Investment Portfolio's Efficiency Evaluation for Markowitz Model and the Index Model

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Abstract. In the financial market, the investment portfolio is always a popular topic that investors are interested in and look forward to exploring as well. Since it does affect the expected return and risk of their investment. In this paper, focusing on two of the main models that are Markowitz model and the index model. The paper also collect data from the stock market to analyze different models under different constraints that have their own practical meaning. The result of 5 constraints is concluded below by conducting an efficient frontier and quick ratio. And it finds out that the index model simplified the calculation of variance matrix but assumption under the Markowitz model is more sustainable in reality. The initial result suggests that both models have their advantages and shortcomings, suggesting it is hard to determine which one is more favorable. However, after further discussion between models. the study found that as long as the original data is accurate, Markowitz model gives better results than index model. If investors want to attain very accurate data, Markowitz model is the best choice. In this paper, the research on the two models of investment portfolios can be a consideration when people are choosing their portfolios. It can be used to create a higher return portfolio as well as a lower risk one, which brings people more thoughts on investing portfolios in promoting the finance market in sense.

Keywords: component; Markowitz Model, the Index Model, stock market.

1. Introduction

The stock market in the United States attracts investors across the world who each day pay close attention to the variation of stock indexes, as the blooming and recession of the stock market entail the profits of corporations on a tremendous scale. Generally speaking, the stock market is known to have a high level of instability and subject to the influence of several outside factors. Therefore, it is crucial for investors to have a good grasp of the construction of an investment portfolio in order to maximize returns while reducing risks, with the optimal combination of stocks. In fact, the study of the theories pertinent to and application of market investment portfolio has been the focus of ongoing researches for economists over decades.

Among all theories currently available, the Markowitz Model and the Index Model are two of the most popular, and scholars so far still have disputes over which model explains the economic phenomenon better. Hence, our study compiles the historical data of the price of different assets on the market, in an effort to find out which one of the two can be better applied to the current market.

Index model is utilized commonly in the existing investment decisions. Hristache, Juditsky and Spokoiny found [1] that Single-index modeling is widely applied in, for example, economic studies as a compromise between too restrictive parametric models and flexible but hardly estimable purely non-parametric models. By such model-ling, the statistical analysis usually focuses on estimating the index coefficients. Mandal[2]suggests that the simplified model proposes that the relationship between each pair of securities can indirectly be measured by comparing each security to a common factor 'market performance index' that is shared amongst all the securities. Mary and Rathika believed [3] that The single-index model is based on the assumption that stocks vary together because of the common movement in the stock market and there are no effects beyond the market that account for the stock co-movement. The expected return, standard deviation and covariance of the single index model represent the joint movement of securities. Lillo and Mantegna [4] found the single-index model is a basic model of price dynamics in financial markets. It assumes that the re- turns of all stocks are controlled by one factor, usually called the "market". Varghese and Joseph show [5] that as per this model, founded by William Sharpe, expected return of the portfolio is the weighted average of the market-related and non-market related component of the expected return of the individual security.

There are also a lot of literature related to Markowitz model. and Millegard found [6] Markowitz model to minimize risk subject to a given expected return is an optimization problem. One wants to find the optimal amount of money to invest in each asset. Varghese and Joseph found [7] Most people agree that holding two stocks is less risky than holding one stock. As per the model introduced by Harry Markowitz, the expected return of a portfolio of securities is the weighted average expected return of its component securities. The proportion of the component securities in the current value of the portfolio is used as weight. Lestari [8] suggests Markowitz model is a method that can be used as an alternative by investors as a basis for making investment decisions. The basic components in calculations using the Markowitz Model are less than calculations using the Single Index Model. Marling and Emanuelsson found [9] Markowitz's portfolio theory provides a method to analyze how good a given portfolio is based on only the means and the variance of the returns of the assets contained in the portfolio. Mangram found [10] an investment framework for the selection and construction of investment portfolios based on the maximization of expected portfolio returns and simultaneous minimization of investment risk.

In this paper, we search 20 years of historical daily total return data for six stocks, which belong in pairs to three different industry groups, one (S&P 500) equity index (a total of seven risky assets) and a proxy for risk-free rate (1-month Fed Funds rate). Then we use several full Markowitz Model ("MM"), alongside the Index Model ("IM"). We set four different constraints and free constraints separately and find the Permissible Portfolios Region, which combines the Efficient Frontier, the Minimal Risk or Variance Frontier, and the Minimal Return Frontier in five constraints. In order to reduce non-Gaussian effects, we aggregate the daily data to the monthly observations. As a result, the effective boundary of the exponential model is higher than that of the Markowitz model, and the standard deviation increases, the effective boundary of the Markowitz model is higher than that of the exponential model. If the raw data is accurate, the Markowitz model is superior to the exponential model. If we want to get very accurate data, the Markowitz model is the best choice.

The remainder of the paper is organized as follows: Section 2 describes the data and graphs; Section 3 performs Markowitz model and index model to identify minimum risk portfolio and maximum sharp ratio portfolio in the case of having constraints and not having constraints; Section 4 analyzes data and graphs to explain how constraints affect Markowitz model and index model; Section 5 compares Markowitz model and index model to determine the effectiveness of the two models and shows the out-of-sample forecasting; The last section presents our conclusions.

Document and are identified in italic type, within parentheses, following the example. Some components, such as multilevel equations, graphics, and tables, are not prescribed, although the various table text styles are provided. The format will need to create these components, incorporating the applicable criteria that follow.

2. DATA

We collect six different stocks, the rate of return for the S&P 500 index, and the one-month, risk-free annual Fed Funds rate from Yahoo Finance in our research, for 20 years from November 9th, 2000, to November 10th, 2020. The selected stocks basically fall into three distinct categories: KO(Coca-Cola) and MCD(McDonald's) are related to the food industry, NVDA(Nvidia) and CSCO (Cisco Systems) to high-tech, CVX(Chevron), and XOM(ExxonMobil) to the industry of oil. The stocks within each category could potentially have more correlations to each other due to the relevance of product in the market which the affiliated companies produce.

SPX is the standard & Poor's 500 index, in which is a stock market index in the United States contains 500 publicly traded domestic companies. It performed as the best overall measurement of American stock market for most investors, as known for public (https://www.britannica.com/topic/SandP-500).

NVDA operates as a visual computing company worldwide, including Graphics and Compute & Networking. Graphics offers GeForce GPUs for gaming and PCs. The Compute & Networking offers Data Center platforms and systems for AI, HPC, and accelerated computing (Yahoo finance, 2021).

CSCO works on design, manufactures, and sells internet protocol based networking and other products related to the communications and information technology industry around the world. CSCO is focusing on infrastructure platforms, such as networking technologies of switching, routing, wireless, and data center products that are used to help networking capabilities (Yahoo finance,2021).

XOM explores for and produces crude oil and natural gas in the united states and internationally. The company operates through the upstream, downstream, and chemical. The upstream focus on crude oil and natural gas. The downstream works on manufactures and trades petroleum products. The chemical offers petrochemicals (https://www.wsj.com/market-data/quotes/XOM/company-people).

CVX is the company through its subsidiaries, engages in integrated energy, chemicals, and petroleum operations worldwide. The company also has upstream focus on exploration, development, and production of crude oil and natural gas, and downstream works on refining crude oil into petroleum products (Yahoo finance, 2021).

KO is a beverage company, manufactures, markets and sells various nonalcoholic beverages worldwide. The company provides sparkling soft drinks, water, enhanced water, and sports drinks, juice, dairy and tea and coffee, and energy drinks. The company operates through a network of independent bottling partners, distributors wholesalers, and retailers, as well as through bottling and distribution operators (Yahoo finance, 2021).

MCD operates and franchises McDonald's restaurants in the United States and internationally. its restaurants offer various food products and beverages, as well as a breakfast menu (Yahoo finance, 2021).

Table 1. The annualized mean and the annualized standard deviation of the excess return

	SPX	NVDA	CSCO	XOM	CVX	KO	MCD
Annualized average return	6.16%	35.04%	3.82%	2.39%	7.76%	5.28%	12.34%
Annualized Standard Deviation	15.10%	58.92%	33.03%	19.87%	21.58%	16.54%	19.04%

We calculated for these companies to get the average return and standard deviation. Table 1 showing the annualized mean and the annualized standard deviation of the excess return The

SPX&500 average return is 6.16%. NVDA and CSCO are similar types of corporations that operate in internet and computing. The average return is quite different, NVDA is 35.04% and CSCO is 3.82%. XOM and CVX are in the same industry which focuses on natural energy and resources. The average return for XOM is only 2.39%, but 7.76% for CVX. Last pair is KO and MCD Which operate on beverage and fast food. The average return for KO is 5.28% and 12.34% for MCD. The standard deviation of NVDA is the largest and the second large is CSCO. Other companies are all around 15%-20%.

And we were also computing the correlation between different stocks and index in Table 2

Table 2. correlations between different stocks

	SPX	NVDA	CSCO	XOM	CVX	KO	MCD
SPX	1.0000	0.5095	0.6400	0.5801	0.6057	0.5028	0.5157
NVDA	0.5095	1.0000	0.429	0.2738	0.2881	0.1250	0.1354
CSCO	0.6400	0.4329	1.0000	0.2527	0.2824	0.2529	0.2967
XOM	0.5801	0.2738	0.2527	1.0000	0.8294	0.3677	0.3646
CVX	0.6057	0.2881	0.2824	0.8294	1.0000	0.4025	0.3960
KO	0.5028	0.1250	0.2529	0.3677	0.4025	1.0000	0.4989
MCD	0.5157	0.1354	0.2967	0.3646	0.3646	0.4989	1.0000

All of the stocks are positively related to SPX &500, suggesting the co-movement which also showing in Figure 3. Except for SPX, the companies from the same industry have a relatively strong correlation than across industries. Such as 0.4329 between NVDA and CSCO, 0.8294 between XOM and CVX, 0.4989 between KO and MCD. This correlation table is used for Markowitz model to calculate portfolio standard deviation.

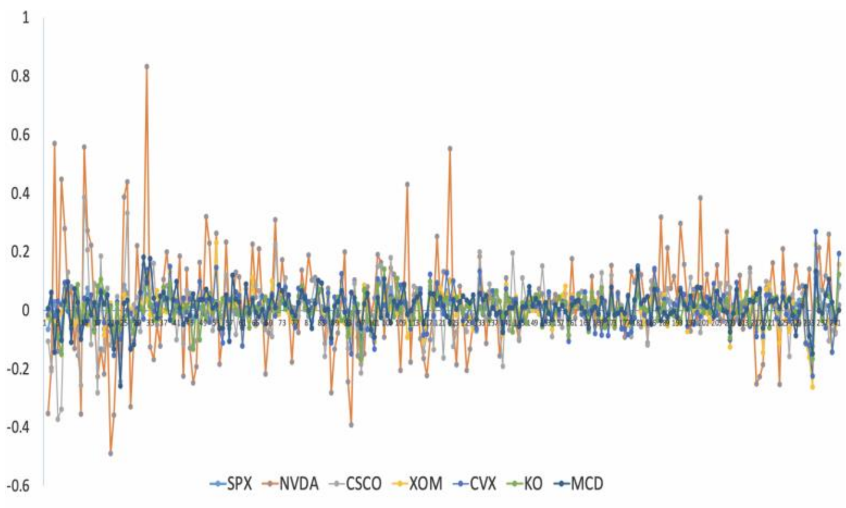

Fig. 1 the excess return changed over period

The movement of SPX and other six cooperation stocks. Positive related with SPX, and co-movement between the same industry companies.

Table 3. beta, alpha, and residual standard deviation for different stocks

	SPX	NVDA	CSCO	XOM	CVX	KO	MCD
Beta	1	1.9884	1.4002	0.7635	0.8659	0.5510	0.6505
Annualized alpha	0%	22.795%	- 4.803%	- 2.310%	2.426%	1.887%	8.334%

Annualized residual Standard Deviation	0%	50.70%	25.38%	16.18%	17.17%	14.30%	16.31%

Beta is the slope between SPX and other six stocks. Since it is used for the calculation of index model. $\sigma_i^2 = \beta_i^2 \sigma_m^2 + \sigma_{ei}^2$

The beta of stock NVDA and CSCO (1.4002) is larger than other stocks, which NVDA is the largest one (1.9884) showing the strong relationship of variance between NVDA and SPX market index. KO has a relatively low beta (0.5510) suggesting a weaker relationship of variance with SPX. Alpha is the intercept, NVDA also has the largest alpha (22.7949%). The residual standard deviation is also important for this calculation. NVDA has the largest residual Standard Deviation as well (50.70%). The bigger beta and residual standard deviation is, the larger the portfolio standard deviation.

3. Methodology

Our study is based on two different models: the Markowitz model, and the Index Model, and the goal is to compare the above models, and determine which one could yield the optimal portfolio with minimal returns / maximum Sharpe ratio.

3.1 Markowitz Model

The Markowitz model is an optimization model for a portfolio with different weights of stocks. It was proposed by American economist Harry Markowitz in 1952, with the aim of minimizing the risk of investment. The Markowitz model has assumptions as following:

The investor is risk-averse and rational

The risk is based on the variabilities (standard deviation) of the return;

The investors' utility function is concave and increasing

An investor either maximizes the return for a given level of risk or minimizes the risk for a given return;

The analysis is based on a single-period model.

The Markowitz Model can be done in typically three steps:

(1). Determine the 'opportunity set'(minimal variance frontier): allowed risk-return combinations

(2). Identify the optimal risky portfolio as the steepest CAL tangent to the opportunity set

(3). Choose the appropriate complete portfolio by mixing with the risk-free asset given risk aversion*

3.2 Index Model

The index model is one of the most commonly used models to measure the return and the risk, and partially simplifies some of the intricate calculations in the Markowitz Model, particularly the estimation of the covariance matrix. This model was developed by William Sharpe in 1963. It has the main assumption that only one macroeconomic factor is considered for the market risk, and such factor can be represented by the return on a market index, for instance, S&P 500 in our study.

3.3 Constraints

On top of that, we conduct analysis on the returns and risks (the risk can be shown in the form of standard deviation) of the portfolio, given 5 different constraints, and each constraint under several frontiers: the efficient frontier and the minimum risk frontier.

The five constraints are:

The sum of absolute values of weights of all stocks smaller than or equal to 2:

$$\sum_{i=1}^{7}|w_i| \leq 2$$

designed to simulate the Regulation T by FINRA, which essentially allows broker-dealers to let their customers to have positions, 50% or more of which are funded by the customer's account equity. Essentially preventing brokers from overly trading and engaging in commission actions.

The absolute value of each weight of the stock smaller than or equal to 1

$$w_i \leq 1, \forall i$$

simulate some arbitrary "box" constraints on weights, which may be provided by the client. This is a common method to avoid high risks in the stocks.

No constraints on the weights at all to illustrate how the area of permissible portfolios in general and the efficient frontier in particular look like in such scenarios.

The weight on each of the stock is greater than or equal to zero

$$w_i \geq 0, \forall i \tag{3}$$

Aim at the simulation of the typical limitations existing in the U.S. mutual fund industry, as a U.S. open-ended mutual fund is not allowed to have any short positions;

The weight of the first stock (broad index) is zero:

$$w_1 = 0 \tag{4}$$

This is intended to find out what effects the inclusion/exclusion of the broad index (SPX) shall bring about.

4. Construction of Portfolios

We are initially provided with the historical daily indexes for the stocks of our interest. The daily index is first converted to monthly, then to the return and excess return with respect to the risk-free rate via inputting and expanding the corresponding formula in the relevant cells. With all these, we can derive the correlation coefficients. Subsequently, the annualized standard deviations and returns can be derived

The efficient frontier, maximum Sharpe frontier, as well as other needed, basic inputs, such as beta, annualized alphas, residuals. Finally, we can use these to construct the return, standard deviation, and Sharpe value of the portfolio, and solve for the maximum and minimum values.

4.1 Minimum Risk

The minimum risk is solved by the Excel solver by changing weights across different stocks, and finding the optimal combination which yields the lowest standard deviation of a portfolio, on a constant level of returns.

Here, it is the graph of the lowest possible variance that can be attained for a given portfolio's expected return. The efficient frontier is the part of the minimal variance frontier that is above the global minimum variance portfolio, which provides the combinations of best returns.

The Markowitz and Index model have the same formula for the return, which takes three parameters as input: α, β, and e. α stands for the difference between the fair (CAPM-predicted) and actually expected rates of return on a stock; β represents the component of a return due to movements in the overall market, and βi is the security's responsiveness to market; movements; e corresponds to the firm-specific risk in the portfolio, that is the unexpected component or return due to unexpected events that are relevant only to this security.

Relevant Formulae:

Standard Deviation for Markowitz Model:

$$\sigma_p = \sqrt{\frac{1}{n}\sum_{i=1}^{n}\frac{1}{n}\sigma_P^2 + \sum_{i=1}^{n}\sum_{j=1, i\neq j}^{n}\frac{1}{n^2}Cov(r_i, r_j)} \tag{5}$$

Standard Deviation for Index Model:

$$\sigma_p = \sqrt{\beta_P^2\sigma_M^2 + \sigma^2(e_p)} \tag{6}$$

Return for both Models:

$$R_p = \frac{1}{n}\sum_{i=1}^{n}\alpha_i + (\frac{1}{n}\sum_{i=1}^{n}\beta_i)\cdot R_M + \frac{1}{n}\sum_{i=1}^{n}e_i \tag{7}$$

4.2 Maximum Sharpe Ratio

The Maximum Sharpe again is derived from the calculation in Excel, dividing each return by the standard deviation. The solver generates the adequate combination which could provide us with a maximal Sharpe ratio.

Sharpe ratio measures the performance of an investment compared to a risk-free asset, herein denoted by the treasury bill rate.

Relevant formulas：

Sharpe Ratio:

$$S = \frac{R_P}{\sigma_P} \tag{8}$$

5. Results analysis

According to the following descriptive statistical analysis (Table 1) of the main variables, the volatility of portfolios. An annualized average return of 7 stocks ranges from 2.39% to 35.04%. Annualized Standard Deviation of these between 15.10% to 58.92%. The statistic of beta is from 0.55 to 1.99 approximately and data in alpha is from -4.8% to 8.3%. Annualized residual Standard Deviation(Standard Deviation) of these are from 0% to 50,70% . In the meanwhile we find the correlation among 7 stocks in table 2.

Table 4. Markowitz model

	Min Standard Deviation	Min return	Min Sharpe ratio	Max Standard Deviation	Max return	Max Sharpe ratio
Constraint 3	12.9813%	0.0475	0.366	34.38%.	33.32%	0.969
Constraint 1	12.9813%	0.0475	0.366	23.32%	21.56%	0.924
Constraint 2	12.9813%	0.0475	0.366	34.38%	33.32%	0.969
Constraint 4	13.3300%	0.0623	0.468	21.10%	17.43%	0.826
Constraint 5	19.8735%	0.1800	0.906	34.11%	33.06%	0.969

Table 5. Index model

	Min Standard Deviation	Min return	Min Sharpe ratio	Max Standard Deviation	Max return	Max Sharpe ratio
Constraint 3	12.18%	0.0525	0.431	24.05%	20.45%	0.850
Constraint 1	12.98%	0.0475	0.366	34.38%	33.32%	0.969
Constraint 2	12.18%	0.0525	0.431	24.05%	20.45%	0.850
Constraint 4	10.05%	0.0735	0.732	13.39%	13.21%	0.986
Constraint 5	12.65%	0.0583	0.460	23.34%	19.82%	0.849

Using both models and limiting them, we come to the following conclusion. Without constraint(free constraint), we found that Minimal risk portfolio lies on the standard deviation of 12.98% and the return of 4.75%. Maximum Sharpe portfolio, the return in this model is 33.32% and the standard deviation in this is 34.378%.Index model has nearly the same efficient frontier, minimal portfolio and Maximum Sharpe as the Markowitz model: Minimal portfolio(return: 5.25% Standard Deviation: 12.18%). Maximum Sharpe portfolio(return: 20.45% Standard Deviation: 24.05%).

Firstly, we set the constraint that the sum of the absolute value of 7 assets' weight is less than 2, we can find that compared with no constraint, Minimal risk portfolio is the same (return: 4.75%, Standard Deviation 12.9813%, Sharpe ratio 0.3659), the constraint 1 does not change the point minimal risk, since the point is the least risky no matter how weights are changed. Maximum Sharpe ratio has changed not only weights but return, standard deviation and also Sharpe. The constraint changes the return and standard deviation, then lowers the Sharpe ratio by controlling the risk of the portfolio. Efficient Frontier is flatter, since the controlling on the constraint, the increase in return by increasing in risk is less realizable. Then broker cannot expand profit by trading as much as possible with high risk. CAL is flatter as the Efficient Frontier becomes flatter. In Index Model. Compared with FM the expected return is estimated to be higher in the index model Index model simplifies the estimation of covariance matrix problems in Markowitz Model but the assumptions of index model can be perfectly broken down when it comes to the real world.

Secondly, we set the constraint that the absolute values of w are no more than one.

In Markowitz model, minimal portfolio lies on the Standard Deviation of 12.98% and the return of 4.75%, which is also similar to free constraint. And in Maximum Sharpe portfolio, its return and Standard Deviation are equal to no constraints . In index model, : Minimal portfolio’s return is 5.25% and Standard Deviation is 12.18%. Maximum Sharpe portfolio’s return is 20.45% and Standard Deviation is 24.05%,which is also similar to free constraint. It shows this constraint has little influence on result.

Thirdly, we set the constraint that the weight on each stock is greater than or equal to zero.

In Markowitz Model, The Minimal Risk portfolio has a Min (Standard Deviation) of 13.3300 %, with a return of 6.2319%. The Minimal Return portfolio has a return of 2.3930%, along with Standard Deviation of 19.8675% and a

Sharpe ratio of 12.045%. The maximal return is 35.0434%, with Standard Deviation = 58.9167%, Sharpe ratio 0.59479. In the Maximal Sharpe Portfolio, the Sharpe ratio equals to 0.82586, with Return = 17.4261%, Standard Deviation = 21.1006%.In Index Model. The Minimal Risk portfolio has a Min(Standard Deviation) of 10.0511%, with a return of 7.3527%(overall lower risk and higher return). The Minimal Return portfolio has a return of 2.3930%(the same as previous), along with Standard Deviation 18.1627%(slightly lower risk) and a Sharpe ratio of 0.13175. The Maximal Return is 35.0434%(doesn't change either), with Standard Deviation = 57.9711%(slightly lower), Sharpe ratio of 0.60450. In the Maximal Sharpe Portfolio, the Sharpe ratio equals to 0.98622, with Return = 13.2054%, Standard Deviation = 13.3898%(less risky but less return).

Lastly, we set that constraint 5: w_1 is equal to 0, we find that the minimal risk portfolio and maximal Sharpe portfolio is very similar to constraint 1 introduced previously, suggesting that without the index the two extreme portfolios have not been affected; the efficient frontier is parallel to the Efficient frontier without constraint but lower, showing a higher expected return with index in the portfolio than without. For Index Model, the results also are similar to constraint 1.

6. Discussion

In this section, we will compare the efficient frontier, as well as the Sharpe ratio of Markowitz model and index model to determine which model is more favorable. We will do this by showing you the graphs of the efficient frontier and data of the Sharpe ratio.

We will compare Markowitz model and index model to determine which model would be better, and we will do this by observing the efficient frontier as well as the maximum Sharpe ratio.

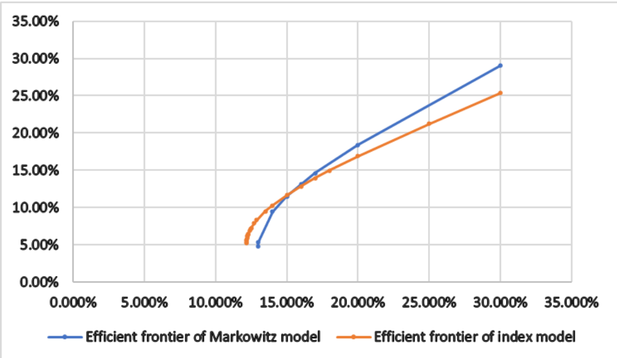

Fig. 2 Efficient frontier of the models (no constraint)

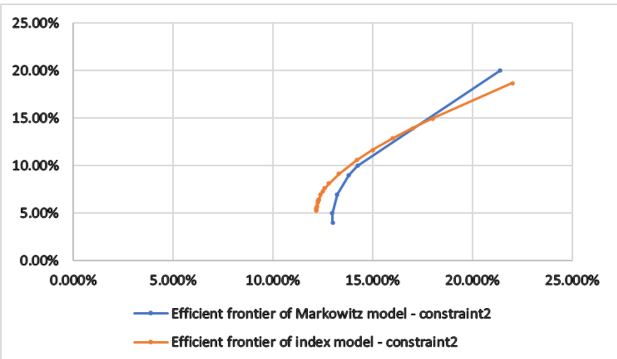

Fig. 3 Efficient frontier of the models (constraint 2)

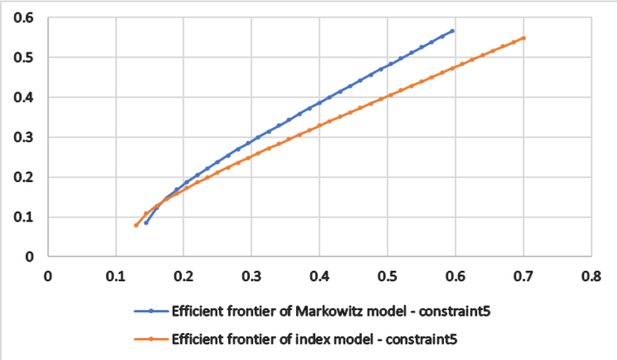

Fig. 4 Efficient frontier of the models (constraint 5)

Figs. 1-3 describe the efficient frontier of both Markowitz model and index model. Figs.1 shows the models with no constraints, while constraints are available for models in Figs. 2-3. By observing these three graphs, it is obvious that they share the same characteristics: efficient frontier of index model lies above the efficient frontier of Markowitz model, and as standard deviation becomes larger, the efficient frontier of Markowitz model goes further above the efficient frontier of the index model. That is, when the standard deviation is large enough, the Markowitz model always has a higher return than that of the index model. It can also be interpreted as the Markowitz model has further smaller risk to attain the same return when the standard deviation is large enough.

Table 6. Sharpe ratio of Markowitz model and index model

models	Sharpe ratio
Markowitz model (no constraint)	0.969
Index model (no constraint)	0.850
Markowitz model (constraint 2)	0.969
Index model (constraint 2)	0.850
Markowitz model (constraint 5)	0.969
Index model (constraint 5)	0.849

Table 6 shows maximum Sharpe ratio of both Markowitz model and index model. We can see from the table that, whether the models have a constraint or not, the Sharpe ratio of Markowitz model is always larger than the Sharpe ratio of index model. The Sharpe ratio of a fund can be obtained by subtracting the risk-free interest rate from the average growth rate of the fund's net value and dividing it by the standard deviation of the growth rate of the fund's net value. The larger the Sharpe ratio, the higher the risk-return of fund unit risk. Thus, with the higher return for each unit of risk, the Markowitz model is more favorable.

These comparisons reveal the defect of the index model. Although the index model can largely simplify the process of calculating, it is not always accurate. It is because the factors ignored by the index model will affect the result, and make the result inaccurate. This idea can be derived by analyzing the data mentioned above. We can see that the index model looks very different from the Markowitz model because of the ignorance of some inessential data. So as long as the original data is accurate, Markowitz model gives better results than index model. If we want to attain very accurate data, Markowitz model is the best choice.

7. Conclusion

In this paper, we have mainly compared the two models which are Markowitz model and index model of analysis of 6 stocks and SPX&500 investment portfolio. We considered 5 constraints for 2 models application.

In short, our study is done through the calculation, manipulation and analysis of the data related to the given 6 stocks' indexes in Excel. We build the portfolio on the basis of two models, the Markowitz model and the Index model. In order to achieve this goal, we take advantage the calculation function of Excel to get all the needed data, including the annualized returns, alphas, betas, standard deviations, and residuals.

The effective boundary of the exponential model is higher than that of the Markowitz model, the standard deviation increases, and the effective boundary of the Markowitz model is further higher than that of the exponential model. At the same time, the Sharpe ratio of the Markowitz model is always greater than the Sharpe ratio of the exponential model, regardless of whether the model has constraints. Thus, the higher the return per unit of risk, the more favorable the Markowitz model. These comparisons reveal flaws in the exponential model. While the indexing model can greatly simplify the calculation process, it is not always accurate. This is because factors that exponential models ignore can affect the results and make them inaccurate. So as long as the raw data is accurate, the Markowitz model is better than the Index model. If we want to get very accurate data, the Markowitz model is the best choice.

The deficiency of this paper is that the sampling is not extensive enough, lack of universality, and the additional constraints are too few and simple, so that it only stays at the theoretical level, away from the actual situation. In future research, we will be committed to more practical data analysis, and use more diverse constraints to enrich our research results. We will also be dedicated to studying many other kinds of index models in addition to the single-index model and find out more about the relationships.

References

[1] Hristache, M., Juditsky, A., & Spokoiny, V. (2001). Direct estimation of the index coefficient in a single-index model. Annals of Statistics, 595-623.

[2] Mandal, N. (2013). Sharpe's single index model and its application to construct optimal portfolio: an empirical study. Great Lake Herald, 7(1), 1-19.

[3] Mary, J. F., & Rathika, G. (2015). The single index model and the construction of optimal portfolio with cnxpharma scrip. International Journal of Management, 6(1), 87-96

[4] Lillo, F., & Mantegna, R. N. (2001). Empirical properties of the variety of a financial portfolio and the single-index model. The European Physical Journal B-Condensed Matter and Complex Systems, 20(4), 503-509.

[5] Varghese, J., & Joseph, A. (2018). A Comparative Study on Markowitz Mean-Variance Model and Sharpe's Single Index Model in the Context of Portfolio Investment. PESQUISA, 3(2), 36-41.

[6] Amu, F., & Millegard, M. (2009). Markowitz portfolio theory. Luettavissa: http://ci, 1-6

[7] Varghese, J., & Joseph, A. (2018). A Comparative Study on Markowitz Mean-Variance Model and Sharpe's Single Index Model in the Context of Portfolio Investment. PESQUISA, 3(2), 36-41.

[8] Lestari, N. P. (2021). SIMULATION OF OPTIMAL PORTFOLIO USING SINGLE INDEX MODEL AND MARKOWITZ MODEL ON LQ-45 INDEX SHARES FOR 2018. JBMP (Jurnal Bisnis, Manajemen dan Perbankan), 7(1), 93-140.

[9] Marling, H., & Emanuelsson, S. (2012). The Markowitz Portfolio Theory. November, 25, 2012.

[10] Mangram, M. E. (2013). A simplified perspective of the Markowitz portfolio theory. Global journal of business research, 7(1), 59-70.