

Comparison of Risk Minimizing and Return Maximizing Portfolio Models

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Abstract. The fundamental purpose of investing in stocks is to make a profit. But in the stock investment, the income always accompanies the risk. In order to reduce the risk of greater returns, the investor will be two or more of the stock portfolio together to invest. This study examines the return and risk of a stock portfolio using a minimal variance and maximum Sharpe ratio model, based on the Markowitz mean-variance theory, in order to identify the best stock portfolio for a given risk preference. The mean-variance model is best for risk-averse investors and just considers risk instead of return. While the introduction of Sharpe ratio enables investors to consider both return and risk simultaneously in the objective functions of the optimization problems. The empirical study based on the stock and fund data of five United Kingdom stocks and one America stock demonstrates the model based on maximum Sharpe ratio criterion is more suitable for risk-seeking investors and can produce more active replacing strategies than minimum Mean-variance model.

Keywords: Portfolio return, Risk, Mean-variance Model, Sharpe ratio Model.

1. Introduction

1.1 Research Background and Motivation

Portfolio theory and practice has been one of the hot issues in recent years, which plays an important role in asset value preservation and risk management. Global assets under management had reached more than \$112 trillion by the end of 2022, according to BCG [1]. Therefore, with the deepening of financial integration and economic globalization, the complexity and diversity of financial market behaviour become more and more obvious. Investors allocate their available cash to assets based on a variety of factors, and through carefully chosen investment methods, they aim to maximize the value of their investments while removing the risk of losses. Investors must thoroughly research the market and make investment selections without missing out on an opportunity. Furthermore, investments must carry risk, and how to effectively balance the tension between the benefit and the risk is the complexity of making investment decisions when confronted with a huge number of financial assets on the market with varying characteristics. To assist in making investment decisions, a variety of asset valuation models that explain the link involving risk and return with relation to a certain investment can be utilized. One of the most popular methods for developing approaches and portfolios is the Modern Portfolio Theory (MPT) [2]. Even if it is based on oversimplifying assumptions, it may be used to characterise the relationship between the return and risk of individual portfolio components in asset allocation. What's more, the selection of optimal portfolio is crucial in defining the investment portfolio strategies. Maximizing portfolio return and reducing portfolio risk are the objectives of investors who optimize their portfolios. Investors must strike a balance between the conflict between risk and return for their investment because return depends on risk. Therefore, no single optimized portfolio can meet the needs of all investors. The investor's preferences for risk and return determine the best portfolio [3].

1.2 Literature Review

Portfolio Choice, a seminal study in the development of contemporary microfinance theory, was published by Markowitz in 1952 [4]. The theoretical groundwork for addressing the issue of the ideal allocation of capital percentage in investment objects throughout the process of investment decision-making is created in this article, which also explains the quantitative way of assessing the degree of

return and risk. In an attempt to lessen the portfolio variance, the main goal of portfolio theory is to build a portfolio with a variety of assets and to allocate various assets in the portfolio in varying proportions. Subsequently, there is a large amount of literature focusing on the optimization criteria for improving the portfolio optimization model, for instance, the mean-absolute deviation, mean-semi-variance, and mean-variance-CVaR criteria and so on [5-7]. The Markowitz model has undergone numerous modifications and additions, yet it is still extensively employed.

However, the requirement to calculate the covariance matrix of all assets in the mean-variance model severely limits its practical application. Additionally, the lowest variance optimization seeks to reduce yield volatility in order to limit risk, but it is unable to produce the high projected return. It is worth noting that the point of minimum variance is the point of minimum return on the efficient boundary. So in 1963, the William F. Sharpe proposed a single-factor model, the Sharpe model, that allowed simplified estimates of covariance matrix, giving a big boost to the practical application of portfolio theory[8]. The maximum Sharpe ratio model is an optimal approach to minimize the risk and maximize the benefit. The Sharpe ratio is defined as the ratio of a portfolio's excess return to its standard deviation. It is easy to see that by maximizing the Sharpe ratio, investors can take into account both portfolio returns, and risk, in the objective function of the optimization problem, and have a good economic explanation [9]. Hence, based on the fact that the stock market is a market where returns and risks coexist, this paper will try to analyse how to allocate wealth in a reasonable way among various assets, while this paper discusses the change of portfolio ratio under the minimum variance model and the maximum sharp ratio model by case study as well.

1.3 Research Contents and Framework

Based on this, this study conducts a theoretical and empirical investigation of the best portfolio strategy, and selects six different stocks in the fields of finance, information technology, new energy, real estate and health care in the UK, compare their different portfolio ratios in the minimum variance model and the maximum Sharpe ratio model. Combining this with the constraints, such as not allowing short selling, that each stock must have a weight greater than or equal to zero, this paper try to compare similarities and differences according to the results. This article is organized as follows: the first section is an introduction, the second section contains conceptual and practical analysis, the third section is based on the results and discussion of theoretical and empirical analysis, and the fourth section is the conclusion.

2. Methodology

2.1 Data Sources

The data of six stocks selected in this paper are from Investing.com. This article has selected outstanding from around the world companies from different industries, shares: Topdanmark A/S (0QCQ), U Blox Holding AG (0QNI), Galp Energia Nom (0B67) , Merlin Properties SA (0QVM), Faes Farma SA (0K9H), Apple Inc (AAPL). Data were collected from July 6,2021, to July 7,2022.

2.2 Parameters according to historical stock price data

2.2.1. Covariance and Correlation Data of Stock Prices

Covariance is a statistical metric used to assess the connection between two random variables. The covariance on different assets X and Y is calculated using the following formula:

$$Cov(X, Y) = E[(X - E[X])(Y - E[Y])] \quad (1)$$

The correlation on different assets X and Y is computed using the following formula:

$$\rho_{XY} = \frac{Cov(X, Y)}{\sigma_X \sigma_Y} \quad (2)$$

2.2.2. Rate of Return

The rate of return is defined as the ratio of the money gain or loss from an investor's investment. The money that was put into the investment might be referred to as an asset, capital, principle, or the investment's cost basis. An investor's expected rate of return can be computed using both current data and projections for future investors. Typically, the rate of return's output is presented as a proportion. The formula determines the arithmetic rate of return R_t on investments made in an asset between time t and time $t-1$ (see below):

$$R_t = \frac{P_t - P_{t-1}}{P_{t-1}} \quad (3)$$

R_t represents for current stock yield, P_t stands for current closing price, and P_{t-1} stands for current opening price.

2.2.3. Expected Return

This paper has n number of assets. Then the expected return μ_i on asset i , $i = 1, \dots, n$ is calculated by

$$\mu_i = E(r^i) = \frac{\sum_{t=1}^m r_t^i}{m} \quad (4)$$

where r_t^i is the return on asset i between periods $t - 1$ and t , $t = 1, \dots, m$ and m is the number of periods for which we have computed the return.

2.2.4. Variance and Standard Deviation

Variance gauges how far a random variable deviates from its mathematical expectation. A measurement of the dispersion of the many potential rates of return in relation to the anticipated rate of return is the variance of the rate of return. Typically, the variation of the rate of return is used to assess the asset's risk:

The variance on portfolio is calculated using the following formula:

$$\sigma_p^2 = \sum_{i=1}^n \sum_{j=1}^n \omega_i \omega_j \sigma_{ij} \quad (5)$$

σ_p^2 is the variance of the portfolio.

2.3 Establishment of the Mean-variance Model

2.3.1. Data Selection

The analysis of data selection and calculation of portfolio investment determine the arithmetic mean return, variance, standard deviation, and covariance of each stock and each stock in the portfolio, as well as their correlation. The following data were obtained:

Table 1. The expected returns of six stocks

	0QCQ	0QNI	0B67	0QVM	0K9H	AAPL
Expected Return	0.053%	0.182%	0.098%	0.017%	0.084%	0.007%

It can be demonstrated that U Blox Holding AG(0QNI), an information technology stock in the United Kingdom, has the highest expected return, which is over 0.1%. While, there is a lowest information technology stock in the America, Apple Inc(AAPL), which is less 0.1%.

Table 2. Portfolio Variance- Covariance Matrix

Variance- Covariance Matrix						
	0QCQ	0QNI	0B67	0QVM	0K9H	AAPL
0QCQ	0.000294	-0.000007	0.000012	-0.000011	0.000005	0.000011
0QNI	-0.000007	0.000861	-0.000086	0.000028	0.000011	0.000051
0B67	0.000012	-0.000086	0.000555	0.000028	0.000008	-0.000004
0QVM	-0.000011	0.000028	0.000028	0.000297	0.000062	0.000050
0K9H	0.000005	0.000050	0.000008	0.000062	0.000229	0.000025
AAPL	0.000011	0.000011	-0.000004	0.000050	0.000025	0.000431
Contributes to Variance	1.84E-05	6.32E-06	9.70E-06	1.24E-05	1.78E-05	9.70E-06

It can be seen that U Blox Holding AG (0QNI), Galp Energia Nom (0B67) and Apple Inc (AAPL) stocks have higher variance than others, which means that those have higher risk than others. Hence, it can be predicted that those three stocks have lower weights in the proportion of optimal portfolio.

2.3.2. Optimal Portfolio

The selection of an optimal portfolio can be based on the mean-variance criterion with the following objective functions and constraints: We aim to minimize the risk, that is, the variance. The variance on portfolio is calculated using the following formula

$$\sigma_p^2 = \sum_{i=1}^{n=6} \sum_{j=1}^{n=6} \omega_i \omega_j \sigma_{ij} \quad (6)$$

The constraint is computed using the following formula:

$$\omega_1 + \omega_2 + \omega_3 + \omega_4 + \omega_5 + \omega_6 = 1 \quad (7)$$

If short selling is allowed, the weight of the stock can be negative. So this paper adds some constraints here, and assumes that short selling is prohibited, resulting in a weight higher than or equal to zero for each stock [10].

Table 3. The optimal portfolio and weights calculated figures

Sum of weights	1.00
Portfolio expected return	0.000652941
Portfolio SD	0.008623378
Portfolio Var	7.43627E-05
Optimal weights	
0QCQ	24.76%
0QNI	8.25%
0B67	13.04%
0QVM	16.74%
0K9H	24.45%
AAPL	12.76%

As predicted that U Blox Holding AG (0QNI), Galp Energia Nom (0B67) and Apple Inc (AAPL) have lower weights, while Topdanmark A/S (0QCQ), Merlin Properties SA (0QVM), Faes Farma SA (0K9H) have larger proportions, which means it has lower risk under minimum mean-variance model. Risk is the unique criterion to be measured.

2.4 Establishment of the Maximizing Sharpe Ratio Model

2.4.1. Sharpe Ratio

When comparing the return and risk on an investment, the Sharpe ratio is a risk-adjusted return measurement. It represents the excess return generated by each unit of total risk taken:

$$Sharpe = \frac{E(R) - r_f}{\sigma} \quad (8)$$

$E(R)$ is expected return of the portfolio, r_f is risk-free interest rate, σ is standard deviation of excess return. According to the fundamentals of the Sharpe ratio definition, the bigger the Sharpe ratio, the better the stock. The point reflected in the graph, that is, directly above it, is the best point.

2.4.2. Data Selection

The data collected of this paper are daily data. Hence, risk-free rate is assumed as 0.05%.

Table 4. Variance-covariance Matrix

Variance- Covariance Matrix						
	0QCQ	0QNI	0B67	0QVM	0K9H	AAPL
0QCQ	0.000294	-0.000007	0.000012	-0.000011	0.000005	0.000011
0QNI	-0.000007	0.000861	-0.000086	0.000028	0.000011	0.000051
0B67	0.000012	-0.000086	0.000555	0.000028	0.000008	-0.000004
0QVM	-0.000011	0.000028	0.000028	0.000297	0.000062	0.000050
0K9H	0.000005	0.000050	0.000008	0.000062	0.000229	0.000025
AAPL	0.000011	0.000011	-0.000004	0.000050	0.000025	0.000431
Contributes to Variance	1.38E-07	1.33E-04	3.30E-05	0.00E+00	2.40E-05	0.00E+00

Under Sharpe ratio model, both risk and return are criterions to be measured. So, optimal portfolio should combine Sharpe ratio with variance to get it.

Table 5. The optimal portfolio calculated figures

Sum of weights	1.00
Portfolio expected return	0.001266374
Portfolio SD	0.01379785
Expected Sharpe ratio	0.055542989

Compared with minimum mean-variance model, portfolio expected return of maximum Sharpe ratio model is greater than that of minimum mean-variance model. It can be shown that Sharpe ratio model takes return into account. Correspondingly, portfolio standard deviation is larger than that of minimum mean-variance model as well. Due to some stocks might have higher return, but those might not have less risk

Table 6. The optimal weights of portfolio calculated figures

Optimal weights	
0QCQ	1.87%
0QNI	39.80%
0B67	0
0QVM	0
0K9H	30.97%
AAPL	0

Contrast with minimum mean-variance model, Faes Farma SA (0K9H) has always accounted for a larger share of whatever model it was under. Besides, for U Blox Holding AG (0QNI), it has highest return among six stocks, as a consequence, it increases dramatically between minimum mean-variance model and maximum Sharpe ratio model. Other stocks may also increase or decrease in response to investor preferences, taking into account both return and risk factors.

3. Results and Discussion

To be able to explore the change of the proportion of the optimal portfolio strategy under the two models, the paper describes it in detail from both theoretical and empirical aspects. The Sharpe ratio model and the mean-variance model are conceptually analysed, and the empirical data of six stocks in different fields for one year are analysed. Some results are as follows. First of all, the empirical analysis shows that the max-sharp model can reflect the relationship between risk and return more scientifically, and provide reference for investment. Then, it can be shown that whether under mean-variance model or maximum Sharpe ratio model, there is a large proportion of all securities in Faes Farma SA, which is a medical treatment corporation. It represents clearly that as the medical field continues to evolve, the pace of scientific and technological innovation will accelerate significantly in the next few years. What's more, as the entire medical industry moves toward digital, medical "Big data" is set to explode. Relatively, from a portfolio perspective, these trends are equally far-reaching. Trends per share are available for investment, while providing investors with an attractive portfolio allocation. Last but not least, portfolios of Topdanmark A/S (0QCQ), Galp Energia Nom (0B67), Merlin Properties SA (0QVM), Apple Inc (AAPL), there is a decrease from minimum variance model to Sharpe ratio model, it demonstrates that when considered portfolio expected return, it will fall in proportion of all securities. While portfolio of U Blox Holding AG (0QNI), which is information technology, has increased weights of all securities when considered expected return. It can be shown that in the globe today, the transformation of human production and life is intensifying due to the information technology sector's rapid expansion, which is driving significant changes across all industrial sectors. The latest wave of significant information technology innovation will support the information industry's value chain, advance economic and social progress, and continue to fulfil peoples' desires for a better living.

4. Conclusion

In terms of Two-Fund theorem, the optimal investment portfolio does not depend on an investor's risk preferences or personal characteristics. However, when choosing weights of each stocks in the optimal portfolio, it should consider investor's tolerance for risk. For risk-averse investors, they have large proportions of choosing minimum mean-variance model, which considers risk firstly. While for risk-neutral or risk-seeking investors, they are more likely make a decision to choose maximum Sharpe ratio model when considering both risk and return factors. In reality, people choose maximum Sharp ratio model, which an optimal portfolio weights that portfolio can bring more return, though it has a certain amount of risk.

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