0, the minimum variance is also increased, which means that the introduction of a Broad Index can further reduce the risk of the portfolio.

However, when investing, in addition to considering risk, return is also an important factor. So, the author again considered the maximum Sharpe ratio portfolio under 5 scenarios. Again, using the Markowitz Model and Index Model yielded the same conclusion. Constraint designed to simulate the Regulation T by FINRA has a very weak limiting effect on a well-diversified portfolio, while the constraint designed to simulate some arbitrary "box" constraints on weights has no limiting effect on the portfolio, and the prohibition of holding short positions weakens the maximum profitability of the portfolio, while the data of constraint that there is no broad index in portfolio shows that the introduction of Broad Index is a good choice for increasing the maximum profitability of the portfolio.

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