Shenzhen Composite Index based on GARCH class model
Research on Stock Market Volatility

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Abstract. Based on the review of ARCH/GARCH models, this paper uses the GARCH model to empirically study the stock market volatility of Shenzhen Composite Index, uses the GARCH-M model to analyze the risk premium, and uses the EGARCH model to empirically study the asymmetry of stock market volatility. The results show that the GARCH model can eliminate the heteroscedastic property of the residuals, the stock market volatility has a strong impact, the return risk premium is not significant, the volatility caused by bad news in the stock market is much larger than that caused by the same size of good news, there is obvious leverage effect. The GARCH model has a good prediction effect. Finally, some relevant conclusions and policy suggestions are given.

Key words: ARCH/GARCH model; Risk premium; Leverage effect; Impact curve.

1. Introduction

Financial time series often appear the phenomenon of volatility aggregation, that is, the variance in a certain period is relatively small, but in another period is relatively large. The conditional variance of residuals is no longer a function of time change, but will be affected by the volatility of the previous period [1]. In order to describe this feature of financial series, Engle first proposed ARCH model in 1982. The main idea of this model is the disturbance term $U_t$, the conditional variance of depends on its previous value $u_{t-1}$ The magnitude of [2]. However, in practical applications, ARCH models often need a high order to obtain a good fitting effect, which increases the instability of the model and the difficulty of model estimation. Therefore, Bollerslev proposed an improved ARCH model in 1986, which was extended to GARCH model [3].

In order to overcome some weaknesses of GARCH model in dealing with financial time series, Nelson proposed EGARCH model [4]. Specifically, in order to allow the asymmetric effect of positive and negative returns to be reflected in the model, weighted information is considered to be introduced. In recent years, many Chinese experts and scholars have used this model to analyze the Chinese stock market, such as using GARCH model to predict the volatility of Shanghai and Shenzhen stock markets; GARCH model is used to predict stock prices; The ARCH model is used to analyze the effectiveness of Chinese stock market, measure the systematic risk of stock market, and help the government to formulate and improve financial policies.

Based on the review of ARCH/GARCH models, this paper uses GARCH model to analyze the volatility of Shenzhen stock market, GARCH-M model to analyze the risk premium, and EGARCH model to analyze the asymmetry of stock market volatility.

2. Empirical test of stock market volatility model

This paper selects 973 data of daily closing price index of Shenzhen Composite Index from January 1, 2018 to December 31, 2021. All data processing and graph generation in this paper were completed by Eviews8.0. In the analysis, the closing index of Shenzhen Composite index was represented by [SZ]. In order to reduce the rounding error, the natural logarithm processing [LSZ] was performed on [SZ] during the estimation, that is, the series [LSZ] was estimated as the dependent variable [5].
2.1 Descriptive statistics of data

For descriptive statistical analysis of the closing index of Shenzhen Composite Index, first draw the line chart of the closing index, as shown below. Then the common statistics, such as mean, median, maximum, minimum and standard deviation, were calculated. The median of the data was 11389.63, the median was 10765.62, the maximum was 15962.25, the minimum was 7089.441, and the standard deviation was 2458.480. Line chart of the closing index is shown as figure1.

2.2 Build a preliminary model

The stock closing price series is often a random walk process. The following model is established by using EViewS8.0 software [6]:

$$\ln S_{Z_t} = \ln S_{Z_{t-1}} + \mu_t$$ (1)

T = 19423.00
$$R^2 = 0.9953$$, DW = 1.978672
Log likelihood =2706.070, \(\ln S_{Z_{t-1}}\) P value =0.000.

It can be seen that the statistics of this equation are very significant and the fitting effect is good, which further confirms that the stock closing price series is consistent with the random walk model.

(1) The stationarity of the residuals, residual statistical graph and residual sequence graph are carried out.

The ADF stationarity analysis of the residuals was performed on the fitted model above, as shown in Table 1[7].

<table>
<thead>
<tr>
<th>Test critical values</th>
<th>t-Statistic</th>
<th>Prob*</th>
</tr>
</thead>
<tbody>
<tr>
<td>Augmented Dickey-Fuller test statistic</td>
<td>-30.79486</td>
<td>0.0000</td>
</tr>
<tr>
<td>1% level</td>
<td>-2.567346</td>
<td></td>
</tr>
<tr>
<td>5% level</td>
<td>-1.941150</td>
<td></td>
</tr>
<tr>
<td>10% level</td>
<td>-1.616480</td>
<td></td>
</tr>
</tbody>
</table>
The null hypothesis of the ADF test is the existence of a unit root. The alternative hypothesis is that there is no unit root. From $P=0.000$, we can see that $P<0.05$, so there is no unit root\[8\]. The ADF value is less than all critical values, and the series is stationary. The statistical plot of the residuals is shown in Figure 1, and the sequence plot is shown in Figure 2.

![Figure 2. Residual statistics](image)

**Series: RESID01**
Sample 1 973
Observations 972

- Mean: $1.62e-15$
- Median: $0.000102$
- Maximum: $0.053717$
- Minimum: $-0.088596$
- Std. Dev.: $0.014956$
- Skewness: $-0.581848$
- Kurtosis: $5.949302$
- Jarque-Bera: $407.1289$
- Probability: $0.000000$

![Figure 3. Residual sequence diagram](image)

It can be seen from Figure 3 that the statistical properties and characteristics of the residuals show obvious sharp peaks and thick tails. According to the JB test of normal distribution, the distribution of the residual does not meet the requirement of normal distribution[9]. In the following, the distribution of the residual is set as T-distribution. At the same time, volatility aggregation occurs in the residual sequence diagram. The clustering phenomenon of volatility means that the fluctuation is relatively small in a certain period and relatively large in another period, which indicates that the fluctuation of the series is relatively large, and the residual term may have conditional heteroscedastic property.

(2) ARCH-LM test was performed on the residuals
The residuals may have heteroscedastic property, and the ARCH-LM test can test whether the residuals have conditional heteroscedastic property. The ARCH-LM test with lag order $P=3$ was selected according to the partial correlation plot and autocorrelation plot of the squared residuals. At 95% confidence level, $F$ statistic is $3.545453$, $P$ value is $0.0142$, less than 0.05, Chi-square $P$ value is $0.0143$, less than 0.05, indicating ARCH effect (heteroscedastic), and the lag order is relatively high, trying to establish a GARCH model[10].
2.3 GARCH model was established

Due to the high ARCH effect in the residuals of the model, the GARCH model was established directly. The GARCH(1,1) model was established for estimation, and the following results were obtained by Eviews8.0.

The mean equation is obtained as follows:

\[ \ln \text{SZ}_t = \ln \text{SZ}_{t-1} + \mu_t \]  

(2)

Variance equation:

\[ \sigma^2_t = 7.95 \times 10^{-6} + 0.0653 \mu^2_{t-1} + 0.900 \sigma^2_{t-1} \]  

(3)

\[ R^2 = 0.9953, \text{ DW}=1.976, \text{ log-likelihood } = 2772.006 \]

The coefficients of ARCH term and GARCH term in the variance equation were 0.0010 and 0.0000, respectively, which were both very significant at the significance level of 0.01, indicating that the GARCH (1,1) model had a good fitting effect on the data. Arch-Im was used to test the heteroscedastic property of the residual error of the above models. The lag order was set to 1, the F-statistic value was 0.900, and the P-value of F-statistic was 0.3428, which was greater than 0.1. The P-value of Chi-square was 0.3423, greater than 0.1. At this time, there is no ARCH effect in the residual. (Heteroscedastic property)

Looking at the residual autocorrelation diagram, the probability of Q-statistic of the residual does not have P less than 0.05, which is a stationary white noise, indicating that the model is sufficient to extract the information of the original sequence.

2.4 Risk premium analysis based on GARCH-M model

Risk premium refers to the fact that investors demand higher returns to offset greater risk. In efficient financial markets, higher returns go hand in hand with higher risks. People use the variance or standard deviation of the return rate to measure the risk of financial assets. In the garch-m model, conditional variance is put into the mean equation to describe the relationship between risk and return, that is, the risk premium. The garch-m (0,1) model is considered to analyze the risk premium, and the conditional variance term in the mean equation in the model is not significant. \( \sigma^2 \) This may be due to the selection of research objects or the establishment of the model.

2.5 Leverage effect analysis based on EGARCH model

Stock price fluctuation is often manifested as leverage effect phenomenon. Negative information of the same unit often has a greater impact on volatility than good news. This asymmetric effect is called leverage effect. EGARCH model is one of the effective methods to study the leverage effect. Using the EGARCH model to analyze the leverage effect, the EGARCH (1,1) model is established, and the results are as follows:

Mean equation:

\[ \ln \text{SZ}_t = \ln \text{SZ}_{t-1} + \mu_t \]  

(4)

Variance equation:

\[ \ln(\sigma^2_t) = -0.775 + 0.926 \ln(\sigma^2_{t-1}) + 0.190 \left| \frac{u_{t-1}}{\sigma_{t-1}} \right| - 0.06 \frac{u_{t-1}}{\sigma_{t-1}} \]  

(5)

\[ R^2 = 0.9953, \text{ DW}=1.978, \text{ log-likelihood } = 2773.858. \]
According to the residual correlation graph of EGARCH model, the fitting effect of this model is very good, the $P_{|u_{t-1}|}$ of P value of is 0.0002, the $\frac{u_{t-1}}{\sigma_{t-1}}$ of P-value of is 0.01, the two coefficients are significant, and the residual is white noise, the information extraction is sufficient. The existence of leverage effect can be tested by the hypothesis. $\gamma$(this value $-0.06) < 0$. At that time, good news ($u_t > 0$) and bad news ($u_t < 0$) had different effects on the conditional variance: good news had a 0.13 (0.19-0.06) shock; There is a 0.25 (0.19-0.06* (-1)) shock for bad news. It follows that bad news has a bigger shock than good news.

As shown in figure 4, it can be seen intuitively that the information impact is asymmetric at first, and the information impact on the left is obviously steeper than that on the right. It means that when there is a negative shock (bad news), the fluctuation of the shock on the Shenzhen Composite Index is significantly larger than that caused by the positive shock (good news), that is, the market has a stronger reaction to the bad news. Therefore, there is obvious leverage effect, and the volatility caused by the bad news in the stock market is larger than that caused by the good news of the same size.

2.6 Evaluation of prediction ability of the model

According to the comparison between the predicted curve and the original curve, it can be found that the two curves are basically close. Therefore, using the modified GARCH family model to fit the return rate of the index has a good effect. Forecast chart of Shenzhen Composite Index is shown as figure5.
3. Evaluation of the GARCH model

The GARCH model can well describe the sharp and thick-tailed characteristics of the fluctuation of the closing price index of large stocks. The ARCH-LM test shows that the heteroscedastic property of the closing price index is indeed eliminated. Therefore, GARCH models can be more widely used in the analysis of stock market quotations.

In this paper, GARCH model is used to analyze the volatility, risk premium and volatility asymmetry of Shenzhen Composite index series, and the following conclusions can be drawn:

1. The Shenzhen composite index series has obvious GARCH effect, and the residuals show non-normal distribution, with the characteristics of sharp peaks and thick tails.

2. The GARCH-M effect of Shenzhen Stock market is not significant.

3. There is obvious leverage effect in Shenzhen stock market, and the volatility caused by bad news is much larger than that caused by the same size of good news.

4. Application of the model in financial market research

The ARCH family econometric model has a wide range of application and value in China. Theory of the ARCH compared with the traditional theory of CAPM, APT, it is a kind of dynamic non-linear stock pricing model, it broke through the traditional methodology and thinking mode, abandon the risks and benefits the assumption of a linear relationship, reflect a special properties of a random process, the characteristics of the variance changes over time, and has the unconditional obey the width of the tail distribution characteristic, It can exactly describe the basic behavior of prices and returns in the financial market. Therefore, ARCH models can be applied in many fields of economics and finance. The ARCH model can be used to verify the regularity description of economic and financial theory, the hypothesis of market efficiency, measure the risk of market changes, explore the optimal dynamic risk-free decision (or small risk decision), and carry out hedging. Especially in asset pricing, the multivariate ARCH model is more effective in optimizing the portfolio. ARCH models are best at dealing with securities volatility, inflation rate, exchange rate and interest rate. Through them, we can study the exchange rate fluctuation in foreign exchange market and its relationship with monetary policy, formulate domestic foreign exchange policy, and ensure the smooth operation of foreign exchange market.
The application of ARCH family econometric models in China's financial market research is also of great significance and function for investment and management subjects, as well as research and development departments. For example, corporate investors and individual investors can use ARCH family measurement model to predict the volatility of Chinese stock market, evaluate the prediction effect of different models, analyze the value of securities, formulate strategies to participate in market transactions, and avoid market risks. Of course, which model is more effective to predict the volatility of Chinese stock market should be carefully compared, analyzed and evaluated on their preconditions and prediction results. Experts and scholars in research and practice departments can use the nonlinear exponential ARCH model (EARCH model) to analyze whether Chinese stock market returns show cluster effect and heteroscedastic effect. The asymmetric ARCH model can be used to analyze whether there is leverage effect and feedback effect in Chinese stock market when the macroeconomic trend is good and the negative news is more likely to cause stock market volatility. Government departments can also use ARCH family econometric model to formulate stock market intervention policies and financial policies, improve the ability to supervise the stock market to avoid the fluctuations of China's stock market.

5. Conclusion

In this paper, GARCH family model is used to empirically study the volatility of Shenzhen Composite Index, GARCH-M model is used to analyze the risk premium, and EGARCH model is used to empirically study the asymmetry of stock market volatility. Although the GARCH model has a good fitting effect, the volatility of the stock market is still large, and the change of China's stock market is affected by many factors, so investors need to make careful decisions and cannot blindly follow the trend of stocks.

References

[2] Engle, Robert F. Autoregressive Conditional Heteroskedasticity with Estimates of