

# Portfolio Investment Strategy Based on Markowitz Model and Single Index Model

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**Abstract.** Investing with the highest return and the lowest risk has always been an ideal situation for every investor in the bond markets. Therefore, financial scholars developed lots of models to estimate the optimal risk portfolios of several bonds. Markowitz Model and Single Index Model are both classical financial models used for this estimation. However, they have different advantages and disadvantages, which makes it difficult to determine which one to use under different circumstances. In this essay, the effectiveness of two regular financial models in establishing the optimal risk portfolio under 5 common constraints is assessed. Based on the data of 7 firms within twenty years, both models are applied to construct their corresponding optimal risk portfolios, and visualized the results through Excel. Then, their results are compared to determine the more suitable model in our case by assessing the expected rate of return, predicted risk, and the accuracy of results.

**Keywords:** Model; Single Index Model; comparison.

## 1. Introduction

How to choose the optimal risk asset portfolio is always a hot topic. In order to better measure the effectiveness of portfolio, scholars put forward many models, among which the two mainstream models are the Markowitz Model (MM) and the Single Index Model (IM). Using the data of six listed companies, this paper makes a comparison between MM and IM under different restrictions by taking the US Treasury bond and the S&P 500 as the estimates of the risk-free yield respectively.

The effectiveness of the portfolio is a portfolio theory based on the advantages and disadvantages of a portfolio of assets evaluation. That is, the effective asset portfolio refers to the asset portfolio with the least risk when the return of the asset portfolio is certain or the asset portfolio with the largest return when the risk of the asset portfolio is certain.

Portfolio selection theory proposed by Markowitz [1][2] is one of the mainstream methods to measure the effectiveness of portfolio. This model believes that the goal of investors is to pursue high returns while avoiding high risks as much as possible. Therefore, the optimal portfolio is to maximize the expected returns under the given risks or minimize the risks under the given expected returns. On

this basis, Tobin [3] relaxed the conditional assumption that short selling was not allowed, introduced the concept of risk-free assets, proposed the "separation theorem of two funds", and improved the theory of portfolio selection. However, Markowitz Model has two defects. First, the Model requires a large amount of estimated data to calculate the covariance matrix. Second, the model cannot provide a method to predict the security risk premium, and thus cannot construct an effective boundary. What's more, as Rubinstein [4] mentioned, Markowitz Model does not consider the market behavior of consumers/investors participating in the game.

In order to simplify the covariance matrix estimation, Sharpe established the capital asset pricing model (CAPM) on basis of rational expectations on the basis of the theory of Markowitz, and decomposed CAPM into a simple linear relationship between systemic risk and company-specific risk and expected return and the risk of the assets [5]. Later, Merton and Samuelson [6] studied and concluded that the proportion of risky assets held by investors has nothing to do with variables such as age, income, and term, but is only determined by investors' risk preference.

Many scholars are interested in the differences and similarities between MM and IM. West G. [7] introduced Markowitz, CAP-M, APT and Black-Litterman model in his paper. Paudel R B and Koirala S [8] showed their application of IM and MM using the data from the Nepalese Stock Market. Sinha and Tripathi [9] focused on stocks selected from NSE and found that the IM is better than the MM but only slightly. Varghese and Joseph [10] also made some comparisons between Markowitz and Sharpe Model.

In order to explore the advantages and disadvantages of the MM model and IM model in different situations, we chose the weekly data of six listed companies, which include Adobe Systems Incorporated (ADBE), International Business Machines Corporation (IBM), Bank of America (BAC), Citigroup (C), Southwest Airlines (LUV) and Alaska Airlines (AKL), and the 1-month Fed Funds rate in the United States from 2001 to 2020. Minimal Risk Portfolio The Efficient Risky Portfolio and Capital Allocation Line are calculated under five different restrictions: Minimal Risk or Variance Frontier, Minimal Return Frontier, Efficient Frontier, Minimal Risk Portfolio, and Minimal Risk or Variance Frontier. We found that although the academic community generally believes that IM model has a smaller variance than MM model under large samples due to the limitation of sample size, MM model has a smaller variance than IM model based on our analysis.

The structure of this paper is as follows. The second part briefly introduces the corresponding enterprises of the stocks and the data used in this paper. In the third part, different asset portfolios are constructed by using the Markowitz portfolio selection model and single index model under different constraint conditions. The fourth part compares the results of the two models. The fifth part is the conclusion.

## 2. FIRMS' BACKGROUND

### 2.1 Introduction of Companies

The Adobe Systems Incorporated (ADBE), an American multinational computer software company. The annual turnover of 2019 is 11.1713 billion dollars. ADBE has several product lines. The Adobe Creative Cloud includes Photoshop, Dreamweaver and PDFs. The products of Adobe help us to edit directly in PDF, without re-entering to use the content, providing a secure and efficient document management experience for the individuals and the business.

The second company is IBM, standing for International Business Machines Corporation, which was built by Thomas·J·Watson in 1911. IBM is the largest information technology and business solutions company of the world. Their annual turnover of 2020 is 77.147 billion dollars. The global capabilities of IBM include services, software, hardware systems, research, and related financing support.

BAC, which stands for Bank of America, is the largest joint-stock commercial bank in the United States by assets. Their annual turnover of 2020 is 1135.89 billion dollars. The retail banking business of BAC focuses on depositing money with individuals and businesses. The online banking of Bank

of America is the leader in online financial services and has gained worldwide recognition for its online banking services. The main personal financial services provided by Bank of America Online include savings, Loans and Credit, investment, Professional Financial Services, and Insurance.

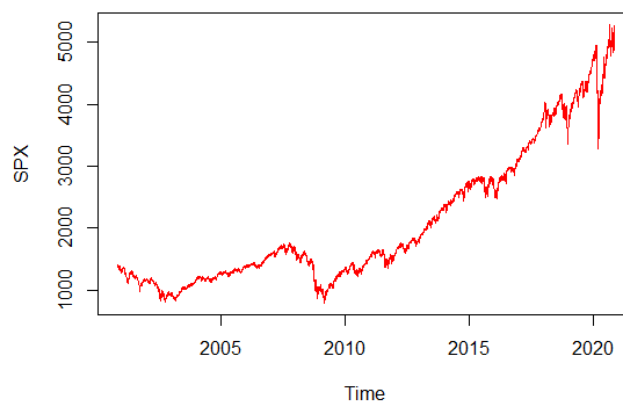
Citigroup is the first financial group in the United States that integrates commercial banking, investment banking, insurance, mutual funds, securities trading, and other financial services. Their annual turnover of 2020 is 1034.49 billion dollars.

LUV is an American airline based in Dallas. It is the third largest airline in the world in terms of passengers carried and flies to more cities than any other airline in the United States. Their business strategies are improvement of equipment utilization rate and cost reduction. Their annual turnover of 2020 is 224.28 million dollars due to the COVID-19 pandemic.

Alaska Airlines began to operate on May 6, 1944, is an airline based in Washington. Their annual turnover is 87.81 billion dollars.

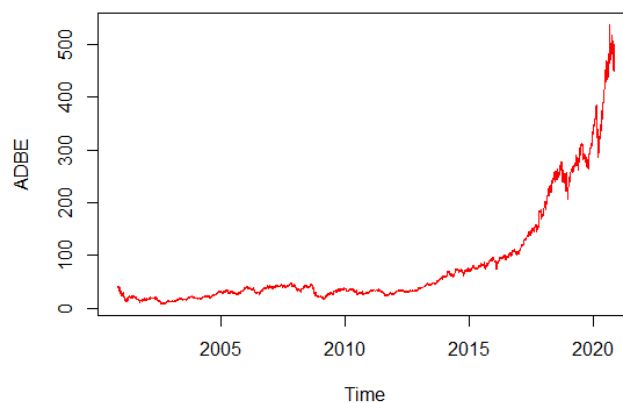
## 2.2 Data description

Data about the daily closing price and treasury bond rate from 2000 to 2020 are collected from Yahoo Finance.



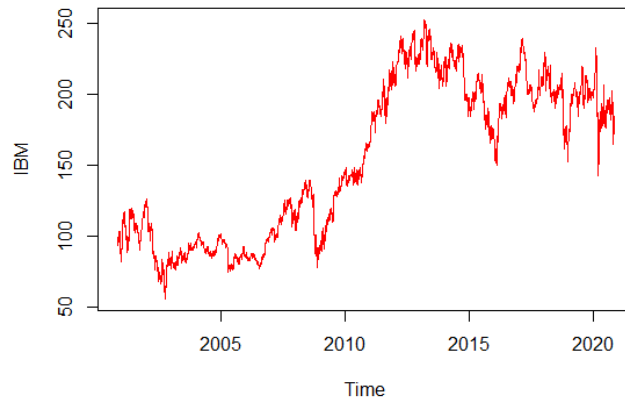
**Fig. 1** SPX from 2000 to 2020

Fig. 1 shows that the SPX from 2000 to 2020 is keeping increasing most of the time but the sudden decline due to the financial crisis in 2008 and the aftermath of COVID-19 pandemic.



**Fig. 2** Stock prices of ADBE from 2000 to 2020

Fig. 2 shows that the ADBE stock price from 2000 to 2015 remained in slight fluctuation, and from 2016 to 2020 it had a sharp increase.



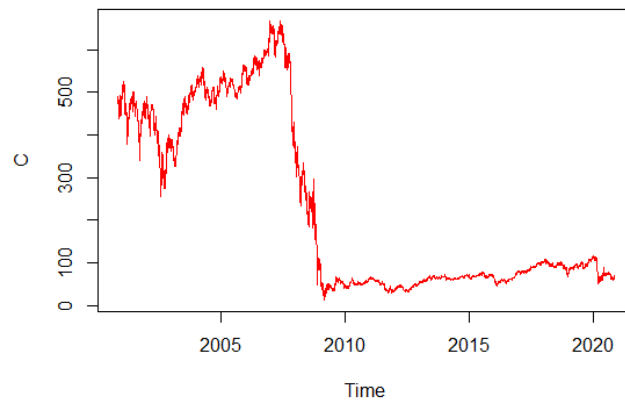
**Fig. 3** Stock prices of IBM from 2000 to 2020

Fig. 3 shows that from 2000 to 2009 the IBM stock price fluctuations had been relatively small and had remained at a low level. From 2009 to 2013, the volatility of IBM stock price increased. From 2013 to 2020, the stock price started fluctuating severely.



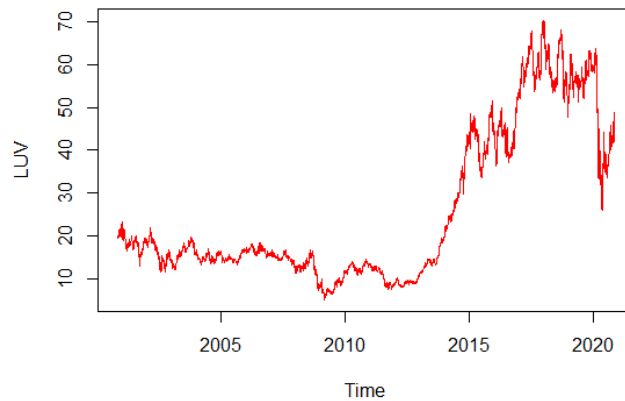
**Fig. 4** Stock prices of BAC from 2000 to 2020

Fig. 4 tells that the fluctuations of the stock price of BAC from 2000 to 2006 remained at a slight level and kept increasing. However, their stock price seriously declined due to the financial crisis in 2008. From then on, the stock prices fluctuated sharply from 2010 to 2020 and have never peaked again.



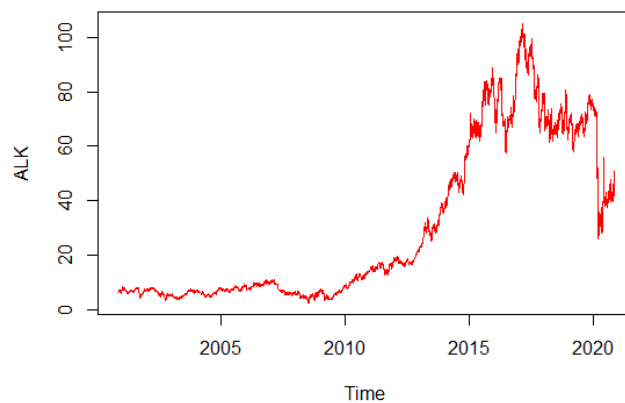
**Fig. 5** Stock prices of C from 2000 to 2020

Fig. 5 tells that the stock price volatility stabilized gradually from 2000 to 2004, and increased a lot from 2004 to 2008. However, similar to BAC, the stock price of it seriously declined due to the financial crisis in 2008 and 2009. Since 2009, stock price fluctuations have been relatively small and have remained at a low level.



**Fig. 6** Stock prices of LUV from 2000 to 2020

From 2000 to 2014, the stock price of LUV volatility stabilized gradually. From 2014 to 2016, the LUV stock price fluctuations had been relatively small and remained at a low level, while from 2016 to 2020, the fluctuations were also relatively small and remained at a high level. Due to the COVID-19 pandemic, the price of it has dropped a lot, but it is gradually increasing these days.



**Fig. 7** Stock prices of ALK from 2000 to 2020

From 2000 to 2010, the stock prices of ALK stabilized gradually. From 2011 to 2016, the stock prices of ALK fluctuated sharply, and then the volatility increased. From 2016 to 2020, the fluctuations were volatile and remained at a high level. Due to the COVID-19, the price of it has also dropped a lot, but it is also gradually increasing these days.

According to the following descriptive statistical analysis (Table 1) of the main variables, the volatility of the prices of SPX ranges from 788.4 to 5293, with a wide range and intense volatility. As for the price volatility of ADBE and IBM, the range exceeds 528.8, whereas for the price volatility of LUV and ALK, the range is relatively limited, the least possible value is 65.2. The results support the notion that the stock prices of computer market fluctuates more dramatically than that of airlines.

The mean and variance of C stock price volatility are much higher than that of airlines stock price volatility, indicating that the average stock prices of C are much higher than that of airlines. Additionally, the difference in C stock price volatility is far greater than that of airlines.

**Table 1.** Descriptive statistics of variables.

Variable	Mean	Std.dev	Min	Max
SPX	2092.2	1101.3	788.4	5293
ADBE	82.7	100.2	8.4	537.2
IBM	154.1	54.7	55.6	252.8
BAC	35.6	15.9	4.6	70
C	228	208.1	13.35	667.8
LUV	25.6	17.8	5.1	70.3
ALK	29.9	29.1	2.7	105

### 3. Portfolio Establishment

Our project aims to get Minimal Risk or Variance Frontier, Efficient Frontier, Minimal Return Frontier, Optimal Sharpe ratio portfolio, Minimal Risk portfolio, and Capital Allocation line. Since we have five limits with Markowitz Model (MM Model) and Index Model (IM), each one contains 10 outcomes.

#### 3.1 Illustration of constraints

Aside of IM and MM model difference, since our clients have different demands and fund constraints, we have another five different constraints on invest weights.

The combination of invest weights of six stocks and SPX has to be smaller or equal to two.

$$\sum_{i=1}^7 |w_i| \leq 2 \tag{1}$$

For any stock and SPX, the invest weights has to be smaller than 1

$$|w_i| \leq 1, \text{ for } \forall i \tag{2}$$

No constraints. We are looking for the most ideal case and our client has adequate fund.

For any stocks and SPX, we have to put investment on each one of them. Our client may not have fund constraint, but client does not want too much risk and we want diversification with all options.

$$w_i \geq 0, \text{ for } \forall i \tag{3}$$

No investment in SPX to see the effects. The client also does not have fund constraint, but it is willing to take higher risk which means we do not want to spend money in risky free asset.

$$w_1 = 0 \tag{4}$$

Especially, in order to show the effects of change of constraints on our portfolio, we put all of the outcomes of one kind into one graph to show the more transparent changes. Our analysis part will show this format.

#### 3.2 The Markowitz Portfolio Optimization Model

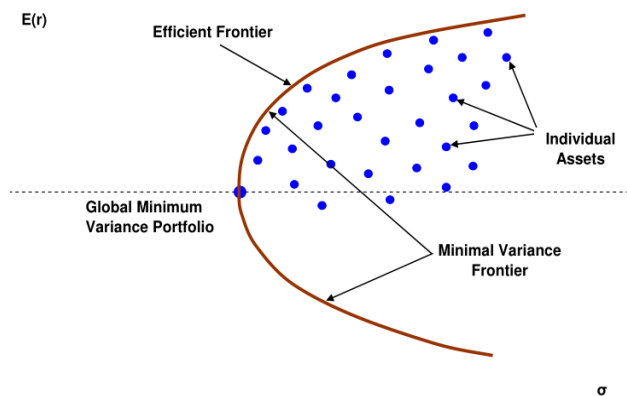
In this part, we are going to give a brief sketch of the Markowitz Portfolio Optimization Model, which we prefer to simplify as the MM Model in the later sections. Our article will be narrated from the following aspects: Basic information about the model, the type and amount of data to be entered, the applicable scope, and how to use it.

### 3.2.1 Introduction of the MM Model

Markowitz Portfolio Optimization Model is used commonly in the establishing of investment portfolio. First, we are willing to talk about some features in the MM Model.

When Markowitz established this model, he based on the following assumptions:

1. Investors consider every investment option according to the probability distribution of the return of a security during a given holding period.
2. Investors estimate the risk of a portfolio based on the expected yield of the securities.
3. Investors' decisions are based solely on the risk and return of the securities.
4. At a certain level of risk, investors expect the maximum return.

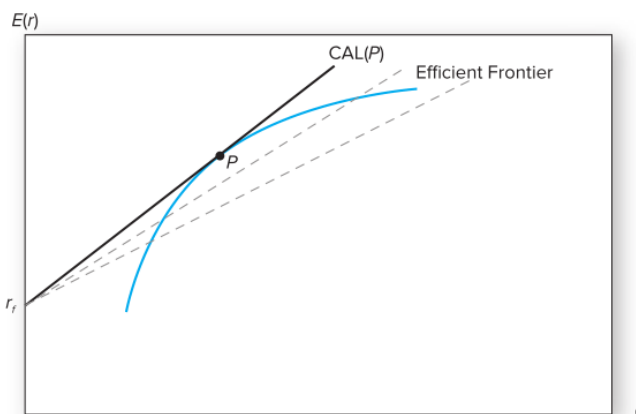


**Fig. 8** The Minimal Variance Frontier

Correspondingly, at a certain income level, investors expect the least risk. According to all the above assumptions, Markowitz gave us an instrument which took the variance as a measure of risk.

Then using the MM model, we generalize the portfolio construction problem to the case of many risky securities and a risk-free asset, which has three parts as follows.

The first step is to evaluate investors' risk premium options. These are summarized by the minimum-variance frontier of risky assets. Second, we identify the optimal portfolio of risky assets by finding the portfolio weights that result in the steepest the Capital Allocation Line (CAL: also known as the capital market link (CML), is a line created on a graph of all possible combinations of risk-free and risky assets. The graph displays the return that investors might possibly earn by assuming a certain level of risk with their investment. The slope of the CAL is known as the reward-to-variability ratio). Finally, investors themselves can choose an appropriate complete portfolio by mixing the risk-free asset with the optimal risky portfolio.



**Fig. 9** The efficient frontier of Risky Assets with the Optimal CAL

### 3.2.2 Input data of the MM model

Here we need some estimates to compose the input list of MM Model. Assuming that we have  $n$  stocks, and we just need to calculate some data which can be referred to by the subsequent operations in this step.

According to the  $n$  stocks we have, we require  $n$  estimates of the expected return of each asset, denoted by  $r_i$ ; then  $n$  estimates of the variances denoted by  $\sigma_i^2$  are still needed; Also a  $n \times n$  matrix is what the investment manager want, and due to the property—the main diagonal symmetry, the amount of  $Cov(r_i, r_j)$  we just need to estimate is  $\frac{n(n-1)}{2}$ .

What is more, when we know the weights of them, we can have the expected return of the portfolio, denoted by  $E(r_p)$ , which is a weighted average of expected returns, using formula (5):

$$E(r_p) = \sum_{i=1}^n w_i E(r_i) \quad (5)$$

And also, we can use formula (6) to have the final  $\sigma_p^2$ , which refer to the variance of the  $n$ -asset portfolio:

$$\sigma_p^2 = \sum_{i=1}^n \sum_{j=1}^n w_i w_j Cov(r_i, r_j) \quad (6)$$

Therefore, we can use the data to find the combination with the highest expected return. In other words, we can use this to confirm the effective boundary set. So now the efficient set of portfolios, or the efficient frontier of risky assets have been affirmed.

Afterwards we can get multiple sets of minimum variances by assigning different weights, and then we can use these variances to plot the Minimum Variance Frontier. The part of minimal-variance frontier above the Global Minimum-Variance Portfolio, is called the Efficient Frontier of risky assets: they provide the best risk-return combinations. Now we have an effective frontier curve.

### 3.3 Determine the optimal risky portfolio

In this part, we should first consider the additional constraints. For instance, Prohibition on short positions, A minimal level of expected dividend yield from the optimal portfolio, Socially responsible investing, Tax-related constraints, etc.

Now we search for the CAL with the highest Sharpe ratio. The CAL generated by the optimal portfolio, P, is the one tangent to the efficient frontier. This CAL dominates all alternative feasible lines. Portfolio P, therefore, is the optimal risky portfolio.

At this point, our portfolio manager is done. Portfolio P is what the manager's clients want.

### 3.4 Single index model

In this part, we are going to talk about the differences and advantages over the MM model from the perspective of the IM model. Since we have already talked about the theories of calculating the permissible regions in the previous part, we will talk about how to calculate them practically using Excel Solver, a popular tool for calculating investment weights in a portfolio giving constraints.

#### 3.4.1 A brief Sketch of the IM Model

The MM Model introduced above has two defects. First, the model requires a large amount of estimated data to calculate the covariance matrix. Second, the model does not provide a way to predict the security risk premium, which is necessary to construct an efficient boundary. We cannot predict the future returns solely on historical returns, presenting a serious shortcoming.

Now, we introduce the Index Model, simplify the estimation of covariance matrix, and strengthen the estimation of security risk premium. By breaking down risks into systemic and firm-specific risks, index models enable readers to understand the power and limitations of diversification, and to measure these risk components for specific securities and portfolios.

To begin with, we will introduce what do we need for the input data set of the IM model. Suppose we now have a portfolio with  $n$  stocks in it, we will need  $n$  estimates of the extra-market expected excess returns  $\alpha_i$ ,  $n$  estimates of the sensitivity coefficients  $\beta_i$ ,  $n$  estimates of the firm-specific variances  $\sigma^2(e_i)$ , one time estimate for the market premium  $E(R_M)$ , one time estimate for the variance of the (common) macroeconomic factor  $\sigma_{M^2}$ .

the index model could be expressed as the following form:

$$R_j(t) = \alpha_i + \beta_i R_M(t) + e_i(t) \tag{7}$$

The vertical intercept of this equation (denoted by the Greek letter  $\alpha$ ) is the security's expected excess return when the market excess return is zero. The slope of the line is the security's beta coefficient,  $\beta_i$  which is the amount by which the security return tends to increase or decrease for every 1% increase or decrease in the return on the index, and therefore measures the security's sensitivity to the market index.  $e_i$  is the zero-mean, firm-specific surprise in the security return in month  $t$ . The greater the residuals (positive or negative), the wider is the scatter of returns around the best-fit line as shown in the graph above.

### 3.4.2 Illustration of specific methods

As mentioned before, our main purpose is to get efficient frontier, minimal risk portfolio, optimal portfolio, and minimal return portfolios frontier. While in section B, we have already discussed the theory part of the project, we will discuss the limit settings in Excel Solver, which we applied as our main tool for the project, for different results we want. Excel solver is a giant calculator to assign investment positions for different stocks in a portfolio based on the limits and conditions it was been given.

The first two ones, efficient frontier and minimal return portfolios frontier, both have the condition of constant standard deviation; but we are looking for minimum return (as shown in formula (8)) for minimal return portfolios frontier and maximum return (as shown in formula (9)) for efficient frontier.

$$\begin{cases} r(\vec{w}) \rightarrow \min_{\vec{w}} \\ \text{subject to: } \sigma(\vec{w}) = \text{const} \end{cases} \tag{8}$$

$$\begin{cases} r(\vec{w}) \rightarrow \max_{\vec{w}} \\ \text{subject to: } \sigma(\vec{w}) = \text{const} \end{cases} \tag{9}$$

Following the constraints of these two frontiers, we could find out that these two curves are like mirror images to each other, and they are symmetrical. In order to combine these two curves and test whether we were right on the former calculations, we could keep our return constant and seek for the minimum standard deviation for the minimal risk frontier according to formula (10).

$$\begin{cases} \sigma(\vec{w}) \rightarrow \min_{\vec{w}} \\ \text{subject to: } r(\vec{w}) = \text{const} \end{cases} \tag{10}$$

After completing the most important step (efficient frontier), the rest of the two portfolios points are more accessible. We will enter the limits in formula (11) to get the minimal risk portfolio and enter limits in formula (12) to get the maximum Sharpe optimal portfolio (which is tangent to CAL line).

$$\{\sigma(\vec{w}) \rightarrow \min_{\vec{w}} \tag{11}$$

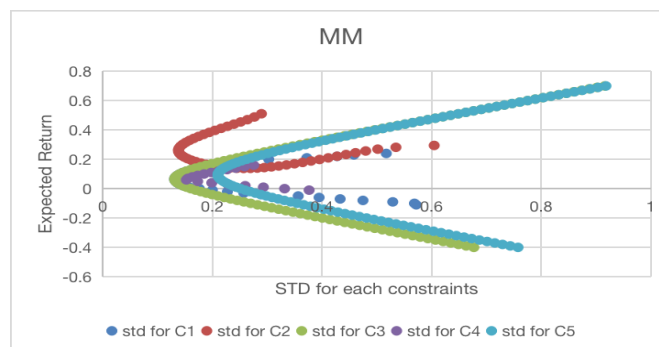
$$\begin{cases} r(\bar{w}) \\ \sigma(\bar{w}) \end{cases} \rightarrow \max_{\bar{w}} \quad (12)$$

## 4. Portfolio Results

So far, we have already covered all the methods and theorems to get to our conclusion. In this part, we applied the two models and five constraints to our data, and got our results. Then, for both of the models, we are going to demonstrate our results towards the three conditions: minimal variance frontier, minimal return frontier, and Optimal Sharpe Ratio Portfolio.

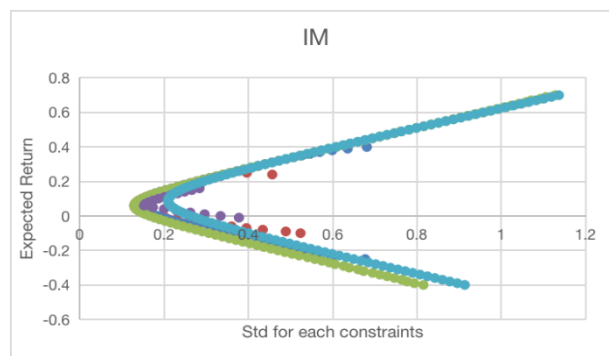
### 4.1 Minimal Variance Frontier

We hold the expected return fixed, pursuing a certain set of weights that provides the minimal variance. We then set up multiple expected returns within a range and being separated by equal partitions to portrait this frontier. Noticing that this curve is equivalent to the maximal return frontier which is also the whole picture, while the efficiency frontier is the upper part of this curve above the minimal variance portfolio. The result is shown as follows.



**Fig. 10** Minimal Variance Frontier of MM

From Fig. 10, the curve of all five constraints follows the C-shaped pattern. The data plausible for Constraint 3 and Constraint 5 are more than the other three constraints. Despite some overlapping, we can still observe that the expected return is generally higher for the fixed risk under Constraint 2. Also, under Constraint 5, the risk tends to be larger for certain values of expected return.

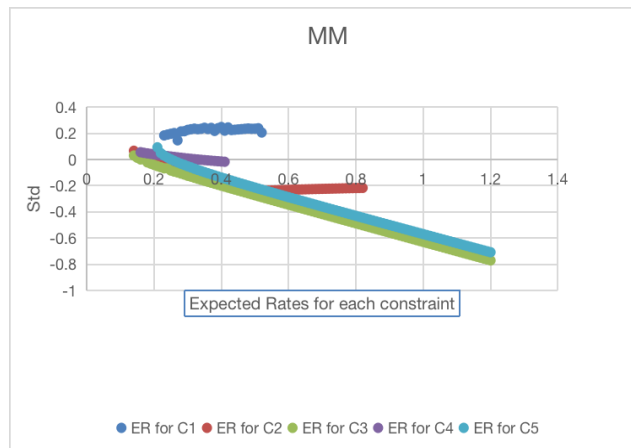


**Fig. 11** Minimal Variance Frontier of IM

In Fig. 11, the C-shaped curves are still obvious. Same as the conclusion above, the risk tends to be larger for certain values of expected return under Constraint 5. Under Constraint 4, the change of expected return for an incremental unit is smaller than the other curves.

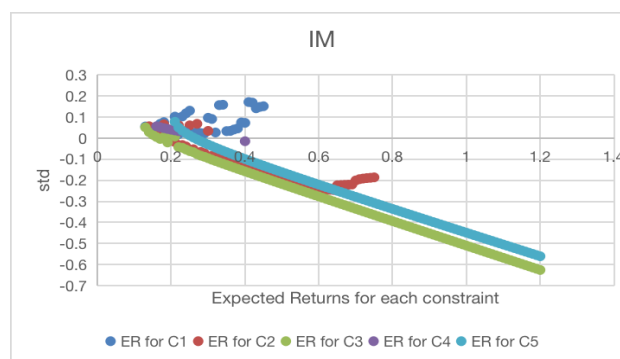
### 4.2 Minimal Return Frontier

This frontier is calculated by holding the variance fixed and pursue a certain set of weights that provides the minimal return. Different from 4.1, this curve is the lower part of the efficiency frontier below the minimal variance portfolio. The result is shown as follows.



**Fig. 12** Minimal Return Frontier of MM

As the results demonstrated in Fig. 12, we observed that most of the curves go downward as they are supposed to be, and their linearity becomes more and more suitable as the expected return gets larger. It is abnormal for Constraint 1 that its curve tends to be a horizontal line.



**Fig. 13** IM Minimal Return Frontier of IM

In Fig. 13, the plot of constraint 1 is still abnormal, and looks like a scattered plot. So does the beginning part of Constraint 2 when the expected return is relatively small. Under these circumstances, IM is inaccurate predicting the risk when the expected return is small.

### 4.3 Optimal Sharpe Ratio Portfolio

This is a portfolio with maximum Sharpe Ratio. It is the tangent point of the Efficiency Frontier and the Capital Allocation Line (CAL). Efficiency Frontier follows a marginal diminishing effect. Therefore, before the tangent point, the increment of expected return will increase as variance increases by the same numerical amount. After the tangent point, the increment of expected return will decrease as variance increases by the same numerical amount. This makes the tangent point the portfolio most desired (the lowest risk per percentage of return). Therefore, we can find the most desired portfolio by simply maximizing the Sharpe Ratio. The result is shown as follows.

**Table 2.** Sharpe Ratio Portfolio of IM

IM	SPX	ADBE	IBM	BAC	C	LUV	ALK
C1	<b>0.96</b>	<b>0.28</b>	<b>-0.09</b>	<b>0.00</b>	<b>-0.41</b>	<b>0.04</b>	<b>0.22</b>
C2	<b>1.00</b>	<b>0.30</b>	<b>-0.15</b>	<b>0.00</b>	<b>-0.43</b>	<b>0.04</b>	<b>0.23</b>
C3	<b>1.02</b>	<b>0.30</b>	<b>-0.15</b>	<b>0.00</b>	<b>-0.43</b>	<b>0.04</b>	<b>0.23</b>
C4	<b>0.04</b>	<b>0.51</b>	<b>0.00</b>	<b>0.00</b>	<b>0.00</b>	<b>0.07</b>	<b>0.39</b>
C5	<b>0.00</b>	<b>0.72</b>	<b>-0.00</b>	<b>0.14</b>	<b>-0.57</b>	<b>0.21</b>	<b>0.51</b>

Under this circumstance (when pursuing the maximal Sharpe ration under the five constraints), we got several results from the charts above for both IM and MM.

In the case of IM (Table 1), for the first constraint, SPX has the largest weight of 0.9608 for long position. Other than that, ADBE has the largest weight of 0.2818 for long position, and C has the largest weight of 0.4066 for short position.

For the second constraint, SPX has the largest weight of 1.0000 for long position. Other than that, ADBE has the largest weight of 0.3015 for long position, and C has the largest weight of -0.4313 for short position.

For the third constraint, SPX has the largest weight of 1.0162 for long position. Other than that, ADBE has the largest weight of 0.3015 for long position, and C has the largest weight of -0.4313 for short position.

For the forth constraint, ADBE has the largest weight of 0.5054 for long position, and IBM, BAC, and C are all given up.

For the last constraint which does not diversify the investment through SPX, ADBE has the largest weight of 0.7184 for long position, and C has the largest weight of -0.5724 for short position.

Based on the results above, the prediction of IM shows that ADBE has the best investment prospect, while C has the worst.

**Table 3.** Sharpe Ratio Portfolio of MM

MM	SPX	ADBE	IBM	BAC	C	LUV	ALK
C1	<b>0.80</b>	<b>0.21</b>	<b>0.00</b>	<b>0.30</b>	<b>-0.50</b>	<b>0.00</b>	<b>0.18</b>
C2	<b>0.98</b>	<b>0.29</b>	<b>-0.16</b>	<b>0.50</b>	<b>-0.76</b>	<b>-0.10</b>	<b>0.25</b>
C3	<b>0.98</b>	<b>0.29</b>	<b>-0.16</b>	<b>0.50</b>	<b>-0.76</b>	<b>-0.10</b>	<b>0.25</b>
C4	<b>0.00</b>	<b>0.56</b>	<b>0.00</b>	<b>0.00</b>	<b>0.00</b>	<b>0.00</b>	<b>0.44</b>
C5	<b>0.00</b>	<b>0.74</b>	<b>0.02</b>	<b>0.91</b>	<b>-1.14</b>	<b>-0.08</b>	<b>0.55</b>

Under this circumstance (when pursuing the maximal Sharpe ration under the five constraints), we got several results from the charts above for both IM and MM.

In the case of MM (Table 2), for the first constraint, SPX has the largest weight of 0.8010 for long position. Other than that, ADBE has the largest weight of 0.2142 for long position, and C has the largest weight of -0.5000 for short position.

For the second constraint, SPX has the largest weight of 0.9758 for long position. Other than that, ADBE has the largest weight of 0.2910 for long position, and C has the largest weight of -0.7580 for short position.

The third constraint shares the same result with constraint 2.

For the forth constraint, ADBE has the largest weight of 0.5599 for long position, and IBM, BAC, LUV, SPX, and C are all given up.

For the last constraint which does not diversify the investment through SPX, BAC has the largest weight of 0.9108 for long position, and C has the largest weight of -1.1350 for short position.

Based on the results above, the prediction of MM also shows that ADBE has the best investment prospect, while C has the worst.

Table 3 and Table 4 provide us with the maximum Sharpe Ratio, and we can conclude that MM predicts a larger Sharpe Ratio for each of the five constraints.

**Table 4.**Maximum Sharpe Ratio of IM

IM	Return	StDev	Sharpe
C1	0.1466	0.1996	0.7345
C2	0.1527	0.2075	0.7358
C3	0.1520	0.2065	0.7358
C4	0.1520	0.2692	0.5645
C5	0.2380	0.3452	0.6895

**Table 5.** Maximum Sharpe Ratio of MM

MM	Return	StDev	Sharpe
C1	0.1512	0.1840	0.8216
C2	0.1943	0.2261	0.8595
C3	0.1943	0.2261	0.8595
C4	0.1613	0.2776	0.5813
C5	0.3103	0.3819	0.8124

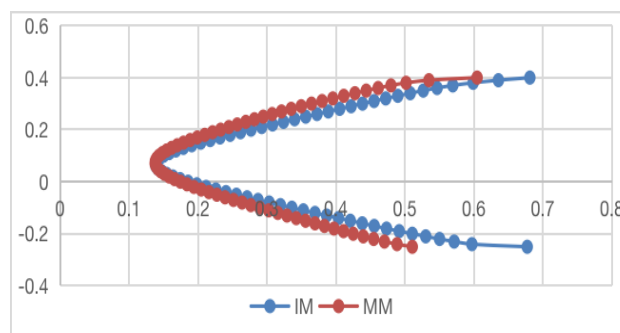
## 5. Analysis

### 5.1 Comparison across the two models

Compared with MM, IM simplified the risk. The risk in IM is made up of macroeconomics risk and firm-specific risk. This ignores the industrial specific risk. On the other hand, the risk calculated by MM is more comprehensive but less precise. That is because MM needs more input data than IM, and every input data is a point estimate which accumulates a more significant error than IM.

We anticipate a difference between the two variances. The difference might be caused by the following reason. Some of the risks might offset one another, rather than a simple linear superposition. Basically, our stocks comes from the industries of technology, finance, and airline, which means the portfolio itself is not very well-diversified. If two of these industries have a negative linear correlation, the risk can be undermined with the same return rate.

Therefore, we construct a comparison experiment between the risk measured by IM and MM, holding expected return fixed. The result is as follows.



**Fig. 14** Comparison between IM and MM

From the picture, we can observe a larger risk estimated by IM for every fixed expected return. The difference of risk will enlarge when expected return gets far from the minimal variance portfolio. A hypothesis test can be constructed in order to prove that if there is a statically significant difference between the risk estimated by IM and MM in our case.

Since the data set is not very large, and the input data for each model are not too much either (less than 40), due to the rule of thumb. And also considering the fact that Index model adds another

variable and assumptions. As a conclusion, we believe that MM is a better model to estimate our most desire portfolio.

## 6. Conclusion

In this paper, we use 20 years of daily total returns data to calculate all proper optimization inputs for two optimization problems: MM and IM. For each optimization problem, we implement and solve with five additional optimization constraints, which makes us change the proportion we invest in risk-free stock and the other six stocks.

For starters, we aggregate the daily data to the monthly observations in order to reduce non-Gaussian effects. With the prepared data above, we first calculate the average, standard deviation, beta, alpha, and correlation matrix of 7 stocks. Secondly, by changing the 7 stock's weight, under 5 different constraints, we solve with the minimum standard deviation and maximal Sharpe Ratio. Besides, we use the control variable method to produce three key frontiers: by finding the minimal standard deviation of each return, we get the minimal risk or variance frontier and next, the efficient frontier, together with the minimal return frontier are received, which two using the same step, but one needs to find the maximal return, the other is the minimum. Finally, we analyse all the results with the purpose of comparing the different constraints for each optimization problem, MM and IM, and the differences between the two optimization problem solutions under the same constraint.

Based on the results of pursuing the maximal Sharpe ratio under the five constraints, since ADBE has the largest weight for the long position, while C has the worst, the prediction of both IM and MM shows that ADBE has the best investment prospect. Besides, compared with IM, MM predicts a larger Sharpe ratio for each of the 5 constraints.

To sum up, as the foundation of IM, MM can be used to accurately manage the risk of a small amount of assets, which is more suitable for the circumstances that are similar to ours for the reasons above. Contrarily, as an improvement of MM, IM has its superiority when analyzing a large amount of security portfolio, it has been widely used in effective markets throughout the world.

The shortcomings of this paper mainly include two aspects. First is the data we are dealing with is not large enough, specifically, less than forty. Second, we haven't dealt with various portfolios, which may cause errors because of specific securities. In the future, we would implement the two models into more circumstances in order to receive more analyzing results for an improvement of our paper.

## References

- [1] Markowitz H. Portfolio Selection[J], *The Journal of Finance*, 1952, 12: 77-91.
- [2] Markowitz H M. Foundations of portfolio theory[J]. *The journal of finance*, 1991, 46(2): 469-477.
- [3] Tobin, J. (1958): Liquidity Preference as Behavior Towards Risk, *Rev. Econ. Studies* 25, 65– 86.
- [4] Rubinstein M. Markowitz's" portfolio selection": A fifty-year retrospective[J]. *The Journal of finance*, 2002, 57(3): 1041-1045.
- [5] Sharpe, W. F. (1964): Capital Asset Prices: A Theory of Market Equilibrium Under Conditions of Risk, *J. Finance* 19, 425– 442.
- [6] Samuelson, P., & Merton, R. C. (1969). A complete model of warrant pricing that maximizes utility. *IMR; Industrial Management Review* (pre-1986), 10(2), 17.
- [7] West G. An introduction to modern portfolio theory: Markowitz, CAP-M, APT and Black-Litterman[J]. Parktown North: Financial Modelling Agency, 2006.
- [8] Paudel R B, Koirala S. Application of Markowitz and Sharpe Models in Nepalese Stock[J]. *Journal of Nepalese Business Studies*, 2006, 3(1): 18-35.
- [9] Sinha, R., Tripathi, P. K. A comparison of markowitz and sharpe's model of portfolio analysis[J]. *Wealth*, 2017, 6(1): 18-24.
- [10] Varghese J, Joseph A. A Comparative Study on Markowitz Mean-Variance Model and Sharpe's Single Index Model in the Context of Portfolio Investment[J]. *PESQUISA*, 2018, 3(2): 36-41.