Pricing and manufacturing strategy of dual-channel green supply chain under common product competition

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Abstract. As consumers' green demand continues to rise, the competition between ordinary products and green products is becoming more and more intense. In order to study the impact of product competition and consumer green demand on product price and product greenness, as well as the optimal manufacturing strategy of dual-channel green supply chain, for a manufacturer and a retailer that can produce common products and green products, a set of The dual-channel supply chain composed of traditional retail channels and online direct sales channels, according to the different production modes of manufacturers, established a dual-channel supply chain game model of traditional production mode, green production mode and mixed production mode, and compared the products under the three production modes Price, greenness, and manufacturer's profit are verified with examples. The results show that: based on the consistent pricing strategy, consumer channel preference directly affects product price and greenness; product competition and consumer green demand sensitivity coefficients both promote product prices and product greenness; the manufacturer's optimal manufacturing strategy for the mixed production mode.

Keywords: Dual channel green supply chain; Pricing decision; Production mode; Product competition.

1. Introduction

With the rapid development of e-commerce, online retailing has opened up a new market for the global retail industry. 2021 saw China's online retail sales of physical goods exceed $1 billion for the first time, and with such huge market demand, more and more manufacturing companies are opening online direct sales channels to further expand their interests. Manufacturers are also developing green products with environmental attributes while adopting online direct sales strategies to attract groups with green needs, thus further enhancing their competitive advantage. How to make strategic pricing in the dual-channel green supply chain and how to choose the manufacturing model of upstream companies have become urgent issues for companies to solve, and also become a hot topic of academic research.

For the dual-channel supply chain decision problem, many scholars have conducted in-depth studies from different perspectives, which are mainly divided into the following two aspects: on the one hand, the impact of different consumer traits on the dual-channel supply chain has been studied from the perspective of consumers. Huang et al [1] argued that consumer green preference would lead to higher product pricing and more profits for supply chain members; Li et al [2] studied the impact of consumer transfer on the dual channels and found that the consumer transfer rate directly affects commercial credit, which is the main factor stimulating retailers to order; Wang et al [3] studied dual-channel supply chain with heterogeneous preference consumers and found that retail and wholesale prices of products are lower when there are agents with effective risk control; Liu et al [4] found that luxury goods suppliers are less willing to open direct sales when facing only loss-averse consumers when they are only willing to open direct sales channels; on the other hand, the impact of channel differences on the supply chain was considered from a channel perspective. Liu [5] analyzed the impact of the degree of channel environmental protection on the supply chain, and the impact of the degree of channel environmental protection on the profits of supply chain members differed under different decision-making approaches; Xie et al [6] explored fresh food price order volume and greenness under three dual-channel sales models and found that the role of channel preference on them is the same regardless of the model.
The above studies on dual-channel supply chains have mainly focused on a single product, and few have considered product competition, but competition among multiple goods has commonly occurred in real life. Amin Saghaeian and Ramezanian Reza [7] established a three-stage, multi-product supply chain using an efficient hybrid genetic algorithm to solve the large-scale problem; Shao and Liu [8] consider a complementary product supply chain that takes into account consumers' environmental awareness and green subsidies provided by the government, and show that both the greenness of complementary products and members' profits increase with the green innovation spillover effect; He et al [9] study the problem of product substitution rates in fresh produce supply chains and find that retailers choose products with higher substitution rates and that high-quality substitutes increase product prices; Pal Brojeswar [10] studied a dual-channel supply chain of common and recycled products and found that an increase in production costs would make manufacturers more inclined to recycle materials to produce recycled products.

In summary, this paper considers selling both common products and green products in direct sales channels and retail channels, establishes a mixed production model dual-channel supply chain game model, and compares it with the traditional production model of only common products and the green production model of only green products, studies the influence of product competition and consumer green demand on supply chain pricing under different production models and the manufacturer's optimal production model choice, and seeks to give reference to dual-channel green supply chain decision makers.

2. Problem Description and Notation Assumptions

In this paper, we take a secondary supply chain consisting of a manufacturer M and a retailer R. The manufacturer, as the dominant player, has a direct network sales channel and can choose to produce either ordinary products or green products; the retailer, as the follower, has a traditional retail channel. As shown in Figure 1, the manufacturer is divided into the traditional production mode of producing only common products, the green production mode of producing only green products, and the mixed production mode of producing both green and common products according to its choice of different production modes. The manufacturer first determines the wholesale price \( W_1, W_2 \), and the greenness \( g \) of the common and green products according to the production model, and the retailer determines the retail price \( P_1, P_2 \) of the common and green products according to the wholesale price. Thus, this paper discusses the influence of product competition and consumer green demand sensitivity coefficient on the pricing of dual-channel supply chain and the optimal manufacturing strategy of the manufacturer concerning the above three models.

![Fig. 1 Dual-channel supply chain under different models](image)

For easy differentiation, normal and green products are indicated by subscripts 1 and 2; offline and online are indicated by subscripts 3 and 4. It is further assumed that.

(1) To reduce channel conflicts, this paper adopts the consistent pricing method, i.e., the retail prices of products in the direct network sales channel and the traditional retail channel are equal, and the wholesale prices, retail prices, and product greenness of ordinary and green products are positive.
(2) Assuming that the basic market volume is a, product demand is influenced by channel, greenness preference, and price sensitivity coefficient, where \( s (0 < s < 1) \) denotes consumer preference for offline channels, denotes consumer preference for online channels; denotes the influence of product greenness on market demand, where \( \theta (\theta > 0) \) denotes consumer demand sensitivity coefficient of green products; \( \rho (\rho > 0) \) denotes consumer price sensitivity coefficient.

(3) Due to the expanding demand for green products in the market, manufacturers are willing to invest more money to increase the greenness of their products. Among them, manufacturers invest in green R&D expenses for product greenness enhancement to increase the market demand for their products [18], where \( \mu (\mu > 0) \) denotes the green input cost coefficient.

(4) \( \beta (0 < \beta < 1) \) indicates the degree of competition between green and common products, and the greater the competitiveness between the two products, it also means the higher the substitutability of the two products.

(5) The difference in production cost between the two products is mainly reflected in the green R&D cost, assuming that the cost of the ordinary product is 0 and the cost of the green product is equal to the green R&D cost.

3. Model Solutions and Discussions

References are cited in the text just by square brackets [1]. (If square brackets are not available, slashes may be used instead, e.g. /2/.)

3.1 Traditional Production Model Supply Chain Pricing Model (Model N)

Under decentralized decision making, the demand functions for each channel of the traditional production model that produces only common products:

\[
D_{N1}^N = a - \rho P_{W1}^N \\
D_{N1}^N = (1 - s)a - \rho P_{W1}^N
\]

Expression for the manufacturer-retailer profit function:

\[
\pi_{MN} = W_{N1}^N D_{N1}^N + P_{W1}^N D_{N1}^N \\
\pi_{MN} = (P_{N1}^N - W_{N1}^N) D_{N1}^N
\]

Theorem 1 The optimal price of the product and the manufacturer’s optimal profit in the supply chain pricing model under the traditional production model:

\[
W_{N1}^* = \frac{a(1-s)}{3\rho} \\
P_{W1}^* = \frac{a(1+2s)}{6\rho} \\
\pi_{M1}^* = \frac{a^2 A}{12\rho} \\
\pi_{R1}^* = \frac{a^2 B^2}{36\rho}
\]

Which \( A = 1 + 4s - 8s^2; \ B = 4s - 1 \)

By Theorem 1, when \( D_{N1}^N > 0 \) and \( P_{W1}^N - W_{N1}^N > 0 \), i.e. \( 1/4 < s < 5/8 \), the manufacturer opens a direct sales channel and the retailer accepts a consistent pricing strategy, and all the theorems in this paper are based on \( 1/4 < s < 5/8 \).

Proposition 1 In the ordinary production model, the price of the ordinary product \( W_{N1}^N \) decreases with increasing \( s \) and \( P_{N1}^N \) increases with increasing \( s \).

Proposition 1 suggests that consumer preference for offline channels is negatively related to the wholesale price of the average product, but positively related to the retail price of the average product. Consumers’ channel preferences are directly related to the demand for each channel. Since manufacturers hold the online direct channel and determine the wholesale price of the product, when consumers’ preferences for the offline channel increase, the demand for the online channel decreases, making the wholesale price lower, and accordingly, retailers hold the offline channel and determine the retail price of the product, and when consumers’ preferences for the offline channel increase, the retail price increases.
3.2 Green Production Model Supply Chain Pricing Model (Model G)

Under decentralized decision making, the demand functions for each channel of the green production model that produces only green products:

\[
\begin{align*}
D_1^G &= sa - \rho P_1^G + \theta g^G \\
D_2^G &= (1 - s)a - \rho P_1^G + \theta g^G
\end{align*}
\]

Expression for the manufacturer-retailer profit function:

\[
\pi_M^G = W_1^G D_3^G + P_1^G D_4^G - \frac{1}{2} \mu g^{G^2} - \frac{1}{2} \mu g^{G^2}
\]

Theorem 2 When \(3\rho - 2\theta^2 > 0\), \(\pi_M^G\) is a concave function with respect to \(W_1^G, g^G\). The optimal price of the product, the optimal greenness and the optimal profit of the manufacturer under the green production model:

\[
\begin{align*}
W_2^G &= \frac{a(s-1)\mu}{2\theta^2 - 3\mu\rho} \\
P_2^G &= \frac{a(\theta^2(R-3) - \mu \rho(2s + 1))}{2\theta^2 - 3\mu\rho} \\
g^G &= \frac{2a\theta(s-1)}{2\theta^2 - 3\mu\rho}
\end{align*}
\]

Proposition 2 Under the green production model, \(W_2^G, P_2^G, g^G\) increase with \(\theta\); \(W_2^G, P_2^G\) increase with \(s\); \(g^G\) decrease with \(s\).

Proposition 2 shows that the demand sensitivity coefficient of consumers for green products is positively correlated with product greenness and product price, where a higher demand sensitivity coefficient of consumers for green products means that the same product greenness will attract more green demand in the market. The greater the demand, the higher the enterprises' investment in green R&D and product pricing, thus forming a virtuous closed loop of green products; the response of green product price to consumers' offline channel preference is the same as the response of ordinary product price in the traditional production model, as the increase of offline channel preference will reduce the offline channel demand, manufacturers will reduce the green input of products to control the wholesale price of green products.

Proposition 3 Comparing the prices of traditional and green production models, it was found that \(W_2^G > W_1^N\); \(P_2^G > P_1^N\).

Proposition 3 shows that the price of the green product is always higher than the price of the ordinary product regardless of the variation of the parameters in the model. This is because the green product has greener R&D costs than the ordinary product in terms of cost, so the cost of the green product is always greater than the cost of the ordinary product.

3.3 Hybrid production model supply chain pricing model (Model NG)

The manufacturer produces both ordinary and green products, and there is substitutability between the two products, when the dual channels sell both products. The demand functions for the two products in different channels in the mixed production model:

\[
\begin{align*}
D_3^NG &= sa - \rho P_1^{NG} + \beta P_2^{NG} \\
D_4^NG &= (1 - s)a - \rho P_1^{NG} + \beta P_2^{NG} \\
D_3^NG &= sa - \rho P_1^{NG} + \beta P_1^{NG} + \theta g^{NG} \\
D_4^NG &= (1 - s)a - \rho P_2^{NG} + \beta P_1^{NG} + \theta g^{NG}
\end{align*}
\]

The expression for the manufacturer-retailer profit function:

\[
\begin{align*}
\pi_M^{NG} &= W_1^{NG} D_3^{NG} + W_2^{NG} D_4^{NG} + P_1^{NG} D_3^{NG} + P_2^{NG} D_4^{NG} - \frac{1}{2} \mu g^{NG^2} - \frac{1}{2} \mu g^{NG^2}
\end{align*}
\]
Theorem 3 When $3\theta (\beta^2 + \rho \beta) + 9(4\rho \beta^2 - 2\mu^2 + \rho \theta^2) > 0$, $\pi_M^{NG}$ is a concave function with respect to $W_1^{NG}, W_2^{NG}, g^{NG}$. Optimal price, optimal greenness, and optimal manufacturer's profit for the common and green products in the mixed production model.

$$W_1^{NG} = \frac{a(s - 1)(-2\theta^2 + 3\beta\mu + 3\mu \rho)}{9\beta^2 \mu + 6\theta^2 \rho - 9\mu^2}$$

$$W_2^{NG} = \frac{a\mu(s - 1)(\beta + \rho)}{3\beta^2 \mu + 2\theta^2 \rho - 3\mu \rho}$$

$$P_1^{NG*} = \frac{a[8\theta\beta^2(s - 1) + 2\theta^2 \beta \mu + 3\beta^2 (\mu + 2x\mu) (1 + 2s) \rho (2\theta^2 - 3\mu \rho)]}{6(\beta - \rho)(2\theta^2 \rho + 3\mu (\beta^2 - \rho^2))}$$

$$P_2^{NG*} = \frac{a(-\beta^2 (\mu + 2x\mu) + \rho (\theta^2 (2 - 4s) + (1 + 2s) \mu \rho))}{2(\beta - \rho)(2\theta^2 \rho + 3\mu (\beta^2 - \rho^2))}$$

$$g^{NG*} = \frac{2a\theta(s - 1)(\beta + \rho)}{3\beta^2 \mu + 2\theta^2 \rho - 3\mu \rho}$$

$$\pi_M^{NG*} = \frac{a^2 (4\theta^2 (s - 1)^2 - 3\beta^2 \mu + 2\theta^2 \rho \mu C + 3A\mu \rho)}{6(\beta - \rho)(2\theta^2 \rho + 3\mu (\beta^2 - \rho^2))}$$

$$\pi_M^{NG} = \frac{a^2 (-3\beta^2 \mu (2F + 3B \beta \theta) - 4\beta \theta^2 D(F + 3\mu \beta \theta)) + 2(2\theta^2 E + 3B \mu \beta \theta + 9B \mu \beta \theta) \rho^2 + 12\theta^2 \mu (BD - 9B \mu \beta \theta) \rho^4}{18(\beta - \rho)(2\theta^2 \rho + 3\mu (\beta^2 - \rho^2))}$$

Which $C = 1 - 8s + 10s^2$, $D = 5s^2 - 2, E = 26s^2 - 22s + 5$, $F = 20^2 (s - 1)$

Proposition 4 In the mixed production model, $W_1^{NG*}, W_2^{NG*}, P_1^{NG*}, P_2^{NG*}, g^{NG*}$ is positively correlated with $\theta$ and $\beta$.

Proposition 4 is similar to Proposition 2 in that the price and greenness of green products in the mixed production model and the green production model respond equally to the consumer's green product demand sensitivity coefficient. Interestingly, the price of the ordinary product also increases with the increase of the consumer's green product demand sensitivity coefficient, at which time the ordinary product appears as a free-rider; independent of the production product, when the consumer's offline channel preference is higher, the manufacturer will lower the wholesale price and product greenness, and the retailer will increase the retail price; when the degree of product competition is higher, the substitutability of the two products is higher, at which time the dual oligopolistic firms will raise product prices and greenness; conversely, the substitutability of the two products is low, and raising pricing at this time may cause negative consumer sentiment.

Proposition 5 Comparing competitive and non-competitive supply chains yields:

1. $g^{NG*} - g^G$
2. $W_1^{NG*} > W_1^{NG}, W_2^{NG*} > W_2^{NG}, P_1^{NG*} > P_1^{NG}, P_2^{NG*} > P_2^{NG}$
3. $\pi_M^{NG*} > \pi_M^{NG*} > \pi_M^{NG}$

Proposition 5 shows that the product price, greenness, and manufacturer profit of the hybrid production model are higher than those of the traditional and green production models. This is because there is only one manufacturer and one retailer in the supply chain, and when product competition arises, firms create price increases by raising the prices of both products at the same time.

4. Computational Results

The following will analyze and compare the decision variables of the supply chain of traditional production model, green production model and hybrid production model by arithmetic examples to reveal the effects of product competition intensity and consumer green product demand sensitivity coefficient on product greenness, product price and manufacturer's profit, where the conclusions obtained according to the assumptions and each theorem of this paper set $a = 100, \theta = 2, \mu = 5, \rho = 2.5, s = 0.4, \beta = 0.6$ and assume $\beta \in (0.1), \theta \in (0.4)$.

4.1 Analysis of the influence of different parameters on the greenness of the product

The relationship between product greenness $g^{NG*}, g^{NG*}$ and $\theta$ and $\beta$ in different models is shown in Fig 2.
The comparison in Figure 2 reveals that the product greenness of model G is smaller than that of model NG. When $\beta=0$, the greenness of both models is the same, and as $\beta$ increases, green products need to improve their product advantages by increasing greenness; when $\theta=0$, the greenness of products in both models is also 0, which indicates that manufacturers are willing to invest in green R&D only when there is a demand for green products in the market. With the increase of $\theta$ the greenness of both models will increase, and the larger $\theta$, is the more significant the greenness.

4.2 Analysis of the influence of different parameters on product prices

The relationship between product price $w^{N}$, $w^{G}$, $w^{NG}$, $p^{N}$, $p^{G}$, $p^{NG}$ and $\theta$ and $\beta$ in different models is shown in the figure.

Figure 3 shows that the prices of ordinary products in model NG are all higher than those in model N; the prices of green products in model NG are all higher than those in model G; the prices of green products in model NG are higher than those of ordinary products. When $\beta = 0$, the prices of the same goods are the same, and only when $\beta$ appears, $\beta$ stimulate the product prices to rise. This is a marketing strategy of the duopoly market to create the phenomenon of the price increase by raising the prices of multiple products with substitutability at the same time. When $\theta = 0$, the prices of green products...
and ordinary products in a similar model are the same. Combined with Figure 2, when there is no green demand in the market, manufacturers will not invest in green R&D for green products at this time, and both products have the same cost and naturally the same price. When $\theta$ increases, the higher the manufacturer's cost investment in the green product, the higher the price, and the common product appears to increase in free-riding behaviour along with the price of the green product.

### 4.3 Analysis of the impact of different parameters on manufacturers' profits

The relationship between manufacturer's profit $\pi_M^N$, $\pi_M^G$, $\pi_M^{NG}$ and $\theta$ and $\beta$ in different models is shown in the figure.

![Fig. 4 Effect of different parameters on manufacturer's profit](image)

The comparison in Figure 4 reveals that regardless of the change in the values of $\theta$ and $\beta$, the manufacturer can earn more profit when it chooses the hybrid production model. The increase in $\theta$ and $\beta$ better highlights the advantages that the hybrid production model brings to the manufacturer.

### 5. Summary

With the expansion of e-commerce and people's pursuit of green products, this paper studies the dual-channel green supply chain pricing and manufacturing strategy under common product competition, and obtains the following findings by analyzing and comparing the supply chain pricing models of traditional, green, and hybrid production modes.

(1) Based on a consistent pricing strategy, consumer offline channel preference is negatively correlated with product wholesale price and greenness and positively correlated with product retail price, regardless of the production model chosen. When companies choose to open new channels, they should fully investigate consumers' channel preferences and decide on the appropriate channels and pricing according to the market situation.

(2) Product greenness, product price, and manufacturer's profit are positively correlated with the consumer green demand sensitivity coefficient; the free-riding effect makes the price of common products increase with the increase of the consumer green demand sensitivity coefficient. The increase in consumers' green demand, not only encourages enterprises to research and develop the greenness of products but also increases the profits of enterprises; society and enterprises should pay more attention to cultivating consumers' green concepts.

(3) By choosing the mixed production model, manufacturers can gain more profit. The product greenness, product price and manufacturer's profit in the mixed production mode will increase with the increase in product competition. From the perspective of enterprise profitability, the diversity and substitutability of products will have a positive impact on enterprises, and in the future, enterprises can develop diverse products to improve their competitiveness.
Appendix

Proof of Theorem 1: Solving by the inverse order method, first, find the first order partial derivative of $P_i^{N_1}$ with respect to $\pi_{R}^{N_1}$:
\[
\frac{\partial \pi_{R}^{N_1}}{\partial P_i^{N_1}} = \rho W_i^{N_1} - 2 \rho P_i^{N_1} + sa = 0
\]
so that it is equal to zero to obtain $W_i^{N_1}$. Substituting this into the reaction function yields $P_i^{N_1}$. Finally, we obtain $\pi_{M}^{N_1}, \pi_{R}^{N_1}$.

Proof of Proposition 1: If $W_i^{N_1}$, $G_i^{N_1}$ exists for the optimal solution. Then, the first-order partial derivative of $\pi_{R}^{N_1}$ with respect to $P_i^{N_1}$, $G_i^{N_1}$:
\[
\frac{\partial \pi_{R}^{N_1}}{\partial P_i^{N_1}} = \theta g^{N_1} + G_i^{N_1} - 2 P_i^{N_1} = 0
\]
where the Hessian matrix of $\pi_{R}^{N_1}$ is: $H = \begin{bmatrix} \frac{\partial^2 \pi_{R}^{N_1}}{\partial g^{N_1}^2} & \frac{\partial^2 \pi_{R}^{N_1}}{\partial g^{N_1} \partial P_i^{N_1}} \\ \frac{\partial^2 \pi_{R}^{N_1}}{\partial P_i^{N_1} \partial g^{N_1}} & \frac{\partial^2 \pi_{R}^{N_1}}{\partial P_i^{N_1}^2} \end{bmatrix}$ such that $|H| > 0$, $W_i^{N_1}, G_i^{N_1}$ is equal to zero, and the joint solution yields $W_i^{N_1}, G_i^{N_1}$.

Proof of Proposition 2: If $W_i^{N_1} - W_i^{N_1} = 4a \theta S(1 - \theta) > 0$, $G_i^{N_1} - G_i^{N_1} = 4a \theta S(1 - \theta) > 0$

Proof of Theorem 3: First, find the first-order partial derivative of $\pi_{R}^{N_1}$ with respect to $P_i^{N_1}, P_i^{N_1}$:
\[
\frac{\partial \pi_{R}^{N_1}}{\partial P_i^{N_1}} = 2 \beta P_i^{N_1} + as - \beta W_i^{N_1} - 2 P_i^{N_1} \rho - W_i^{N_1} \rho = g^{N_1} \theta + 2 \beta P_i^{N_1} + as - \beta W_i^{N_1} - 2 P_i^{N_1} \rho + W_i^{N_1} \rho
\]

First-order partial derivative of $W_i^{N_1}, W_i^{N_1}, G_i^{N_1}$:
\[
\frac{\partial \pi_{R}^{N_1}}{\partial W_i^{N_1}} = 2 + \frac{\partial \pi_{R}^{N_1}}{\partial G_i^{N_1}} = \frac{\partial \pi_{R}^{N_1}}{\partial W_i^{N_1}} = \frac{\partial \pi_{R}^{N_1}}{\partial G_i^{N_1}} = \frac{\partial \pi_{R}^{N_1}}{\partial P_i^{N_1}} = \frac{\partial \pi_{R}^{N_1}}{\partial G_i^{N_1}} = \frac{\partial \pi_{R}^{N_1}}{\partial W_i^{N_1}} = \frac{\partial \pi_{R}^{N_1}}{\partial G_i^{N_1}}
\]
where the Hessian matrix of $\pi_{M}^{N_1}$ with respect to $W_i^{N_1}, W_i^{N_1}, G_i^{N_1}$ is: $H = \begin{bmatrix} \frac{\partial^2 \pi_{M}^{N_1}}{\partial g^{N_1}^2} & \frac{\partial^2 \pi_{M}^{N_1}}{\partial g^{N_1} \partial P_i^{N_1}} \\ \frac{\partial^2 \pi_{M}^{N_1}}{\partial P_i^{N_1} \partial g^{N_1}} & \frac{\partial^2 \pi_{M}^{N_1}}{\partial P_i^{N_1}^2} \end{bmatrix}$, so that when

$|H| > 0$, $W_i^{N_1}, W_i^{N_1}, G_i^{N_1}$ is equal to zero, and the joint solution yields $W_i^{N_1}, W_i^{N_1}, G_i^{N_1}$. Substituting this into the reaction function yields $P_i^{N_1}, P_i^{N_1}$, and finally $\pi_{M}^{N_1}$.
\[ \frac{\partial W_{N^G}}{\partial \theta} = \frac{4a\beta \theta (1 - s) \mu (\beta + \rho)}{(3\beta^2 + \rho(2\theta^2 - 3\mu^2))} > 0, \quad \frac{\partial W_{N^G}}{\partial \beta} = \frac{a(1 - s) \mu (-4\beta^2 + 3\beta^2 \mu^2 + 2\theta^2 \rho + 6\theta \mu \rho + 3\mu \rho^2)}{(2\theta^2 + 3\mu(\beta^2 - \rho^2))} > 0 \]

\[ \frac{\partial W_{N^G}}{\partial \mu} > 0, \quad \frac{\partial W_{N^G}}{\partial \rho} > 0, \quad \frac{\partial W_{N^G}}{\partial \theta} > 0, \quad \frac{\partial W_{N^G}}{\partial \beta} > 0, \quad \frac{\partial W_{N^G}}{\partial \mu} > 0, \quad \frac{\partial W_{N^G}}{\partial \rho} > 0 \]

Proving Proposition 5:

1. \[ g_{N^G} - g_{G^G} = \frac{2a\beta^2 (1 - s)}{3\mu(3\rho - 3\beta^2 - 2\theta^2)} > 0 \]
2. \[ W_{N^G} - W_{G^G} = \frac{a\beta \mu (1 - s)(\beta + \rho)}{(3\beta^2 - 3\beta^2 \mu - 2\theta^2 \rho)} > 0, \quad W_{N^G} - W_{G^G} = \frac{a\beta \mu (s - 1)(2\theta^2 - 3\beta^2 \mu)}{(2\theta^2 - 3\rho \mu (\beta^2 - \rho^2))} > 0 \]
3. \[ P_{N^G} - P_{G^G} = \frac{a\beta (\beta^2 + 2\theta^2 \mu + \rho (\theta^2 (2 - 4s) + (1 + 2s) \mu \rho))}{2(\beta - \rho)(2\theta^2 + 3\mu(\beta^2 - \rho^2))} > 0, \quad \frac{P_{N^G}}{P_{G^G}} = \frac{4\theta^2 (s - 1)}{(2\rho - 2\theta^2 + 3\mu(\beta^2 - \rho^2))} > 1 \]
4. \[ \pi_{N^G} > \pi_{G^G}, \quad \pi_{N^G} > \pi_{M^G}, \quad \pi_{M^G} > \pi_{M^N}, \quad \pi_{M^N} > \pi_{M^G} \]

References


