

Improved Dynamic Portfolio Selection Model with DCC-GARCH: Evidence from the U.S. Stock Market

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Abstract. As an important financial instrument in real financial market, the selection and management of portfolio has always been a significant and hot issue in the field of economics. This study collects data of stocks daily returns in the real US stock market to test whether DCC-GARCH model can improve the performance of portfolio return compared with the original static model when the assumption of fixed covariance matrix is relaxed. This study also compares the performance of improved portfolio with that of S&P500. The result shows that portfolio returns obtained by the dynamic model outperform the original static portfolio and market performance by a large margin. This shows that, without taking into account transaction costs, an effective way to enhance the traditional mean-variance model is to fit and predict the dynamic covariance using DCC-GARCH model.

Keywords: DCC-GARCH; portfolio selection; mean-variance model; the US stock market.

1. Introduction

As an effective and direct method to raise funds lying idle in society and regulate the distribution of capital allocation, portfolio plays a very important role in financial market. In the process of investing, the most important factors that investors concerned about are return and risk. Therefore, investors often adopt the way of portfolio investment to diversify risks and obtain appropriate return rate. In another word, they invest in several different risky assets at the same time. At this point, investors need to make use of appropriate models for decision-making analysis. For a long period of time, investment decisions mostly rely on the experience summed up in practice and the investment plan developed on this basis and has not had theoretical support. In 1952, Markowitz put forward the "Mean Variance Portfolio" [1], which studies how investors select and allocate various assets on the basis of risks and returns when they are in pursuing of their own utility maximization. Portfolio theory mainly studies how to invest in a variety of risky assets under the condition of uncertainty in the future so as to maximize returns and minimize risks. This theory is such a breakthrough in solving the problem of equilibrium of risks and returns.

In classical mean-variance model, correlation among risky assets and the covariance matrix of them are assumed not to change over time. This assumption is undoubtedly very strong. As a matter of fact, in the real financial market, the correlation among risky assets fluctuates all the time, so the applicability of this original static method will be greatly weakened in the long run. In view of the shortcomings of static models, researchers have come up with a variety of improvements. The paper obtains the dynamic covariance matrix by using Dynamic Conditional Correlational Autoregressive Conditional Heteroscedasticity Model (DCC-GARCH) in fitting process and obtain the prediction of covariance matrix in one day. According to the mean variance approach, construct the dynamic maximum Sharpe ratio portfolio. Then, compare the performances of portfolio got through classic static model, the dynamic portfolio and S&P 500.

The significance of this study is that it empirically tests the improvement effect of DCC-GARCH model in the construction of maximum Sharpe ratio portfolio and test whether it is a good improvement of the original static model according to recent financial data in American stock market. The results show that, without considering the transaction costs, dynamic portfolio constructed through DCC-GARCH model outperforms the portfolio obtained through static model and outperforms S&P 500, which indicates that DCC-GARCH model is a good method to improve the

original static model, and the improved model can obtain higher cumulative returns in the American stock market.

2. Literature Review

In terms of theoretical research, the research on portfolio has always been concerned by economists. Markowitz [1] came up with the famous mean-variance model, which measures returns and risks by mean value and standard deviation respectively, aiming to minimize risk under the condition of expected return or maximize return under certain risk conditions. Sharpe [2] came up with a single index model for portfolio selection in order to reduce the calculation complexity when estimate covariance matrix. The above models consider both positive and negative excess returns as risk. However, the positive excess expected return is actually preferred by investors. In order to adopt a more accurate risk profile, Markowitz [3] and Mao [4] proposed the mean-downside semi variance model, which does not consider the positive deviation from the mean value when calculate the variance. Konno et al. [5] suggested mean-variance-skewness model based on a few key mean variance method models. It is useful under circumstance of asymmetrical income distribution. When the portfolios' income distributions are asymmetrical, they still may differ in skewness even they have the equal mean and variance, the higher skewness the more possible to achieve higher returns. Thus, this model more accurately captures the current status of the financial market. However, this model's fundamental issue is that it is challenging to fix. In light of this, Konno et al. [6] suggested a linear programming model for constructing a portfolio which uses the expected absolute deviation to describe risk. A common name for this is the mean-absolute deviation model. In order to address the issue of computation, Cai et al. [7] employed the maximum expected absolute deviation to describe covariance among assets. Compared to the models listed above, Roy [8] proposed the safety-first model, and it gives another kind of risk-control idea, which is controlling the probability of losses. The study of value at risk (VaR) in the mid-1990s can be regarded as a follow-up formulation of this idea. Additionally, Rockafellar [9] uses the idea of CVaR (conditional value at risk) to partially resolve the problem with the normal distribution assumption.

In addition to the static model mentioned above, many dynamic models have been proposed by previous researchers to adapt to the frequent changes in financial market. The first is utility function model, which includes continuous time model and discrete time model. Merton [10, 11] pioneered the study of dynamic portfolio selection under continuous time and pointed out the difference between dynamic portfolio selection and static portfolio selection. In the process of constructing a portfolio under the premise of discrete time, Balduzzi et al. [12] established a prediction model of income distribution and took transaction costs into consideration. The second is the dynamic mean-variance model. The embedding method was utilized by Li and Ng [13] to convert the multi-stage mean-variance portfolio selection problem into a dynamic programming problem and to derive analytical formulations for both the efficient frontier and optimal strategy.

In terms of application, there are also many relevant studies. In order to study inflation in the incomplete market, Xu and Wu [14] used a continuous time mean-variance model and the stochastic Linear quadratic control technique (SLQ). They were able to determine the effective frontier and optimal strategy. A discrete-time model which separates time into multiple periods and uses mean-variance approach in stochastic market under the condition of bankruptcy was researched by Huiling et al. [15]. The validity of the portfolio selection theory in the Indian stock market was investigated by Dey and Maitra [16]. Kim [17] examined the Markowitz portfolio selection theory's efficacy by applying it to the Korean stock market and obtained encouraging outcomes. In order to better the assessment of dynamic correlations of financial assets, Xu et al. [18] used DCC-MIDAS model in the Chinese stock market and demonstrate its superiority. Miguel et al [19] focused on emerging market industries to study the effects of portfolio selection when considering skewness and found that skewness plays an important role in portfolio allocation. In order to assess VaR (value at risk) of portfolios in those BRICS economies, Bongabonga and Nleya [20] employed multivariate GARCH

models. They discovered that the optimum strategy for limiting losses in the BRICS is to have portfolios that place a greater emphasis on currency and a lesser emphasis on equities.

3. Data

Considering the diversity of industries involved, the paper chooses stocks of five companies in four different industries as risky assets which are Apple and Amazon in technology industry, Netflix in movies and entertainment industry, Coca-Cola in food and beverage industry, and Marriott in hotel industry. The paper only chooses leading companies because they have relatively larger market share, and their performances are more stable. Though there are some small companies achieving rapid growth rate and high return, their risks are always higher than leading companies. Therefore, this paper neglects these stocks.

Data used is collected from Yahoo Finance (<https://www.yahoo.com/author/yahoo-finance>). Trading days' daily data from July 30, 2021, to May 13, 2022, is collected. The paper uses adjusted closing price as the daily price of each stock to calculate the daily return rate because it considers situations of splits and dividends. Here is the descriptive statistics of five risky assets in trading days. It is easy to find that, Netflix has the lowest mean value of its daily return and the highest standard deviation among five stocks. It also has the largest range. The one with the lowest standard deviation and the highest mean value of daily return is Coca-Cola. The max value of daily return for Amazon is the largest among all the five risky assets.

Table 1. Descriptive Statistics

Stocks	Mean	Std. Dev.	Min	Max
Apple	0	0.018	-0.056	0.070
Amazon	-0.002	0.025	-0.140	0.135
Coca-Cola	0.001	0.010	-0.040	0.039
Marriott	0.001	0.023	-0.070	0.076
Netflix	-0.005	0.039	-0.351	0.111

4. Methods

In this paper, two methods are used to calculate the weight vector of the portfolio. The first method is the classical static weight method, and the second method is the dynamic weight method after relaxing the assumption that the covariance matrix of risky assets is fixed. The method used to obtain weight vector of maximum sharp ratio portfolio is mean variance approach, with parameters of return rate and covariance matrix. The linear programming problem of mean-variance method is as follows.

The vector of return rate is $X=(X_1, \dots, X_5)^T$, the covariance matrix is $\Sigma_5 = (v_{ij})_{5 \times 5}$, $i, j = 1, 2, \dots, 5$. Let $P_5 = \Sigma_{i=1}^5 \omega_i X_i$ represent the return rate of the portfolio composed of five risky assets by weighted average method. According to Markowitz theory, here is the linear programming model. To obtain the efficient frontier, it is necessary to minimize the variance of portfolio with the constraint of return and the sum of weight. Then, select the maximum Sharpe ratio portfolio among all portfolios in efficient frontier.

$$\min w^T \Sigma_5 w \quad (1)$$

$$s. t. \quad r_p = w^T \mu, \quad (2)$$

$$w^T I = 1 \quad (3)$$

Where $r_p = E(P_5)$, $\mu = E(X)$, I is the 5x1 unit column vector. As for the classical static weight method, the covariance matrix of five stocks is obtained through the daily return data of the previous days. The average return rate of the stock is obtained by calculating their mean value of daily return. The obtained covariance matrix and return rate vector are used as the parameters of the mean variance method, and the effective frontier of the portfolio is obtained through Monte Carlo Simulation. Then choose the maximum Sharpe ratio portfolio and get its weight vector. The weight is static. In the next 50 days, we calculate the return rate of the portfolio by weight average method with static weight vector of the portfolio.

A major defect of the above static model is that it cannot capture the change of the correlations among different assets' return rate. In order to solve the problem of constant correlation matrix, Engel [21, 22] proposed the Dynamic Conditional Correlational Autoregressive Conditional Heteroscedasticity Model (DCC-GARCH) on the basis of the constant correlational coefficient volatility model. In addition to capturing the dynamic correlations, the model has the following advantages: relatively fewer parameters to be estimated and simpler calculation; The complex technique of estimating covariance matrix is simplified; Large scale correlation matrix can be estimated. Therefore, as for the second dynamic weight method, for the next 50 trading days, the daily return rate of the previous days for all the five risky assets are used. Considering that DCC-GARCH model can capture the change of dynamic correlation, in the following process, DCC-GARCH is used. The daily covariance matrixes for the previous 150 trading days are obtained according to the DCC-GARCH on the basis of GARCH (1, 1) model. After that, predict covariance matrix of five risky assets today. The DCC-GARCH model is as follows.

$$r_t | \Omega_t \sim N(0, H_t) \tag{4}$$

$$H_t = D_t R_t D_t \tag{5}$$

$$R_t = (\text{diag}(Q_t))^{-1/2} Q_t (\text{diag}(Q_t))^{-1/2} \tag{6}$$

$$D_t = \text{diag}(\sqrt{h_{11,t}}, \dots, \sqrt{h_{NN,t}}) \tag{7}$$

$$Q_t = (1 - \eta - \omega) \bar{Q} + \omega Q_{t-m} + \eta \delta_{i,t-n} \delta_{j,t-n} \tag{8}$$

$$R_t = Q_t^{*-1} Q_t Q_t^{*-1} \tag{9}$$

Where r_t is return rate of asset, Ω_t is information set at time t, R_t is dynamic correlation matrix, $h_{11,t}$ is conditional variance fitted through GARCH model with one financial variable. H_t is conditional covariance matrix, Q_t is covariance matrix, \bar{Q} is unconditional covariance after residual standardization. The two-step estimating strategy suggested by Engel is the one used by the DCC-GARCH model. Prior to estimating the parameters of the dynamic structure using the maximum likelihood estimation approach, it is necessary to adopt the univariate GARCH model for each financial risky asset to produce the standardized residual sequence. At the same time, the average return data of the previous 150 days are used as the return value in mean variance method. The effective frontier is drawn by Monte Carlo Simulation, and the weight vector of five risky assets in the maximum Sharpe ratio portfolio today is obtained. Therefore, the daily weight of the portfolio will be dynamically adjusted. Then these dynamic weight vectors are used to calculate the cumulative return of last 50 days that is from March 4, 2022, to May 13, 2022.

5. Results

As for static model, the weight vector of maximum sharp ratio portfolio is shown in table 2. Amazon and Netflix have nearly 0 percent in the portfolio. The stock accounts for the largest share is Coca-Cola.

Table 2. Weight vector of static portfolio

	Weight
Apple	31.78%
Amazon	About 0%
Coca-Cola	64.97%
Marriott	3.25%
Netflix	About 0%

As for dynamic model, for the first day, the weight vector of maximum sharp ratio portfolio is as follows. It can be found that, there are some differences in the weight of stocks. The weight of Coca-Cola changes little while the percentage of Apple decreases by 3%, and Marriott increases from 3% to 6%.

Table 3. Weight vector of dynamic portfolio. (Day 1)

	Weight
Apple	28.27%
Amazon	About 0%
Coca-Cola	65.29%
Marriott	6.4%
Netflix	About 0%

Dynamic portfolio is constructed through DCC-GARCH model, and daily returns are computed by weighted average method. Then it's cumulative net worth is obtained. Here is the figure of cumulative net worth of two portfolio obtained above and S&P 500.

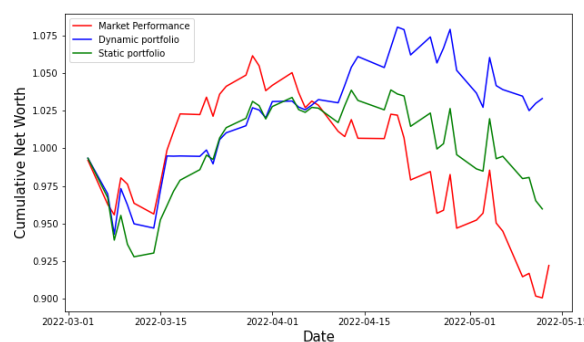


Fig. 1 Cumulative net worth of two portfolios and S&P500

As can be seen from the figure that the blue line, which represents the cumulative net worth of the maximum sharp ratio portfolio using DCC-GARCH model, outperforms the case of static weight which is the green line in the vast majority of trading days. It also shows a far better performance in the cumulative returns compared to the S&P500 in the 50-day forecast period. In fact, for cumulative net worth, a dynamically weighted portfolio outperformed a static portfolio, and both outperformed the S&P500. It is worth noting that in the 50-day forecast period, the cumulative net value of the portfolio obtained by the static method and the S&P 500 is less than 1, meaning that they have actually negative return during the 50 trading days. However, the net value obtained by the dynamic method is greater than 1, achieving a positive profit.

The results show that the dynamic vector obtained by DCC-GARCH model can outperform the S&P 500 by a large margin without considering transaction costs. At the same time, it also shows that using dynamic weight vectors in mean variance approach through DCC-GARCH model is a good improvement method for classical static problems without considering transaction costs.

6. Conclusion

It is found in this study that in the real US stock market, the return of the portfolio obtained by the dynamic model, which uses the DCC-GARCH model after loosening the assumption of fixed covariance matrix, greatly outperforms the original static portfolio and the S&P500. In other words, the result shows that using DCC-GARCH model to simulate daily covariance matrix is a good method to improve the classical mean-variance model without considering transaction costs. One explanation could be that the DCC-GARCH model can use the new and updated data to forecast the daily covariance matrix, thus the daily covariance obtained can better reflect the real correlation and volatility between stocks than the original model. The significance of this study is that it further verifies the effectiveness of DCC-GARCH model to improve the classical mean-variance model by using real U.S. stock market data and provides research support and insights for portfolio selection and management in the future financial market.

Finally, when calculating the return rate, this study does not take into consideration the impact of transaction costs, which may do harm to the significance of the conclusion to some extent. At the same time, while there are many models to simulate the diurnal covariance matrix, only the DCC-GARCH model is considered in this study. In subsequent studies, multiple prediction models can be considered to simulate the covariance and their returns can be compared.

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