

Comparisons to Investment Portfolios under Markowitz Model and Index Model based on US's Stock Market

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Abstract. In order to construct an investment portfolio, it is crucial to select risky assets and arrange the weights to each asset. This article selects 6 stocks to compose the portfolio and compare their performances under 5 constraints that are always considered in real life by using Markowitz Model and Index Model. These two models would produce different investment portfolios as they take different factors of stock into account. By calculating the maximal return rate determined by Sharpe ratio, and the minimal risk rate determined by standard deviation, comparing the two models and conclude which model is more suitable under each constraint. According to the results, Markowitz model proves that under certain constraints, investors' portfolio selection can be simplified to balance two factors, namely, the expected return and variance of the portfolio. In the case of Index Model, the conclusion is more general and regular. The results would play a significant role in determining the stocks' future performance and help investors constructing their portfolios under different constraints.

Keywords: component; Markowitz Model, Index Model, Minimal risk portfolio, Maximal Sharpe ratio).

1. Introduction

Investment portfolio is usually a combination of stocks, bonds and derivatives that investors and financial institutions owned, and it can also contain assets such as foreign currencies, precious metals, real estates. To diversify and minimize the risk, investors put a certain proportion of their capital into different securities. Expressed through an old saying is that people cannot put all their eggs into one basket. This concept, which is called diversification theory, is formally proposed by Harry Markowitz (Portfolio selection: efficient diversification). However, portfolio matching is not easy. Two main aspects need to be considered. One is the importance of risk and return, which means that we should add risk-free assets for safe and risky assets for higher returns. The other one is the correlation between independent portfolios. For the reason that the existence of similar products will make no sense or even enlarge the risk of investment, it is important to add non-homogeneous securities with a low correlation coefficient to the portfolio to ensure effectiveness.

Hence, confirm the proportion of each security will have great significance on hedging risks and maximizing benefits. This paper has carried out related research on finding an optimal complete portfolio.

This article investigated different literature about the development of portfolio investment theory. Roebbers *et al.* [1] found that, in order to optimize the portfolio, investors allocate the funds in the current assets. The quality of the portfolio is assessed by different factors, including returns and risks, for choosing the optimal portfolio from feasible portfolios. Therefore, optimizing the portfolio is a

kind of multiple objective problems, which means that there must be a balance between the risk and return: a higher return with a higher risk and vice versa. This balance is affected by the risk aversion of investors. A standard model for portfolio optimization is the traditional Markowitz mean-variance portfolio problem [2]. In Markowitz model, the weighted combination of return expectation and return variance represents the balance between expected return and risk. However, literally, the Markowitz model is very persuasive, but there are many criticisms about the model, because of the unrealistic setting. Therefore, Scozzari and Tardella [3] found that it is necessary to extend the simple Markowitz model with cardinality and quantity constraints. Scholten and Read [4] found that the Markowitz model predicts different attitudes to choose the risk in numerous experimental studies. For example, Deck and Schlesinger [5], in the wide environment series, found that most people are risk-averse investors, while most papers almost ignored the minority.

Since Markowitz first found the classic mean-variance model, many new methods and elements have been provided by a lot of researchers. For example, there are the minimum-variance model [6], mean-variance-skewness model [7], mean-semi-variance model [8]. These researches are based on the probability and statistics theory, which can be seen as the classic portfolio theory's extension. However, because of the complex security market in real life, Liu [9] found the uncertain measure, then provided the uncertain theory deeply. This theory has been utilized in wide areas, like medical care, environment, insurance, and education. Particularly, Huang [10] found the risk index model by considering the adjusting problem of portfolio. These research works have found that the uncertain variable could be described as the estimation of security returns in the real world.

In this article, we will use these two models to summarize the change trend of seven stocks in recent years, and get the rate of return, Standard Deviation and Sharpe ratio under different constraints. Through the study of different models under different constraints, we compare the minimum risk and maximum Sharpe ratio portfolio calculated by the two models under different weights of each stock. Finally, we conclude that the algorithm of the index model is better in the exploration of the optimal solution of multiple stock portfolios.

For the structure of the article. First of all, we introduce data, and draw the different charts of collected stocks over time. We calculate the specific value of the maximum, minimum, and average of these stocks in Section 2. Then, in Section 3, we list the research methods involved in this paper one by one. We construct different constraints from different aspects in Section 4, and introduce the related constraints in detail in this part. Next in Section 5, we get the advantages and disadvantages of different portfolios under different constraints and models through the combination and analysis of these data. In the next, we set up different portfolios, and carry out in-depth research. We calculate the optimal solution under different constraints by using the formulas of various results under five constraints. Finally, according to these answers, we can get the advantages and disadvantages of the two models in different scenarios and the additional restrictions of the portfolio area in five cases in Section 6.

2. Data

The historical statistics of a stock can help investors predict its performance in the future. Therefore, to select the most suitable investment group, this article employs the 20 years of historical daily total return data for six stocks: Amazon (AMZN), Apple (AAPL), Goldman Sachs (GS), U.S. Bancorp (USB), United Parcel Service (UPS), and FedEx Corporation (FDX), S&P 500 equity index (SPX), and a proxy for risk-free rate (1-month Fed Funds rate). The sampling period starts from 11 September, 2000, and ends on 11 October, 2020. In order to reduce non-Gaussian effects, this article aggregates the daily data to the monthly observations. After extracting the data at the end of each month, 242 statistics for a total of 7 stock indexes came out.

This article creates two figures depicting the stock's prices changing with the time moving forward. Fig. 1 includes GS, USB, UPS, FDX, and AAPL. Fig. 2 includes SPX and AMZN.

Fig. 1 includes 5 stocks per month over 20 years, and all of them showed an upward trend generally. For GS, it first increased in the first 7 years, and then fluctuated sharply during 2018-2015. After a period of rising, it reached its highest price of \$320.94 in 2017, but then fluctuated with a decreasing trend. For USB, it showed a stable upward but slow upward trend during the first 8 years, and after a sudden decline in 2018, it continuously increased until it reached its highest price of \$105.80 in 2019. For UPS, it fluctuated during 2000 – 2008, and then showed an upward trend with slight fluctuations until 2019. It reached its highest price of \$276.76 in 2020 after a significant sudden growth. For FDX, it fluctuated from 2000 – 2008 and experience a large increasement from 2008 – 2017. With a sudden decrease in 2017 – 2019, it finally showed a significant upward trend and reached its highest price of \$302.73 in 2020. For AAPL, it increased slightly but stable during 2000 – 2019, and increased largely in 2019 then reached its highest price of \$149.35.

Fig. 2 includes SPX and AMZN, and both of them appeared a significant upward trend overall. For SPX, after 2-year decrease, it increased with a slight fluctuation until 2017. After a small decline, it increased significantly with more volatile fluctuations, and finally reached its highest price of \$5255.77 in 2020. For AMZN, it increased with a stable trend from 2000 – 2018, after a short period of fluctuations, it increased significantly and finally reached its highest stock price of \$3450.19 in 2020.

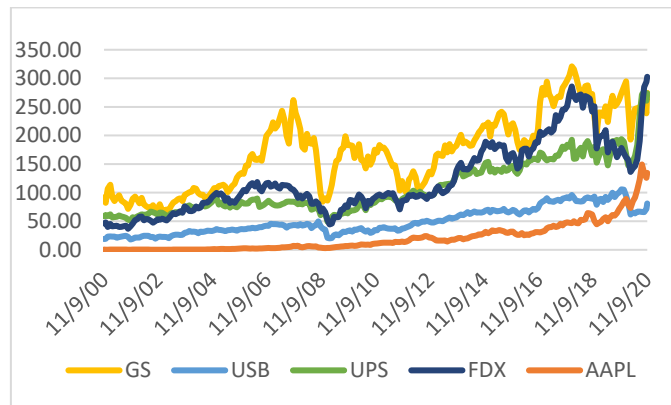


Fig. 1 Monthly stock prices of GS USB UPS FDX AAPL over 20 years

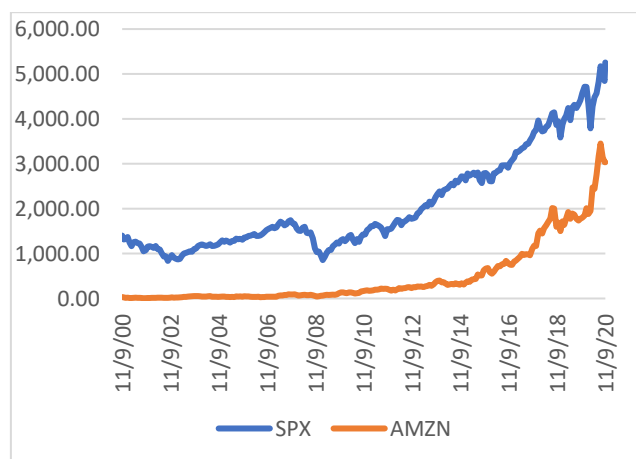


Fig. 2 Monthly stock prices of AMZN and SPX over 20 years

Then, this article discusses the detailed information of different stocks (Table 1) and calculates the correlation coefficient of these stocks (Table 2).

Table 1 lists the highest, lowest, and average stock prices for these 7 stocks. SPX is the highest among the maximal and minimal prices of all of the stocks, at \$5255.77 and \$837.12, respectively. It also reaches the highest price of \$2102.51 when counting all the average stock prices. Besides SPX,

AMZN is the highest among the maximal and average prices of all of the stocks, at \$3450.96 and \$481.02, respectively. GS is the highest among the minimal prices of all the stocks, which is \$66.68.

Table 2 contains the correlation data for these 7 stocks. Besides the correlation between one stock and itself, which is 1, the largest correlation exists between SPX and GS, which is 0.70, and the smallest correlation exists between USB and AMZN, which is 0.08. There are 14 correlations less than 0.50 and 7 correlations larger than 0.

Table 1. Statistic description for 7 stocks

	SPX	AMZN	AAPL	GS	USB	UPS	FDX
Max	5,255.77	3,450.96	149.35	320.94	105.80	276.76	302.73
Min	837.12	5.97	0.25	66.68	18.10	48.16	36.75
Mean	2,102.51	481.02	20.90	171.85	50.00	108.35	122.24

Table 2. Correlations among 7 stocks

	SPX	AMZN	AAPL	GS	USB	UPS	FDX
SPX	1.00	0.50	0.53	0.70	0.57	0.57	0.60
AMZN	0.50	1.00	0.40	0.30	0.08	0.30	0.31
AAPL	0.53	0.40	1.00	0.40	0.14	0.24	0.37
GS	0.70	0.30	0.40	1.00	0.47	0.33	0.41
USB	0.57	0.08	0.14	0.47	1.00	0.43	0.42
UPS	0.57	0.30	0.24	0.33	0.43	1.00	0.68
FDX	0.60	0.31	0.37	0.41	0.42	0.68	1.00

3. Method

In this section, to find the optimal complete portfolio of S&P and the six stocks chosen, we introduce two models. One is Markowitz Model, which is the fundamental model, and the other one is Index Model, which is an optimization of the previous formula. In addition, we add five constraints to simulate the different markets or situations investors faced in reality. We will next explain these concepts in detail.

3.1 A. Markowitz Model

Markowitz Model is originally proposed by Harry Markowitz in 1952. There are several basic assumptions for this model. First, investors know the distribution of rate of investment is a normal distribution. Second, investors chase for maximum expected utility. They make decisions through expected rate of return and risk since the expected utility is a function of these two variables. Third, investors are risk averse and the risk represents by the variance of rate of return. Forth, investors are rational, so they will choose a portfolio with minimum risk when the expected rate of return remains unchanged, and maximize the expected rate of return when risks are the same.

To build this model, we first suppose there are n kinds of stocks in the market, their rate of return is $r_1, r_2, r_3, \dots, r_n$ separately, and the weight of each is $w_1, w_2, w_3, \dots, w_n$ separately, where the sum of weight is equal to 1. Thus, we can get the expected rate of return is:

$$E(r_p) = \sum_{i=1}^n w_i E(r_i) \quad (1)$$

and the variance is:

$$Var(r_p) = \sum_{i=1}^n w_i^2 Var(r_i) + \sum_{i \neq j} w_i w_j cov(r_i, r_j) \quad (2)$$

However, there do exist some drawbacks in Markowitz Model. One is that it needs a huge number of estimations of co-variance matrix, another is that it has no use for predict risk premium because it

is hard to confirm future expected return through historical data. And this is the reason why we also adopt index model.

3.2 Index Model

As an optimization of the Markowitz Model, Index Model simplifies the estimation of the covariance matrix and enhances the analysis of risk premium. To better find the influence of diversification, the risks are separated into non-systematic risks and systemic risks, where the front one can be eliminated through diversification and the latter cannot.

This model is firstly presented by William F. Sharpe in 1963. To build this model, we suppose the excess return of stock i is R_i and the excess return of market is R_M . R_M is equal to market's rate of return (r_M) minus risk free rate (r_f). α_i is the expected excess returns, β_i is the sensitivity coefficients, and e_i is the error term. Thus, we can infer the function of Index Model:

$$R_i(t) = \alpha_i + \beta_i R_M(t) + e_i(t) \quad (3)$$

Take expected value on both sides:

$$E(R_i) = \alpha_i + \beta_i E(R_M) \quad (4)$$

And through the calculation of $Cov(R_i, R_i)$, the function of total risk can be obtained:

$$\delta_i^2 = \beta_i^2 \delta_M^2 + \delta^2(e_i) \quad (5)$$

Where $\beta_i^2 \cdot \delta_M^2$ represents systematic risk and $\delta^2(e_i)$ represents non-systematic risk. Similarity, the excess return for the portfolio of stocks is:

$$R_p = \alpha_p + \beta_p R_M + e_p \quad (6)$$

Take expected value on both sides:

$$E(R_p) = \alpha_p + \beta_p E(R_M) \quad (7)$$

The variance is:

$$\delta_p^2 = \beta_p^2 \delta_M^2 + \delta^2(e_p) \quad (8)$$

3.3 Five optimization constraints

To construct the five constraints, the study mainly consider the weight of each stock.

$$\sum_{i=1}^7 |w_i| \leq 2 \quad (9)$$

It represents that the sum of the weight of all the stocks should less than 2. This constraint simulates the *Regulation T by FINRA*, it allows the broker-dealers' clients to have positions and at least half of the positions are from their own account equity.

$$|w_i| \leq 1, \text{ for } \forall i \quad (10)$$

This constraint represents that the weight of each stock should no less than -1 and no larger than 1, and it simulates those arbitrary 'box' constraints provided by clients.

$$\text{None (free)} \quad (11)$$

This problem does not have constraints, it acts as a control option. Through this the real data illustrate by the model without limitation can be acquired.

$$w_1 \geq 0, \text{ for } \forall i \quad (12)$$

This constraint simulates the general restriction in the American mutual fund industry.

To ensure the interest of those co-investors who purchased mutual funds, short positions are not allowed to exist in those US open-ended mutual funds.

$$w_1 = 0 \quad (13)$$

This constraint supposes that the weight of S&P is equal to 0, which is aimed at studying whether the addition of a broad index has any influence.

By controlling the weight of stocks, this study mainly wants to explore the changes of the optimal portfolio under different scenarios, such as whether short positions are allowed and whether a broad index exists.

Then after the models and constraints are given, the article will now move on to the portfolio building.

4. Portfolio establishment

This section would describe the efficient frontier and maximal sharpe ratio, which were used to build the optimal portfolio.

4.1 Efficient Frontier

The efficient frontier is a kind of optimal portfolio, which is needed to provide the highest expected return with a defined level of risk or the lowest risk, and a particular level of the expected return. The portfolios behind the efficient frontier are optimal, due to the fact that they could not provide the satisfactory returns to the given level of the risk. In addition, the portfolios on the right hand of the efficient frontier are not optimal, because their risk is higher than the portfolio on the efficient frontier for the defined returns.

A portfolio frontier is a graph that maps out all possible portfolios with different asset weight combinations, with levels of portfolio standard deviation graphed on the x-axis and portfolio expected return on the y-axis.

$$E(r_p) = \sum_{j=1}^n w_j E(r_j) \quad (9)$$

$$R_i = r_f + (E[r_m] - r_f) \quad (10)$$

To develop a portfolio frontier, the first step is to decide the values of the expected return of assets, the standard deviation of assets and the correlation of two assets. Then it could calculate the expected return and variance of the portfolio by the formula with the different weight of assets (w_1, w_2, w_3, \dots). Then, the article uses the smooth lines in order to show the expected return and standard deviation of the portfolios. Then, it could get the result, which is the grey curve.

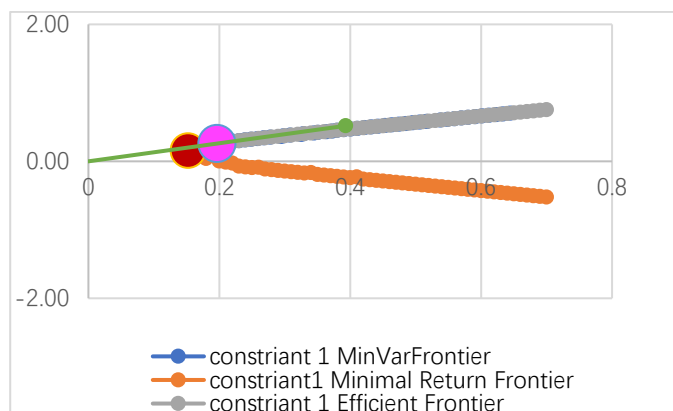


Fig 3. Efficient frontier

From the figure, it could describe that the portfolios on the downward-sloping portion of the portfolio frontier are dominated by the upward-sloping portion. In addition, the points on the upward-sloping portion of the portfolio frontier represent portfolios that have the minimal risk, while the points on the downward-sloping portion represent portfolios that are not efficient.

According to the mean-variance criterion, any investor would optimally select a portfolio on the upward-sloping portion of the portfolio frontier, which is called the efficient frontier, or minimum variance frontier. The choice of any portfolio on the efficient frontier depends on the investor’s risk preferences. A portfolio above the efficient frontier is impossible, while a portfolio below the efficient frontier is inefficient.

4.2 Maximal sharp ratio

Maximum sharpe ratio is also called the efficient risky portfolio. In finance, the Sharpe ratio (also known as the Sharpe index, the Sharpe measure, and the reward-to-variability ratio) measures the performance of an investment (e.g., a security or portfolio) compared to a risk-free asset, after adjusting for its risk. It is defined as the difference between the returns of the investment and the risk-free return, divided by the standard deviation of the investment (i.e., its volatility). It represents the additional amount of return that an investor receives per unit of increase in risk. The ratio describes how much excess return it is, which is the extra volatility that it bears for holding a riskier asset. In order to understand this ratio completely, the following formula is shown,

$$S(x) = \frac{r_x - R_f}{\sigma(r_x)(r_x - R_f)} \quad (11)$$

Where x is the investment; r_x is the average rate of return of x ; R_f is the best available rate of return of a risk-free security; $\sigma(r_x)$ is the standard deviation of r_x

The Sharpe ratio is a measure of return often used to compare the performance of investment managers by making an adjustment for risk.

5. RESULTS

In this study, the article used Markowitz Model and Index Model to compare the minimum investment risk portfolio and the maximum Sharpe ratio portfolio under different constraints.

5.1 Minimal Risk Portfolio

Firstly, the portfolio was established with the minimal risk principle. According to Table 3, the following results can be got. When the variance is the smallest, that is, when the risk is the smallest, the rate of return is 2.79% under constraint 1, and the rate of return is the largest under constraint 5,

which is 12.10%. The standard deviation is also calculated and fluctuates in the range of 13 to 18. The maximum and minimum of Sharpe ratio are 0.201 and 0.679, respectively.

Table 3. Minimal risk portfolio under Markowitz Model

	SPX	AMZ N	AAP L	G5	USB	UPS	FDX	Portfoli o	Portfoli o	Portfoli o
	w_1	w_2	w_3	w_4	w_5	w_6	w_7	Return	St. Dev	Sharpe
Constrain t 1	1.12 5	-0.065	- 0.057	- 0.16 7	0.05 3	0.14 0	- 0.02 9	2.79%	13.89%	0.201
Constrain t 2	1	-0.055	0.001	- 0.13 4	0.06 0	0.23 0	- 0.10 2	4.59%	13.96%	0.330
Constrain t 3	1.16 1	-0.069	- 0.018	- 0.16 4	0.01 6	0.18 3	- 0.10 8	3.38%	13.87%	0.243
Constrain t 4	0.80 9	-6E-09	0	0	0.03 9	0.15 2	2E- 08	6.58%	14.82%	0.444
Constrain t 5	0	0.244	0.640	0.05 6	0.33 7	0.52 1	- 0.06 3	12.10%	17.82%	0.679

By comparing the similarities and differences of the minimum risk portfolio under this model, it can be seen that under any constraints, the smaller the weight of SPX, the larger Standard Deviation, the greater the weight of USB stock, the larger the Sharpe ratio, and the rate of return has little to do with the weight of each stock. In general, under constraint 5, the return is the highest, Standard Deviation is the largest, and Sharpe ratio is the largest. In the case of constraint 1, the 3 are almost the smallest.

Table 4. Minimal risk portfolio under Markowitz Model

	SPX	AMZ N	AAP L	G5	USB	UPS	FDX	Portfoli o	Portfoli o	Portfoli o
	w_1	w_2	w_3	w_4	w_5	w_6	w_7	Return	St. Dev	Sharpe
Constrain t 1	- 0.36 2	0.354	0.854	- 0.13 8	0.07 6	0.21 5	0.00	40.50%	36.30%	0.286
Constrain t 2	1	-0.058	- 0.046	- 0.14 5	0.08 2	0.17 6	- 0.00 7	3.56%	13.93%	0.260
Constrain t 3	1.21 3	-0.329	0.753	0.62 3	0.60 4	1.78 8	- 3.65 3	7.16%	71.27%	0.101
Constrain t 4	0.73 8	0	0.009	0	0.07 6	0.17 6	0	7%	14.84%	0.472
Constrain t 5	0	0.031	0.118	0.05 6	0.33 7	0.52 1	- 0.06 3	12.10%	17.82%	0.679

Based on this model, the above Table 4 can be obtained. From the table, it can be concluded that under different constraints, different return rates, Standard Deviation and Sharpe ratio will be obtained. In the case of constraint 1, the rate of return is the maximum of 40% and the minimum of 3.56% in the case of constraint 2. Constraint 3 has the largest Standard Deviation, which is 71.27%, and the minimum is 13.93% under constraint 2. As for the Sharpe ratio, the maximum is 0.679 for constraint 5 and the minimum is 0.101 for constraint 3.

Through the study of this model which is different from the former, the following general rule can be drawn: when UPS shares have a large weight, the return rate of the whole portfolio will be relatively small, on the contrary, Standard Deviation will be relatively large. Of course, these changes are carried out under certain constraints and within a certain range.

5.2 Efficient Risky Portfolio

In this part, the portfolio with efficient risky principle was analyzed. As can be seen from Table 5, under constraint 3, the return rate is the largest, which is 78.51%, and the rest fluctuates almost in the range of 20%-40%. For Standard Deviation, constraint 3 is also the largest, which is 66.91%. The Sharpe ratio is the same, but the difference between the Sharpe ratios under each constraint is not particularly large, which can be almost ignored.

Table 5. Efficient Risky Portfolio under Index Model

	SPX	AMZ N	AAP L	G5	USB	UPS	FDX	Portfoli o	Portfoli o	Portfoli o
	w_1	w_2	w_3	w_4	w_5	w_6	w_7	Return	St. Dev	Sharpe
Constrain t 1	- 0.39 1	0.355	0.737	- 0.10 9	0.08 4	0.18 5	0.13 9	37.41%	33.72%	1.109
Constrain t 2	- -1	0.377	0.870	- 0.21 4	0.73 0	0.31 3	- 0.07 5	42.46%	36.73%	1.150
Constrain t 3	- 3.45 4	0.792	1.657	- 0.10 9	1.49 9	0.63 3	- 0.01 9	78.51%	66.91%	1.173
Constrain t 4	- 0	0.026	0.539	- 0	0.24 4	0.01 1	- 2.8E -08	27.48%	26.55%	1.035
Constrain t 5	- 0	0.244	0.640	- 0.32 2	0.43 2	0.11 1	- 0.10 5	30.73%	28.41%	1.082

Markowitz shows that under certain constraints, an investor's portfolio selection can be simplified to balance 2 factors, namely, the expected return of the portfolio and its variance. Risk can be measured by variance and reduced by diversification. Portfolio risk depends not only on the variance of different assets, but also on the variance of shares. In this way, the complex multi-dimensional problem of portfolio selection with a large number of different assets is constrained to become a simple 2 times planning problem with a clear concept. That is, mean variance analysis. And Markowitz gives the practical calculation method of the optimal portfolio problem.

Table 6. Efficient Risky Portfolio under Index Model

	SPX	AMZ N	AAP L	G5	USB	UPS	FDX	Portfoli o	Portfoli o	Portfoli o
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	w_1	w_2	w_3	w_4	w_5	w_6	w_7	Return	St. Dev	Sharpe
Constraint 1	0.754	0.007	0.129	0.159	0.146	0.223	0.101	10.52%	15.03%	0.67
Constraint 2	-1	0.461	0.924	0.185	0.202	0.332	0.264	46.56%	40.82%	1.15
Constraint 3	0.439	0.100	0.211	0.013	0.190	0.080	0.002	24.48%	8.52%	1.173
Constraint 4	0	0.305	0.658	0	0	0.017	0.020	32.93%	31.18%	1.056
Constraint 5	0	1.681	0.594	0.204	0.473	0.544	2.497	55.36%	98.71%	0.561

From Table 6, it can be seen that under the index model, constraint 5 has the highest rate of return, 55.36%, while constraint 1 has only 10.52%. At the same time, constraint 5 has the largest Standard Deviation and the smallest Sharpe ratio, which are 98.71% and 0.561, respectively. Under constraint 3, the risk is the smallest, which is only 8.52%. The Sharpe ratios of constraint 2, 3 and 4 are about 1.1, The Sharpe ratio under constraint 5 is 0.67 similar to constraint 1.

To sum up, through the study of Markowitz model and Index Model in the minimum risk portfolio and the maximum Sharpe ratio, it can be found that Markowitz model proves that under certain constraints, investors' portfolio selection can be simplified to balance two factors, namely, the expected return and variance of the portfolio. In the case of Index Model, the conclusion is more general and regular.

6. Conclusion

This paper mainly researched the portfolio choice problem under 5 different constraints by using monthly stock prices of SPX, AMZN, AAPL, GS, USB, UPS, and FDX. Based on those monthly observations, the article calculate all proper optimization inputs for the full Markowitz Model, alongside the Index Model. Using these optimization inputs for MM and IM, it can find the regions of permissible portfolios for five cases of the additional constraints.

Based on the Markowitz Model and Index Model, the study mainly consider the effects of the rate of returns and risks on the optimal portfolio. Moreover, 5 constraints are added to compare the difference between the two models, and finally some conclusions have been drawn.

The article obtains lots of findings. Under constraints 1 and 4, the return rate of the exponential model algorithm is better and the risk is smaller. In the case of constraints 2, 3 and 5, Markowitz model is more intuitive, appropriate and cost-effective. The minimum variance portfolio is the portfolio with the least risk in a series of portfolios, which is suitable for risk adverse investors. Due to the reciprocal relationship between risk and return, the return of this investment method is also the lowest.

The Markowitz model and Index model do precisely conclude the return rate and risk under different constraints that are always considered in real life, and they can be a significant role for the investors to predict the stocks' future performance. This article also helps the investors to define which model is more intuitive under different conditions, which can lessen the investors' workload. The shortcoming of this research is that, after all, the future cannot be precisely predicted, so all the results gotten can only be a significant basis for the stocks' prices trend in the future.

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