Monetary policy responses to supply shocks and demand shocks

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Abstract. Economic shocks, mainly COVID-19, have had a large impact on the global economy, such as the recent extremely high inflation rate in the United States. The monetary policy of the central bank plays a great role in stabilizing the economy. This paper mainly discusses the specific monetary policies implemented by the central bank when the macro economy meets the unexpected shocks and because one of the most important assumptions in this paper is that when the central bank makes decisions, the economy along the “balanced growth path”, the economy discussed in this paper is a smoothly developing developed country, such as the United States, rather than China.

Keywords: Monetary policy, dynamic AS-AD model, Solow model, Monetary policy rule.

1. Introduction

The long-term economic stability is extremely important because a long spurious boom will surely mean a huge recession ahead, such as the Great Depression of 1929 and the subprime mortgage crisis of 2008. What is more, the recession of economy brings foreseeable uncertainty to the whole society. The Great Depression, for example, indirectly led to World War II. So the goal of many central banks is to maintain long-term economic stability, in other words, to minimize the fluctuation of the national economy, which can be achieved through reasonable monetary policy regulation.

When facing supply or demand shocks in the macroeconomic environment, the central bank usually carries out monetary policy to adjust the economy and control the interest rate. In this paper, the specific method for the central bank to do will be illustrated when the economy is along the “balanced growth path” (the proof will be given below) by using the dynamic AS-AD model.

New-Keynesian shows that money neutrality exists in the long run but in the short run there has money non-neutrality (Fischer, 1977), which shows that the central bank can affect the short-run economic growth, but not in the long run. Suppose that the rational policymakers want to minimize the fluctuation of the economy, which means that the central bank wants to minimize this kind of function:

$$\min \sum_{t=1}^{\infty} (Y_t - \bar{Y}_t)^2$$

In this function, the central bank makes decisions from time $t = 1$, and the data $t < 1$ is the past exogenous data, $Y_t$ means the output at time $t$, $\bar{Y}_t$ means the nature output at time $t$.

* In this paper, $t \in \mathbb{Z}$.

From the dynamic AS-AD model, there are two equations: (Mankiw, 2009)

**DAS:** $\pi_t = \pi_{t-1} + \phi (Y_t - \bar{Y}_t) + \nu_t$

**DAD:** $Y_t = \frac{1}{1+\alpha_y} (\pi_t - \pi_t^*) + \frac{1}{1+\alpha_y} \varepsilon_t$.

Long-term economic stability can not only reduce macroeconomic and social problems, but also increase investor confidence and promote further economic development.

One of the contributions of this paper is the innovation of the model, which mixes the short-to-medium term model of DAS-DAD with a long-term model of Solow model. This provides a new way to analyze macroeconomics. At the same time, because the model implies that central bank decisions are affected by multiple variables, central banks and statistical departments are very important for data collection and processing in order to quantify these variables.
2. Assumptions

1. Central bank knows past $Y_t$
e.g., $Y_0, Y_{-1}, Y_{-2}$
2. The supply shock $\nu_t \sim N(0, \sigma_\nu^2)$ and $\{\nu_t\}$ is i.i.d and the central can measure the supply shock $\nu_t$ each period. The central bank also knows the past $\nu_t$ (e.g., War, oil crisis, Covid-19).
3. The demand shock $\varepsilon_t \sim N(0, \sigma_\varepsilon^2)$ and $\{\varepsilon_t\}$ is i.i.d and the central bank can measure the demand shock $\varepsilon_t$ each period.
4. Central bank knows past $\bar{Y}_t$ e.g., $\bar{Y}_0, \bar{Y}_{-1}, \bar{Y}_{-2}$, and can measure the $\bar{Y}_t$ when $t \geq 1$ (When $u_t = \bar{u}$ or $P = P^e$) (the proof will be given later).

- $u_t$ means the unemployment rate at time $t$.
- $\bar{u}$ means the natural unemployment rate at time $t$.
- $P$ means the price level at time $t$.
- $P^e$ means the expected price level at time $t$.
5. Before $t = 1$, $u_t = \bar{u}$ each period.
   In other words, $Y_t = \bar{Y}_t$ when $t < 1$.
6. The economy is along the “balanced growth path” at $t = 1$ in a steady state. (Blanchard, Johnsan, 2013)

3. The Model

3.1 The method to calculate the $\bar{Y}_t$:

When $u_t = \bar{u}$ or $P = P^e$, $Y_t = \bar{Y}_t$

Suppose $Y_t = F_1(A_t, K_t, N_t)$

- $A_t$ means the technological level at time $t$.
- $K_t$ means the number of capitals at time $t$.
- $N_t$ means the number of people with jobs at time $t$.
- $N_t = (1 - u_t)L_t$
- $L_t$ means the labor force.

In this case, the production function is assumed to be:

$Y_t = K_t^\alpha (A_t N_t)^{1-\alpha}$

$\quad = K_t^\alpha (A_t (1 - u_t)L_t)^{1-\alpha}$

When $u_t = \bar{u}$, $Y_t = \bar{Y}_t = K_t^\alpha (A_t (1 - \bar{u})L_t)^{1-\alpha}$

- To calculate $\bar{u}$:
  - Wage determination $W = P^e F_2(u_t, z)$
  - Wage depends negatively on employment rate $u_t$, but positively on the catchall variable $z$. In this case, $z$ is assumed to be a constant.
  - Price determination $P = (1 + m)W$ where $m$ is the markup of the price over the cost. (Blanchard, Johnsan, 2013)

When $P = P^e$, $u_t = \bar{u}$, $Y_t = \bar{Y}_t$.

$\frac{W}{P} = F_2(u_t, z)$ and $\frac{P}{W} = 1 + m$ from the wage determination and price determination.

$F_2(u_t, z) = \frac{1}{1 + m}$ and $u_t = \bar{u}$.

Because the $z, m$ is constant, so $\bar{u}$ is constant and $\bar{u} = F_3(z, m)$.

- To calculate the $A_t, K_t, L_t, N_t, Y_t, \bar{Y}_t$ when $t = 1$:

$A_0, K_0, L_0$ are assumed to be known by the central bank.

This model assumes there are exogenous growth:

$A_{t+1} = (1 + g_A)A_t$ and $L_{t+1} = (1 + g_L)L_t$ and the central bank has known the $g_A$ and $g_L$.

Because $N_t = (1 - u_t)L_t$ and assumption 5 (Before $t = 1$, $u_t = \bar{u}$ each period.)

so $N_{t+1} = (1 + g_L)N_t = (1 + g_L)(1 - \bar{u})L_t$ when $t \leq -1$
\[ Y_t = K_t^\alpha (A_t N_t)^{1-\alpha} \]

Construct \( y_t \equiv \frac{Y_t}{A_t N_t} \), \( k_t = \frac{K_t}{A_t N_t} \)

Thus \( y_t \equiv f(k_t) = k_t^\alpha \)

Solow model recall: \( \Delta K_{t+1} = K_{t+1} - K_t = sY_t - \delta K_t \)

So \( \frac{K_{t+1}}{A_{t+1} N_{t+1}} = \frac{K_t}{A_t N_t} - s \frac{Y_t}{A_t N_t} - \delta \frac{K_t}{A_t N_t} \)

\[ = (1 + g_A) (1 + g_L) k_{t+1} - k_t = s f(k_t) - \delta k_t \]

\[ \approx (1 + g_A + g_L) k_{t+1} - k_t = sk_t^\alpha - \delta k_t \] because \( g_A g_N \) is extremely small.

\[ = (1 + g_A + g_L) (k_{t+1} - k_t) = sk_t^\alpha - (\delta + g_A + g_L) k_t \]

\( s \) means the saving rate, \( \delta \) means the depreciation rate.

When \( k_{t+1} - k_t = 0 \), \( sk_t^\alpha - (\delta + g_A + g_L) k_t = 0 \).

Because \( f'(k_t) > 0, f''(k_t) < 0, s > 0, (\delta + g_A + g_L) > 0 \),

Must exist one positive solution \( k^* \) and because \( s, \delta, g_A, g_L, \alpha \) is constant, \( k^* \) is constant and at this moment the economy is along the “balanced growth path” (Blanchard, Johnsan, 2013). Proof 1

In this case, \( k^* = \left( \frac{s}{\delta + g_A + g_L} \right) \frac{1}{1-\alpha} \), \( y^* = k^* \alpha = \left( \frac{s}{\delta + g_A + g_L} \right) \frac{1}{1-\alpha} \) can be calculated.

Because \( K_t = k^* A_t N_t, Y_t = y^* A_t N_t \) when the economy is along the balanced growth path.

\[ K_t = \left( \frac{s}{\delta + g_A + g_L} \right) \frac{1}{1-\alpha} A_t N_t \text{ and } Y_t = \left( \frac{s}{\delta + g_A + g_L} \right) \frac{1}{1-\alpha} A_t N_t \]

Because of the assumption 6 (the economy is along the “balanced growth path” at \( t=1 \)).

\[ K_1 = \left( \frac{s}{\delta + g_A + g_L} \right) \frac{1}{1-\alpha} A_1 N_1, \quad Y_1 = \left( \frac{s}{\delta + g_A + g_L} \right) \frac{1}{1-\alpha} A_1 N_1 \]

Because of the exogenous growth \( A_{t+1} = (1 + g_A) A_t, A_1 = (1 + g_A) A_0 \).

However, at time \( t = 1 \), this model does not have the assumption 5 anymore (\( u_t = \bar{u} \) each period).

\( N_t = (1 - u_t) L_t \), the growth of \( N_t \) may not be \( g_L \) because it is influenced by the \( u_t \) at time \( t = 1 \).

At first, this model infers that the central bank wants to minimize this kind of function: \( \min \sum_{t=1}^{\infty} (Y_t - \bar{Y}_t)^2 \), the best solution is \( Y_t = \bar{Y}_t \) every period (the specific method will be shown in the next section), so when the economy faces the supply shock \( v_t \) or demand shock \( \varepsilon_t \) at time \( t = 1 \), the central bank can use the monetary policy to prompt \( u_1 = \bar{u} \) and \( Y_1 = \bar{Y}_1 \), and \( N_1 = (1 - \bar{u}) L_1 = (1 - \bar{u}) (1 + g_L) L_0 = (1 + g_L) N_0 \).

\[ = (1 + g_A) N_0 \]

Thus \( A_1 = (1 + g_A) A_0 \)

\[ K_1 = \left( \frac{s}{\delta + g_A + g_L} \right) \frac{1}{1-\alpha} (1 + g_A) (1 + g_L) A_0 N_0 \]

\[ L_1 = (1 + g_L) L_0 \]

\[ N_1 = (1 - \bar{u}) L_1 = (1 - \bar{u}) (1 + g_L) L_0 = (1 + g_L) N_0 \]

\[ Y_1 = \bar{Y}_1 = y^* A_1 N_1 = \left( \frac{s}{\delta + g_A + g_L} \right) \frac{1}{1-\alpha} (1 + g_A) (1 + g_L) A_0 N_0 \]

To calculate the \( A_t, K_t, L_t, N_t, Y_t, \bar{Y}_t \) when \( t > 1 \):

When the economy faces the supply shock \( v_t \) or demand shock \( \varepsilon_t \) at time \( t = 2 \), the central bank also can use the monetary policy to prompt \( u_2 = \bar{u} \) and \( Y_2 = \bar{Y}_2 \) with the purpose of

\[ \min \sum_{t=1}^{\infty} (Y_t - \bar{Y}_t)^2 \]

\( \text{when } t > 1, \text{ we do not have the assumption 6 anymore (the economy is along the “balanced growth path” at } t=1) \)

In macroeconomics, the balanced growth path of a dynamic model is a trajectory such that all variables grow at a constant rate. (Wikipedia, 2022)

When \( u_2 = \bar{u}, N_2 = (1 - \bar{u}) L_2 = (1 - \bar{u}) (1 + g_L) L_1 = (1 + g_L) N_1 \),
\[ N_2 = \frac{(1-\bar{u})L_2}{(1-\bar{u})L_1} = (1+g_L). \] under this condition, repeat the proof of the Solow model above (proof 1), because the economy is along the “balanced growth path” at \( t=1 \), the economy is still along the “balanced growth path” at \( t=2 \) and the growth rate of \( Y_2 \) is \((1+g_A)(1+g_L)\), which is as same as \( Y_1 \) and follow Wikipedia’s definition of “balanced growth path”.

\[ k^* = \left( \frac{s}{\delta+g_A+g_L} \right)^{1-\alpha}, \quad y^* = k^{*\alpha} = \left( \frac{s}{\delta+g_A+g_L} \right)^{1-\alpha}, \quad k^* \text{ same as before, because } s, \delta, g_A, g_L, \alpha \text{ is unchanged.} \]

Thus \( A_2 = (1+g_A)^2A_0 \)

\[ K_2 = \left( \frac{s}{\delta+g_A+g_L} \right)^{1-\alpha}(1+g_A)^2(1+g_L)^2A_0N_0 \]

\[ L_2 = (1+g_L)^2L_0 \]

\[ N_2 = (1-\bar{u})L_2 = (1-\bar{u})(1+g_L)^2L_0 = (1+g_L)^2N_0 \]

\[ Y_2 = \bar{Y}_2 = y^*A_2N_2 = \left( \frac{s}{\delta+g_A+g_L} \right)^{1-\alpha}(1+g_A)^2(1+g_L)^2A_0N_0 \]

when \( t > 2 \), start at \( t = 3 \): repeat the previous inference:

1. When the economy faces the supply shock \( v_t \) or demand shock \( \varepsilon_t \) every time \( t \), the central bank also can use the monetary policy to prompt \( u_t = \bar{u} \) and \( Y_t = \bar{Y}_t \) with the purpose of \( \min \sum_{t=1}^{\infty}(Y_t - \bar{Y}_t)^2 \).

2. Start at \( t = 3 \), because \( u_3 = \bar{u} \) and when \( t = 2 \) the economy is along the “balanced growth path”, the economy is still along the “balanced growth path” at \( t = 3 \).

The conclusion can be reached by mathematical induction:

1. When \( t \geq 1 \), if the central bank can implement appropriate monetary policies, even if the economy faces the supply shock \( v_t \) or demand shock \( \varepsilon_t \) each period \( t \), monetary policy can help \( u_t = \bar{u} \) and \( Y_t = \bar{Y}_t \) each period \( t \).

2. Because of \( u_t = \bar{u} \) each period \( t \) and assumption 6 (the economy is along the “balanced growth path” at \( t=1 \) in steady state), the economy is along the “balanced growth path” each period \( t \).

3. \( Y_t = \bar{Y}_t = y^*A_2N_t \left( \frac{s}{\delta+g_A+g_L} \right)^{1-\alpha}(1+g_A)^t(1+g_L)^tA_0N_0=G(s, \delta, g_A, g_L, \alpha, A_0, N_0, t) \)

\[ s, \delta, g_A, g_L, \alpha, A_0, N_0 \text{ are constant, only } t \text{ changes in time.} \]

Therefore, the central bank has a way to compute \( \bar{Y}_t \).

The method to implement appropriate monetary policies when the economy faces the supply shock \( v_t \) or demand shock \( \varepsilon_t \) each period \( t \), \( t \geq 1 \):

As mentioned above: When the economy faces the supply shock \( v_t \) or demand shock \( \varepsilon_t \) every time \( t \), the central bank also can use the monetary policy to prompt \( u_t = \bar{u} \) and \( Y_t = \bar{Y}_t \) with the purpose of \( \min \sum_{t=1}^{\infty}(Y_t - \bar{Y}_t)^2 \).

From the dynamic AS-AD model, there are two equations: (Mankiw, 2009)

DAS: \( \pi_t = \pi_{t-1} + \phi(Y_t - \bar{Y}_t) + v_t \)

DAD: \( Y_t = \bar{Y}_t + \frac{\alpha\pi_t}{1+\alpha\theta_t}(\pi_t - \pi_t^*) + \frac{1}{1+\alpha\theta_t}\varepsilon_t \).

\( \pi_t^* \) is the target inflation rate.

DAS: use the lag operator:

\[ \pi_t - \pi_{t-1} = \phi(Y_t - \bar{Y}_t) + v_t \]

\( (1-L)\pi_t = \phi(Y_t - \bar{Y}_t) + v_t \)

\[ \pi_t = (1\frac{1}{1-L})(\phi(Y_t - \bar{Y}_t) + v_t) \]

\[ = \phi(Y_t + Y_{t-1} + Y_{t-2} + \cdots - \bar{Y}_t - \bar{Y}_{t-1} - \bar{Y}_{t-2} - \cdots) + v_t + v_{t-1} + \cdots \]

\[ = \phi(Y_t + \sum_{i=1}^{\infty}Y_{t-i} - \sum_{i=0}^{\infty}\bar{Y}_{t-i}) + v_t + \sum_{i=1}^{\infty}v_{t-i} \]

\[ \sum_{i=1}^{\infty}Y_{t-i} \text{ is constant (assumption 1, Central bank knows past } Y_t) \]

\[ \sum_{i=0}^{\infty}\bar{Y}_{t-i} \text{ is constant (assumption 4, Central bank knows past } \bar{Y}_t \text{ when } t \geq 1) \]
Because assumption 2 (the supply shock $v_t \sim N(0, \sigma^2)$ and \{v_t\} is i.i.d) and the central bank can measure the supply shock $v_t$ each period, the central bank also knows the past $v_t$), $\sum_{t=1}^{\infty} v_{t-i}$ is a given constant.

Because \{v_t\} is i.i.d and $\mu = 0 = Ev_t$, $\frac{1}{n} \sum_{i=1}^{n} v_{t-i} \rightarrow 0$ (Shuyuan, 2006).

For simplifying the computation, this model assumes $\sum_{i=1}^{\infty} v_{t-i} = 0$

$$\pi_t = \phi(Y_t + \sum_{i=1}^{\infty} Y_{t-i} - \sum_{i=0}^{\infty} \bar{Y}_{t-i}) + v_t$$

DAD: $Y_t = \bar{Y}_t - \frac{\alpha \theta \pi}{1 + \alpha \theta \pi} (\pi_t - \pi_t^*) + \frac{1}{1 + \alpha \theta \pi} \epsilon_t$

$$= Y_t - \frac{\alpha \theta \pi}{1 + \alpha \theta \pi} (\phi(Y_t + \sum_{i=1}^{\infty} Y_{t-i} - \sum_{i=0}^{\infty} \bar{Y}_{t-i}) + v_t - \pi_t^*) + \frac{1}{1 + \alpha \theta \pi} \epsilon_t$$

Therefore, when there are shocks on $v_t$ or $\epsilon_t$ the central bank can adjust the $\pi_t^*$ to prompt $Y_t = \bar{Y}_t$

$v_t$, $\epsilon_t$ are random disturbances that affect the central bank’s decisions ($\pi_t^*$)

Because $\bar{Y}_t = \frac{s}{\delta + g_A + g_l} (1 + \gamma_A)^t (1 + g_A)^t A_0 N_0$.

Thus $\pi_t^* = \phi (\frac{s}{\delta + g_A + g_l}) 1 + \gamma_A)^t (1 + g_A)^t A_0 N_0 + \sum_{i=1}^{\infty} Y_{t-i} - \sum_{i=0}^{\infty} \bar{Y}_{t-i}) + v_t - \epsilon_t$

$$= G_1 (\bar{Y}_t, \sum_{i=1}^{\infty} Y_{t-i}, \sum_{i=0}^{\infty} \bar{Y}_{t-i}, v_t, \epsilon_t)$$

Setting target inflation $\pi_t^*$ is a kind of monetary policy that can affect the macroeconomy.

Take Covid-19 as an example. Suppose Covid-19 delivers a bad supply shock and a bad demand shock. It is represented in terms of $\epsilon_t < 0$ and $v_t > 0$ in the formula. The best response for the central bank is to increase $\pi_t^*$ to prompt the equation to work again.

Because of the Monetary policy rule: $i_t = \pi_t + \rho + \theta_t (\pi_t - \pi_t^*) + \theta_\gamma (Y_t - \bar{Y})$ (Blanchard, Johnson, 2013)

When $\pi_t^*$ increases, $i_t$ will decrease.

For the GDP, the macroeconomy has this equation (in an open economy):

$Y = C(Y, T) + I(Y, i) + G + X(Y^*, \epsilon) - IM(Y, \epsilon)/\epsilon$

$= C(Y, T) + I(Y, i) + G + X(Y^*, \epsilon) + NX(Y, Y^*, \epsilon) (Marshall – Lerner condition)$

$= C(Y, T) + I(Y, i) + G + X(Y^*, \epsilon) + NX(Y, Y^*, \frac{EP}{p^*})$

If domestic and foreign bonds are perfect substitutes, then there is perfect capital mobility.

Given rising to the interest parity condition:

$$(1 + i_t) = (1 + i^*) \frac{E_t}{E_{t+1}}$$

If the expected future exchange rate is $E_{t+1}$, then:

$$i = (1 + i^*) \frac{E_t}{E} - 1$$

$$E = \frac{(1+i)}{(1+i^*)} \frac{E_t}{E_e}$$

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The economy discussed in this paper is monetary policy when the economy is overheating. When the model is the open economy, \( \frac{(1+i)}{(1+i^*)} \bar{E}^P \frac{P^*}{P} \) decrease because only \( i \) changes under this condition. Thus \( \frac{d\pi^*_t}{d\bar{E}} > 0 \) can offset the negative economic impact of Covid-19 and vice versa.

In general, the central bank can increase the \( \pi^*_t \) to decrease \( i \) because of the monetary policy rule, and the decrease of \( i \) will increase the investment \( I(Y, i) \) and net export \( NX(Y, Y^*, \frac{(1+i)}{(1+i^*)} \bar{E}^P \frac{P^*}{P}) \), which can offset the negative economic impact of Covid-19 and vice versa.

Theoretically speaking, the long-term economic stability brought by reasonable monetary policy can directly affect people’s investment confidence. In the previous formula, investment is affected by the output and interest rate \( (I(Y, i)) \), but investment is also influenced by people’s expectations of future risk. \( \theta \) is the risk premium, so the formula become \( I(Y, i + \theta) \). Economic stability can significantly reduce risk premium, thereby increasing investment and promoting economic development in the long run.

### 4. Conclusion

The correct implementation of the central bank’s monetary policy cannot be separated from the support of a large number of statistical data, so it is extremely critical to sort out and process massive macroeconomic data. In addition, the pros and cons of monetary policy must be carefully considered before it is implemented. Ungrounded interest rate reduction may significantly promote economic growth in the short term but may bring long-term economic instability. In other words, monetary policy should be response to the existing shocks and eliminate the positive or negative influence of unexpected shocks. The rotation of ruling parties in the United States often brings a series of impertinent monetary policy, the parties in order to have a good economic result in their reign, are often implemented some radical monetary and fiscal policies, the development goal of focusing on their eight years rather than the long-term economic growth. Such as investing in infrastructure rather than technology and education, which have a long payback cycle. Therefore, the independence of the central bank is very vital to national economy. The central bank should avoid simply obeying the goal of the government, instead, the central bank should implement monetary policies independently, and correct the inappropriate fiscal policies of the government. In this paper, the influence of the
government's fiscal policies is replaced by $\varepsilon_t$. As President Reagan said: Government is not the solution to our problem, government is the problem. The government itself should not interfere too much in the development of the economy. The government and central bank usually only need to regulate the order of the market and prevent the market from being too cold or too hot. The fundamental driving force of economic development lies in the spontaneous actions of the people. The key to promoting long-term economic growth is improving nature output $\overline{Y}_t$. According to the above inference, nature output is related to technological level $A_t$. Although this paper proposes the exogenous growth $A_{t+1} = (1 + g_A)A_t$, investment by governments and central banks in science, technology and education can significantly improve the technological level in the future, which can promote the growth of nature output. In conclusion, the core of this paper is that the implementation of monetary policy of the central bank is after rather than before the occurrence of shocks. When the central bank observes supply shocks or demand shocks, it will adopt corresponding monetary policy according to the last inference of this paper. From the perspective of implementation, the central bank should strengthen the collection and processing of data in order to have accurate estimation of various complex variables. At the same time, central bank independence is also very important, because the central bank has the ability correct the government’s fiscal policy. Long-term economic stability can not only reduce many social problems, but also increase people’s investment confidence and promote long-term economic development.

References