

Portfolio Optimization Based on 10 UK Stocks

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Abstract. The rise of the interest rate caused turbulence in the UK stock market, raising more attention of investors on portfolio optimization. This study selected 10 UK stocks according to their market capitalization. By using the Monte Carlo simulation, this paper found the efficient frontier and constructed the 1/N, the maximum Sharpe ratio and the minimum volatility portfolio. The result showed that the British American Tobacco possesses the largest proportion of the maximum Sharpe ratio portfolio, and the GSK for the portfolio with the minimum variance. Through comparing the cumulative return of the three portfolios with the FTSE 100 Index, all three portfolios are found to outperform the benchmark index. This result may elucidate the portfolio management for some investors in this especial period.

Keywords: Portfolio optimization; mean-variance; UK stock market.

1. Introduction

Caused by the pandemic and the escalating pressure on food and energy from Russia's invasion of Ukraine, the inflation rate rocketed to 9% in the UK, reaching the highest level since 1991 according to the Office of National Statistics [1]. In response to the high inflation, the Bank of England raised the interest rate to 1.25% which has not been observed since 2009 [2], when the UK was undertaking the economic crisis. The UK stock market has taken a hit. Therefore, optimizing the portfolio is becoming an even more concerning issue for investors to mitigate the risk and improve the revenue at the same time.

Based on the Modern Portfolio Theory pioneered by Harry Markowitz [3], the mean-variance model was proposed and became one of the most efficient models to solve the portfolio management problem. In this model, the portfolio expected return and risk are quantified and invested together and intends to minimize the variance which is usually deployed to quantify the risk constrained by the expected level of return [4]. All possible asset combinations are plotted together according to their risk-expected return profile, constructing a clustered region with a positive-slope boundary called "the efficient frontier". Optimal portfolios can be selected from the efficient frontier based on different strategies. In this paper, 1/N portfolio together with two risk-based strategies, i.e., maximum Sharpe ratio strategy and minimum variance strategy, will be applied.

The 1/N portfolio, which often been quoted as the naïve portfolio, has the advantage of easy implement since it is independent from the approximation of stock return or optimization [5]. Previous studies used it as a benchmark for evaluating the performance of other portfolios, such as DeMiguel et al. [5] and Haley [6], concluding that no single optimizing model outperforms the 1/N portfolio consistently. Therefore, the naïve portfolio is regarded as efficient and will be considered in this paper. The maximum Sharpe ratio strategy applies the well accepted metrics which used to measure the investment portfolio [7]. Theron and Vuuren [8] drawn comparison among 4 portfolios and reached to the result that the cumulative return of the maximum Sharpe ratio portfolio is superior to others at most of the time. The minimum variance strategy satisfies the preference for risk management caused by financial crisis [9]. The estimation risk of the minimum variance portfolio weights is also found to be lower than other portfolios weights because it relies only on the covariance matrix rather than both the matrix and the expected returns [10].

The aim of this paper is to analyse the efficiency of asset allocation optimization and provide the individual investors a reference for portfolio management in this special period. The portfolios consisting of 10 stocks will be constructed using Monte Carlo simulation and derive the assets weights in the minimum volatility allocation and the maximum Sharpe ratio allocation. The portfolio

cumulative returns will then be compared with the 1/N portfolio return and the Index return in the same time period, leading to the conclusion. The result showed that all the three targeted portfolios beat the index in the investigated period. For validation, a robustness check is conducted, proving rationality of the method and results.

This paper is structured as follows, Section 2 introduces the stocks chose and data used in the study. Section 3 describes the method of building the model and sorting out the optimal portfolios. Results and the robustness check will be presented in Section 4 and Section 5 reaches to the conclusion and discussion.

2. Data

This paper selects top 10 representative stocks according to the market capitalization from the Financial Times Stock Exchange 100 (FTSE 100) Index components. The ticker of the 10 stocks are AZN.L, SHEL.L, HSBA.L, ULVR.L, GSK.L, BP.L, DGE.L, BATS.L, RIO.L and GLEN.L. Closing prices from November 23rd 2021 to June 21st 2022 are collected and separated into training set and test set. The training set is used for calculating the average return and covariance matrices in order to construct the efficient frontier. The test set is for evaluating the performance of the selected asset allocations by comparing their cumulative returns to the FTSE Index return. The basic information of the ten chosen stocks is presented in Table 1, Table 2, Figure 1, respectively. According to Table 2, British American Tobacco has the highest average return during the period while GSK has the minimum fluctuation. Figure 1 reveals that Glencore presents the highest cumulative return of 40.24%, whereas the cumulative return of Unilever decreased with fluctuation and reaching -11.30% in the end.

Table 1. Selected stocks

	Company
AZN.L	AstraZeneca p.l.c
BATS.L	British American Tobacco p.l.c.
BP.L	BP p.l.c
DGE.L	Diageo p.l.c
GLEN.L	Glencore p.l.c
GSK.L	GSK p.l.c
HSBA.L	HSBC Holdings p.l.c
RIO.L	Rio Tinto Group
SHEL.L	Shell p.l.c
ULVR.L	Unilever p.l.c

Table 2. Descriptive statistics of the daily return of the 10 stocks

	Max	Min	Mean	Std Dev	Cumulative Return
AZN	0.058	-0.05	0.0025	0.017	26.28%
BATS	0.04	-0.05	0.0028	0.015	30.85%
BP	0.062	-0.079	0.002	0.025	18.92%
DGE	0.06	-0.047	0.0005	0.017	3.32%
GLEN	0.088	-0.059	0.0037	0.023	40.24%
GSK	0.04	-0.03	0.0017	0.013	17.34%
HSBA	0.051	-0.067	0.0022	0.02	21.37%
RIO	0.048	-0.048	0.0032	0.02	34.06%
SHEL	0.081	-0.057	0.0033	0.023	35.03%
ULVR	0.073	-0.07	-0.0011	0.018	-11.3%

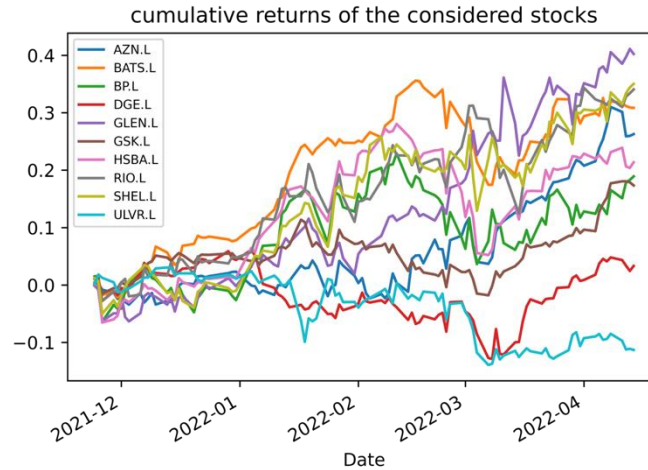


Fig. 1 Cumulative returns of selected stocks

3. Methods

The mean variance model examines the balance between the portfolio expected return and the risk. The portfolio return and the variance are as follows:

$$\mu_p = \sum_i w_i \mu_i \quad (1)$$

Where w_i is the i^{th} component weight of the portfolio, μ_i is the expected return of the i^{th} component.

$$\sigma_p^2 = \sum_i \mu_i^2 w_i^2 + \sum_i \sum_j \sigma_i \sigma_j w_i w_j \rho_{ij} \quad (2)$$

Where σ_i is the standard deviation of the asset i returns, and ρ_{ij} is the correlation coefficient between the returns on assets i and j . Therefore, calculating the average return and the covariance matrix of the stocks is essential.

The Monte Carlo simulation is adopted to create different portfolios. This technique helps imitating the operation process as random ones under certain conditions and using given parameters; the stocks' average return and covariance matrices were used as parameters in this study. By stochastically generating weights for each asset in the portfolio, one is able to obtain the expected return and the portfolios variance under the weights. Through repeatedly simulating one hundred thousand portfolios in the same way, the data pool would be representative for the most possible asset allocations and could be used for plotting out the efficient frontier.

In this paper, three specific portfolios are considered and can be pointed out from the efficient frontier: the 1/N portfolio, the maximum Sharpe ratio portfolio and the minimum volatility portfolio. The investors are found to prefer allocating their money evenly among various assets and the simplest 1/N portfolio is widely used [11], thus take the 1/N portfolio into consideration. The Sharpe ratio is a prevalently used tool for measuring and evaluating the risk-adjusted return in investments [12]. The calculation is as follows:

$$SR = \frac{R_p - R_f}{\sigma_p} \quad (3)$$

Where R_f is the risk-free rate, R_p is the portfolio expected return and σ_p is the standard deviation. Portfolios with the same expected returns will have a higher Sharpe ratio if it's standard deviation decreases. Therefore, the portfolio which has the maximum Sharpe ratio is superior to other portfolios in terms of risk-adjustment performance. The minimum variance strategy helps risk-reverse

investors minimize the equity risk while maintaining the market exposure and consequently the minimum volatility portfolio would provide a clue for investors.

4. Result

100,000 simulations had been performed using the Monte Carlo method. The scatter plot is shown below (Figure 2), and the blue edge is the efficient frontier. The black markers on the plot indicate the positions of a portfolio when it is formed by only one of the stocks. Under the efficient frontier are the sub-optimal portfolios which provides comparatively lower return for the given level of risk.

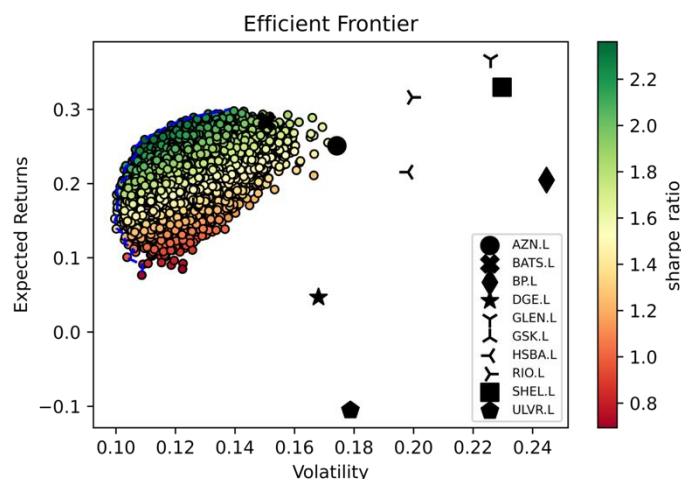


Fig. 2 Efficient Frontier

Then the best portfolios can be selected. The two targeted portfolios are calculated and labeled on Figure 3. The minimum volatility portfolio locates at the left limit of the frontier and the stock weights in the portfolio are: AZN.L: 11.31%, SHEL.L: 0.34%, HSBA.L: 2.15%, ULVR.L: 23.54%, GSK.L: 28.36%, BP.L: 1.75%, DGE.L: 4.03%, BATS.L: 11.88%, RIO.L: 10.11% and GLEN.L: 6.54%. The weights of the chosen stocks in the maximum Sharpe ratio portfolio are AZN.L: 19.85%, SHEL.L: 2.56%, HSBA.L: 0.53%, ULVR.L: 5.43%, GSK.L: 12.28%, BP.L: 0.18%, DGE.L: 0.79%, BATS.L: 24.08%, RIO.L: 20.28% and GLEN.L: 14.02%, shown in Table 3. The volatility is 9.97% for the minimum volatility portfolio and the Sharpe ratio is 236.19% for the maximum Sharpe ratio portfolio, shown in Table 4. For the minimum volatility portfolio, GSK accounts for more than a quarter, while BATS takes up approximately 25% in the maximum Sharpe ratio portfolio. Notice that BP and HSBA possesses only a tiny fraction in both two portfolios.

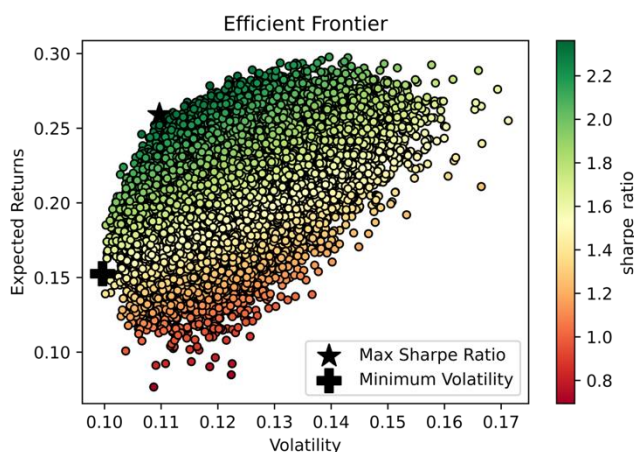


Fig. 3 Portfolios with maximum Sharpe ratio and minimum volatility

Table 3. Weight of each stock in the two optimal portfolios (%)

	Max Sharpe Ratio	Min Volatility
AZN	19.85	11.31
BATS	24.08	11.88
BP	0.18	1.75
DGE	0.79	4.03
GLEN	14.02	6.54
GSK	12.28	28.36
HSBA	0.53	2.15
RIO	20.28	10.11
SHEL	2.56	0.34
ULVR	5.43	23.54

Table 4. Return and volatility of the two portfolios

	Return	Volatility	Sharpe Ratio
Max Sharpe Ratio	25.91%	10.97%	236.19%
Min Volatility	15.25%	9.97%	153.03%

Obtaining the two asset allocations allows the next step: the calculation of the portfolio return. Using the test set from 19th of April 2022 to 20th of June 2022 together with the stock weights, the daily portfolio returns can be gained, and furthermore, the cumulative returns. For comparison, the FTSE 100 Index return data is collected over the same period. From Figure 4 one can observe that the study result outperforms the broad market. The cumulative return of the maximum Sharpe ratio portfolio is -4.03%, that of the minimum volatility portfolio is -1.74% and for 1/N portfolio it is -2.96%, comparing with the FTSE 100 index return of -6.49% (shown in Table 5). Figure 4 indicates that the minimum volatility portfolio and the 1/N portfolio are superior to the FTSE 100 index at most of the time. The cumulative return of maximum Sharpe ratio portfolio rocketed on the last 25 days and surpassed the FTSE 100 index even though it was level pegging with the index before 15th of May 2022.

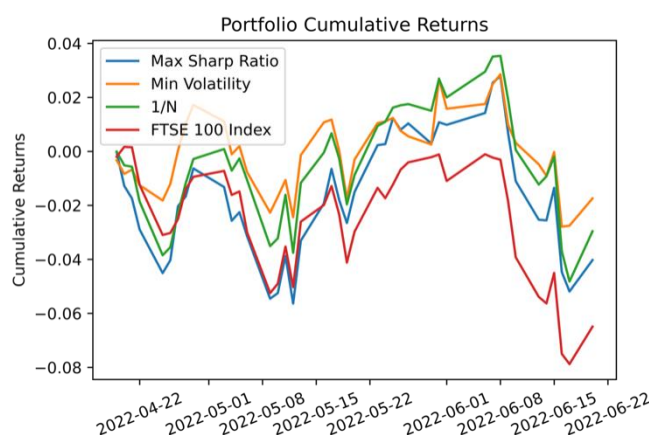

Fig. 4 Comparison between FTSE 100 index return and the portfolio returns

Table 5. Cumulative return comparison

	Cumulative Return
Max Sharpe Ratio	-4.03%
Min Volatility	-1.74%
1/N Portfolio	-2.96%
FTSE 100 Index	-6.49%

For cogency, a robustness check will be conducted. Firstly, eliminate the BP and the HSBA from the portfolio components since they accounted for the least percentage. Secondly, repeat the Monte Carlo simulation and the calculation of cumulative returns. The altered weights for the two optimal portfolios are shown in Table 6, from which one would observe that the largest components stay unchanged in both two portfolios: BATS for maximum Sharpe ratio portfolio and GSK for minimum risk portfolio. Finally, calculate the cumulative returns for the three portfolios and make comparison with the FTSE index. The outcome is presented in Figure 5 and Table 7, showing a similar result with previous data where minimum variance portfolio possesses the highest cumulative return. Therefore, the method and the results are valid and effective.

Table 6. Weight of the remaining 8 stocks in the two optimal portfolios (%)

	Max Sharpe Ratio	Min Volatility
AZN	19.85	11.31
BATS	24.08	11.88
BP	0.18	1.75
DGE	0.79	4.03
GLEN	14.02	6.54
GSK	12.28	28.36
HSBA	0.53	2.15
RIO	20.28	10.11
SHEL	2.56	0.34
ULVR	5.43	23.54

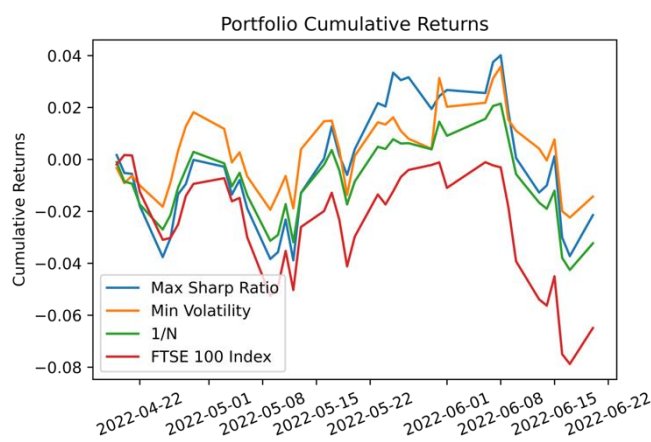


Fig. 5 Comparison between FTSE 100 index return and the altered portfolio returns

Table 7. Altered cumulative return comparison

	Cumulative Return
Max Sharpe Ratio	-2.15%
Min Volatility	-1.43%
1/N	-3.23%
FTSE 100 Index	-6.49%

5. Conclusion and discussion

In summary, this paper investigated the three portfolios (1/N, minimum variance and maximum Sharpe ratio) composed of 10 selected UK stocks. Based on the data from yahoo finance, the cumulative return of the stocks can be calculated over the specific time period. Then, one hundred thousand random portfolios were simulated using the Monte Carlo simulation, followed by the calculation of the portfolio return and volatility. By plotting the portfolio return against the volatility,

the data was visualized, and the efficient frontier can be pointed out. The asset weights in the two optimal portfolios were known. Together with the 1/N portfolio, where all assets weigh equally, the cumulative portfolio returns were accessible using the test set data.

The comparison with the FTSE 100 index revealed that all three portfolios transcend the stock index, with the minimum variance portfolio gaining the highest return. GSK accounts for more than $\frac{1}{4}$ in the minimum volatility allocation before and after the robustness check, indicating that GSK would be a desirable stock to allocate the most wealth for in the perspective of risk-averse investors.

Intuitively, the maximum Sharpe ratio portfolio would provide the highest return accompanied with a higher risk. However, when setting the sight on a broader view, one can notice that the market is undertaking pressure thus appears to be more fluctuant. The minimum variance strategy efficiently hedges risks and benefits the investors through reducing the volatility in their expected return. While all the portfolios give negative returns and the investment environment tends to be not ideal from a macro perspective, the minimum variance portfolio met the target of minimizing the risk during the challenging time. Therefore, it might be a preferable and recommended asset allocation strategy when the stock market becomes pessimistic.

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