

# The Application of Monte Carlo Simulation and Mean-variance in Portfolio Selection

Qinying Li<sup>1,\*,†</sup>, Wenyue Zhong<sup>2,\*,†</sup>

<sup>1</sup>College Of Arts and Science, New York University, New York, United States

<sup>2</sup>School of International Studies, Zhejiang University, Hangzhou, China

\*Corresponding author: ql2164@nyu.edu, 3180106443@zju.edu.cn

†These authors contributed equally.

**Abstract.** This paper manages to construct an efficient portfolio that has a better return than the market index. This portfolio is unique because its five assets have been carefully picked from different industries to diversify risk, which is rarely seen in other research. First, this article uses Monte Carlo simulation to generate a possible portfolio and calculate the weight of each asset in different scenarios. Second, it uses the mean-variance model to measure the risk and return of the portfolio. Based on the accessed data, this study finds that the daily return of the minimum volatility portfolio outruns the market index, and it also determines the weight of each stock in the minimum volatility portfolio. The paper provides feasible ways of constructing an efficient portfolio, such as how to select stocks and allocate assets. The results in this paper may benefit the related investors in financial markets and help them find efficient portfolio that outruns the market index.

**Keywords:** Portfolio; Stock; Monte Carlo simulation; Mean-variance; Efficient frontier

## 1. Introduction

While making an investment, there are two things that people need to keep in mind, return and risk. Many studies document the relationship between risk and return, which is the most fundamental relationship of financial economics. For example, Fama and MacBeth [1] measured a stock's risk as to the co-variance of its return and the return of a market portfolio. French, Schwert and Stambaugh [2] explored the inter-temporal relations between risk and expected returns. The desired investment should be featured with decent return and limited risk; however, such a scenario is hard to achieve with a single asset. Thus, Markowitz [3] proposed the concept of portfolios in 1952, and asset allocation has become a widely-used strategy in investment to realize the diversification of risk and in the meantime, acquire a preferred return. However, the construction of an efficient portfolio is a tricky task that requires investors to study every chosen asset thoroughly. Generally, an efficient portfolio is constructed through a two-stage process, stock selection and asset allocation. Brinson, Hood and Beebower [4] thought that asset allocation is the more important stage. A portfolio should be tailored according to the investor's goals, risk tolerance and investment horizon. There are three main types of assets which are equities, fixed income assets and cash, each of them is featured with different risks and return. Thus, investors need to discreetly select what assets should be included in the portfolio, but this is far from enough. A smart investor would do further research, such as modeling the trend of assets in the future, to find out the best proportion of these assets so as to best balance return and risk. In short, asset allocation has already become a very useful and common strategy in modern investment and by using it properly, one could maximize his expected utility with appropriate risk and return.

Dijkstra [5] pointed out that although efficiency can show the optimal portfolio straightforwardly when the parameters are known, it will not work so efficiently when the parameters are unknown. Sen, Gupta, and Dey [6] developed norm-constrained portfolios based on the efficient frontier developed by Markowitz to reduce the risk when dealing with data in high dimensions. There are higher risks when the dimension of stocks involved in the portfolio is higher. Merritt and Andre de [7] combined efficient frontier and Monte Carlo simulation to figure out the optimal portfolio with reasonable diversification. They draw the efficient frontier and do Monte Carlo simulation to show

more results of portfolio based on different portfolios composed of different proportions of stocks. Mehrjoo, Jasemi, and Mahmoudi [8] promoted that efficient frontier should be the first step to screening portfolio, and Lower Partial Moment (LPM) model is needed as the further step to measure risks in the way more similar to the way perceived by the individual. Lower Partial Moment (LPM) is a set of moments used to estimate financial downside risk. There is more than one LPM. Blanchett [9] incorporated time factors into the basic efficient frontier, “allowing the user to easily adjust the input parameters for mean-variance optimization (MVO) through the use of a simple “time decay” factor, where the annualized standard deviations are reduced at some constant rate,” which shows the changes of the portfolio during different periods. Since stocks’ behaviors change as time passes and they are also influenced by the factors from different periods, time factors should be considered when drawing the efficient frontier. Luo and Wu [10] realized that climate was of great importance to the economy. More specifically, CO<sub>2</sub> is one important element considered to impact the climate to a high degree. Luo and Wu used data of European CO<sub>2</sub> allowance, crude oil market and three stock markets in US, Europe and China from February 25, 2008, to December 5, 2012, and applied two models, Mean-Variance and Mean-CVAR models, to achieve the optimal portfolio of those five components, in order to demonstrate the relationship between climate and economy. Reboredo [11] verified that EU Allowances (EUAs) could reduce portfolio risk based on daily prices for the ICE ECX EUA Futures Contract Emissions Index from March 1, 2008, to July 9, 2011. The author obtained an average VaR(Value at Risk) reduction in the optimal weights and hedge portfolios of about 1.3% and 1.7% compared to the crude oil portfolio without the optimal weights and the hedge portfolio. He got the result of 60% of Amazon and 40% of Apple to obtain the maximum return with the lowest risk. Undoubtedly, the efficient frontier is a useful tool to obtain the optimal portfolio in the stock market. However, it is found that there are a few research on a portfolio for stocks or the application of optimal portfolio in real life. This arouses our interests, and this study chooses several specific stocks to achieve the Maximum Sharp Ratio portfolio and Minimum Volatility Portfolio, and further test if they will work efficiently in the future, instead of merely calculate the optimal portfolio for the past data.

To the best of our knowledge, this paper makes the following contributions to the literature. First, it draws the efficient frontier for the portfolio composed of the following five stocks -- Apple, Johnson&Johnson, Occidental Petroleum, JP Morgan Chase & Co. (JPM), and Costco Wholesale; Second, this study finds the maximum sharp ratio and minimum volatility on the efficient frontier and figures out the proportion of each company -- Costco occupies the largest portion in maximum sharp ratio portfolio and Occidental Petroleum (OXY) occupies the lowest portion in maximum sharp ratio portfolio, while Johnson&Johnson occupies the largest portion in minimum volatility portfolio and JP Morgan Chase & Co. (JPM) occupies the lowest portion in minimum volatility portfolio; Third, we find out the outcomes of those two portfolios based on the data from January 1, 2021, to May 19, 2021, particularly on their average return, and compare the outcomes of those two portfolios with that of the market. This step is to examine if the chosen portfolio works efficiently as expected compared to market index stocks.

This paper is organized as follows. Section 2 shows the data and methodologies used in this paper to describe how to conduct this experiment; Section 3 is the related empirical results about the specific proportion of each company in both portfolios and Section 4 concludes this paper.

## 2. Data

In this paper, for the sake of empirically investigating the portfolio constructions of the American stock market, it collects the data for asset prices from Yahoo Finance (<https://finance.yahoo.com/>). The time scale is from June 1, 2011, to May 31, 2021. Specifically, the concrete assets this study adopted are Apple, Johnson&Johnson, Occidental Petroleum, JPM, and Costco Wholesale. The reasons why we select these assets are as follows. First, these companies are relatively developed with stable earnings and a mature business structures. Second, these companies are in different

industries to diversify the portfolio. More detailed, Apple is in the technology industry, Johnson&Johnson (JNJ) belongs to Healthcare, Occidental Petroleum (OXY) is in the energy industry, JPM is a financial asset, and Costco Wholesale (COST) belongs to the consumer industry. This paper transfers these prices to different returns. Some basic information of these returns is shown in the following Table 1.

**Table 1.** Descriptive statistics of the selected assets

	Mean	Volatility	Max	Min
Apple	0.001	0.018	0.120	-0.129
JNJ	0.001	0.011	0.080	-0.100
OXY	-0.0002	0.027	0.328	-0.365
JPM	0.001	0.018	0.107	-0.112
COST	0.001	0.013	0.100	-0.086

According to Table 1, all of the stocks except OXY have a positive mean of returns. A positive mean of returns indicates that the stock's price has an increase on average. OXY's return faces the largest volatility as 0.027, largest maximum 0.328, and lowest minimum -0.365. OXY's stock price has relatively fluctuated. Although the maximum return of JNJ is 0.080, lower than other stocks, it had the lowest volatility of 0.011. The mean of returns for all the five stocks are close to each other shown in the table: the mean of return for Apple is 0.001, for JNJ is 0.001, for OXY is -0.0002, for JPM is 0.001, and for COST is 0.001.

### 3. Methodologies

#### 3.1 Monte Carlo simulation

This paper adopts the Monte Carlo simulation method to simulate the weights of assets in a certain portfolio. The Monte Carlo simulation is a popular method in portfolio construction and is selected by numerous previous studies. Bilenko and Lavrov [12] used Monte Carlo simulation to perform normal distribution formalization for risk assessment in investment. Korytárová and Pospíšilová [13] has incorporated it in cost-benefit analysis to evaluate investment risks.

Monte Carlo simulation is used to model the probability of a set of outcomes that cannot be easily predicted due to random variables. It helps us to form a better understanding of risk and uncertainty in prediction and forecasting. The method is summarized as follows. Suppose five positive values ( $X_1, X_2, X_3, X_4, X_5$ ) are got, then, the corresponding weight for each asset can be standardized as follows.

$$\omega_i = X_i / \sum X_i \quad (1)$$

where  $\omega_i$  is the standardized weight for the asset.

#### 3.2 Mean-Variance Model

This paper adopts the mean-variance model to measure risk against return. The mean-variance model, first introduced in 1952 by Markowitz [3], is an important strategy used in portfolio investment.

The model is composed of two parameters: expected return and corresponding risk of the expected return of a certain portfolio. Specifically, the expected return of the portfolio is calculated by the following equation.

$$P = \omega'R \quad (2)$$

Where  $P$  is the expected return of the portfolio,  $\omega=(\omega_1 \dots \omega_5)$  is the weight for asset and  $R$  is the expected return for the asset. Variance usually measures how far a set of numbers are spread out from their mean. In this model, variance, expressed as the volatility of returns produced by the selected assets, is used to measure risk in portfolio investment. The variance of the expected return can be calculated as follows.

$$V=D(P)=\omega' \Omega \omega \tag{3}$$

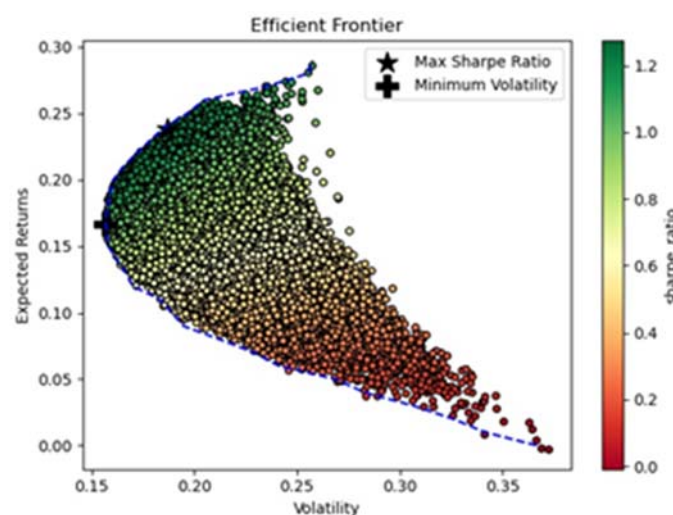
where  $V$  is the variance of the portfolio,  $\Omega$  is the variance-covariance matrix of asset returns. Since this article only includes risky assets in the given portfolio, it adopts the Sharpe ratio to measure the performance of the investment by adjusting for its risk. Sharpe ratio normally explains how much excess returns can be gained if the investor decides to take more risk. If the Sharpe ratio is larger than 1, the excess return outweighs the risk and vice versa. The Sharpe ratio is usually calculated in the following formula,

$$\text{Sharpe ratio} = \frac{R_p - R_f}{\sqrt{V}} \tag{4}$$

where  $R_p$  refers to the expected return of the portfolio;  $R_f$  refers to the risk-free rate of return, and  $V$  refers to the variance of portfolio return.

#### 4. Result

In this experiment, we formed Monte Carlo simulations 100,000 times and calculated their expected returns and volatility for each portfolio. The result is demonstrated as points in Figure 1. The x-axis is volatility, and the y-axis is the expected return. We figured out the efficient frontier based on those results and the two certain important portfolios -- maximum sharp ratio and minimum volatility. The detailed information of the two portfolios is shown in Table 2.



**Figure 1.** Monte Carlo simulations and the efficient frontiers

Figure 1 indicates two points, representing max sharp ratio portfolio and minimum volatility portfolio. The asset weights in the two portfolios are shown in the following Table 2 and Table 3, respectively. The blue dotted line represents the efficient frontier. The efficient frontier is composed of optimal portfolios. The optimal portfolio offers the highest expected return at given volatility or experiences the lowest volatility at a given expected return.

**Table 2.** Maximum Sharp Ratio

company	AAPL	JNJ	JPM	COST	OXY
weight(%)	41.56	13.49	0.37	44.52	0.06
Annualized Return					0.24
Annualized Volatility					0.19

**Table 3.** Minimum Volatility

company	AAPL	JNJ	JPM	COST	OXY
weight(%)	4.73	58.96	0.62	34.90	0.79
Annualized Return					0.17
Annualized Volatility					0.16

Table 2 lists detailed information about the Maximum Sharp Ratio portfolio. In this portfolio, COST occupies the largest portion of 44.52%, AAPL occupies the portion of 41.56%, JNJ occupies the portion of 13.49%, JPM occupies the portion of 0.37%, and OXY occupies the smallest part as 0.06%. The annualized return is 0.24 and the annualized volatility is 0.19.

Table 3 lists detailed information about the Minimum Volatility portfolio. In this portfolio, JNJ occupies the largest portion of 58.96%, AAPL occupies the portion of 4.73%, COST occupies the portion of 34.90%, OXY occupies the portion of 0.79%, and JPM occupies the smallest part of 0.62%. The annualized return is 0.17 and the annualized volatility is 0.16.

The Maximum Sharp Ratio portfolio has both higher expected returns and higher volatility compared to Minimum Volatility portfolio. In both two portfolios, JPM and OXY occupy the relatively low portions--they take 0.37% and 0.06% in the Maximum Sharp Ratio portfolio and 0.62% and 0.79% in Minimum Volatility portfolio. APPL and JNJ behave significantly differently in the two portfolios. More detailed, APPL occupies 41.56% in the Maximum Sharp Ratio portfolio, while it occupies only 4.73% in the Minimum Volatility portfolio. It may be because AAPL offers high expected returns while having high level of risks. On the other hand, JNJ occupies 13.49% in the Maximum Sharp Ratio portfolio, while it occupies 58.96% in the Minimum Volatility portfolio. It may be because JNJ offers relatively lower expected returns while having relatively lower risks.

According to Bednarek and Patel [14], the minimum variance portfolio can outperform several models in the construction of the actual portfolio. Thus, this paper particularly focuses on this model. Specifically, this research compares the model with the market index, NASDAQ Composite, from January 1, 2021, to May 19, 2021. The results are shown in the following Table 4.

**Table 4.** Portfolio Average Daily Return and Market Index Daily Return

Minimum Volatility	NASDAQ Composite (^IXIC)
0.0007	0.0006

This table lists the Average Daily Return for Maximum Sharp Ratio portfolio, Minimum Volatility portfolio, and NASDAQ Composite (^IXIC), the market capitalization-weighted index of over 2,500 common equities listed on the Nasdaq stock exchange. The Average Daily Return for the Minimum Volatility portfolio is 0.0007. It is higher than that of the market index as 0.0006. Thus, it can state that the Minimum Volatility portfolio works efficiently compared to the market index.

## 5. Conclusion

This paper presents the performance of a portfolio, which includes assets from different industries in comparison with the market index. Representative assets of different industries are specifically selected to lower the correlation within the assets and diversify the risk of the portfolio. First, it collects the asset prices from Yahoo Finance and transfers the prices into returns. Second, it uses Monte Carlo simulation to generate possible ways of forming a portfolio and predict the weight of

the assets in each scenario. Then, this study measures return and risk with the help of the mean-variance model. As a result, it finds that the portfolio with minimum volatility exceeds the market index in daily return. The results of this essay might turn out to be useful for investors in financial markets and help investors to make better financial decisions.

However, there is also something that can be improved in this paper. First, it constructs the portfolio based on 10-year data while a data with longer time scale, which might exclude some arbitrary factors, is also worth further studies. Second, the models applied in this paper are only suitable for preliminary studies while more sophisticated models might provide more accurate results.

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