

Embedded Asian Option Pricing in Structured Financial Products

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Abstract. Based on the dual background of the increasingly serious problem of population aging and the re-listing of treasury bond futures market, the market demand for structured pension financial products is gradually expanding, which also puts forward higher requirements for the pricing strategy of embedded Asian options in products. Therefore, this paper designs an inclusive pension financial product with embedded Asian options, and further studies the pricing of the product with embedded Asian options under Monte Carlo simulation. Monte Carlo simulation can simulate the future trend and final value of asset prices under risk-neutral conditions, and can effectively overcome the problem that Asian options embedded in structured financial products have no analytical solution. At the same time, the probability distribution of product returns is calculated by constructing the bear market call option strategy, and the result of product breakeven probability measure is obtained. The data shows that Asian options have a good smoothing effect on the fluctuation of maturity profit and loss, which can improve the breakeven probability of products to a certain extent and reduce investors' financial risks. It is suitable for structured pension financial products.

Keywords: Asian options; Monte Carlo simulation; breakeven probability measure.

1. Introduction

On May 20, 2022, the State Administration of Foreign Exchange issued the 'Notice on Measures for Further Promoting the Foreign Exchange Market to Serve the Real Economy', allowing financial institutions to provide customers with RMB foreign Asian options and their combination products. The successful implementation of the Asian options business also means the re-innovation of China's foreign exchange derivatives market, which helps expand the hedging tool options of real enterprises and reverse the serious homogeneity of structured financial products in China's banking industry. Therefore, this paper designs an inclusive pension financial product with embedded Asian options, and uses Monte Carlo simulation method to analyze its pricing. From the perspective of product structure, the choice of derivative contract options is the basis for the design of structured wealth management products, and it is also the most core part of the whole product structure [1]. In other words, the analysis of pricing, hedging, income distribution and other issues surrounding the embedded Asian option structured financial products can be essentially attributed to the study of embedded Asian options. Since the income of inclusive pension financial products with embedded Asian options mainly depends on the density function of the product, this paper constructs a specific form of Asian option density function, so as to simulate the relative operation of the underlying asset price. At the same time, the standard Brownian motion is used to describe the future random change path of the 5-year Treasury bond futures price, and the arbitrage is carried out by constructing the bear market call option strategy, so as to reduce the extreme distorted profits or losses that may occur on the expiration date of the contract, and better serve the demand for neutral hedging of structured financial products.

2. Theoretical Framework

2.1 Model Construction

Similar to the Black-Scholes option pricing model, the Monte Carlo simulation method assumes that the change in the price of the underlying asset follows geometric Brownian motion in the case of risk-neutral situation, and its expression is as follows:

$$\frac{dS}{S} = \mu dt + \sigma dz \quad (1)$$

Through logarithmic processing of the above formula, and according to ITO lemma, it can be deduced that:

$$d\ln S = (r - \frac{\sigma^2}{2})dt + \sigma dz \quad (2)$$

From this it can be deduced that:

$$\ln S(t + \Delta t) - \ln S(t) = (r - \frac{\sigma^2}{2})\Delta t + \sigma \varepsilon \sqrt{\Delta t} \quad (3)$$

Thus, the price change formula of the underlying asset can be obtained:

$$S(t + \Delta t) = S(t) \exp[(r - \frac{\sigma^2}{2})\Delta t + \sigma \varepsilon \sqrt{\Delta t}] \quad (4)$$

By refining the effective period of the option contract, a series of normal random numbers are generated, and then the values are substituted into the underlying asset prices at different price levels in turn. Finally, the maturity expectation of this structured financial product can be calculated by repeating the above steps. Assume that $R(S_0, S_1, S_2, \dots, S_n)$ represents the maturity return function of the option contract; $E(R)$ represents the expected level of the maturity return function; r represents the risk-free interest rate is constant; T represents time. Then the following formula can be obtained:

$$V = e^{-rT} \cdot E(R(S_0, S_1, S_2, \dots, S_n)) \quad (5)$$

$$E(R) = \frac{1}{N} \sum_{i=1}^n R_i(S_0, S_1, S_2, \dots, S_n) \quad (6)$$

Finally, the sum of the maturity value of the product can be discounted according to the risk-free interest rate, so as to obtain the initial value of the structured financial product. The expression is as follows:

$$P = V + B \quad (7)$$

2.2 Fixed Income Securities Pricing

Based on the discounted cash flow model, this paper estimates the value of 2021 book-entry discounted (fifty-six issues) treasury bonds. Compared with the income discount model, the conclusion obtained by the free cash flow discount model is more accurate [2]. When the future cash

flow, discount rate and discount period are known, the cash flow can be discounted to obtain the value of the theoretical financial product. The mathematical expression is as follows:

$$B = \sum_{i=1}^n \frac{C}{(1+r)^i} + \frac{F}{(1+r)^n} \quad (8)$$

Where B represents the value of the bond; F represents the principal due for repayment; n represents the duration of the bond; C represents the interest paid at maturity; r represents the discount rate of the bond.

In order to simplify the calculation, the above expression can be transformed into a more general form:

$$B = \sum_{i=1}^n \frac{I_i}{(1+r_i)^i} \quad (9)$$

Where B represents the value of the bond; n represents the duration of the bond; I_i represents cash flows in different periods; r_i represents the spot interest rate in different periods.

The new annualized rate of return (R) is calculated as follows:

$$K = \frac{P}{C} = \frac{V-C}{C} = \frac{V}{C} - 1 \quad (10)$$

$$R = (1+K)^N - 1 \quad (11)$$

Substitute K to get:

$$R = (1 + \frac{V}{C} - 1)^N - 1 = (\frac{V}{C})^{\frac{D}{T}} - 1 = (\frac{100}{98.921})^{\frac{360}{182}} - 1 = 2.1576\% \quad (12)$$

The market value at the expiration of the term is equal to its face value, and the calculation process is as follows:

$$P_{\text{expire}} = C + C \times R \times \frac{T}{D} = C \times (1 + R \times \frac{T}{D}) = 98.912 \times (1 + 2.1576\% \times \frac{360}{182}) = 100 \quad (13)$$

Where K represents the rate of return; P represents the income; V represents the corresponding market value after time T ; C represents the principal invested by investors; D represents the effective investment time of one year, and the effective investment time of bonds for one year is calculated as 360 days; D/T represents the number of repeated investments of investors in one year.

2.3 Embedded Asian Option Pricing

The income of inclusive pension financial products with embedded Asian options mainly depends on the density function of the product, so the relative performance of the underlying asset price can be simulated by constructing a specific form of Asian option density function. At the same time, the standard Brownian motion is proposed to describe the future random change path of the 5-year treasury bond futures price. The specific movement process is shown as follows:

The above formula is processed logarithmically, and it can be deduced according to ITO lemma:

$$d\ln S = (r - \frac{\sigma^2}{2})dt + \sigma dz \quad (14)$$

From this it can be deduced that:

$$\ln S(t + \Delta t) - \ln S(t) = (r - \frac{\sigma^2}{2})\Delta t + \sigma \varepsilon \sqrt{\Delta t} \quad (15)$$

Thus, the price change formula of the underlying asset is obtained:

$$S(t + \Delta t) = S(t) \exp[(r - \frac{\sigma^2}{2})\Delta t + \sigma \varepsilon \sqrt{\Delta t}] \quad (16)$$

Where S represents the price of the underlying asset; r represents the risk-free interest rate, which is calculated with the equivalent annualized return of 2.1576% of 2021 book-entry discounted (fifty-six issues) treasury bonds; ε represents the random number of normal distribution; σ represents the annual volatility of futures price.

This paper summarizes the historical data used in the pricing process of the structured financial product, as shown in Figure 1. These data are mainly from the daily closing price of the 5-year treasury bond futures TF2206 contract published by China Financial Futures Exchange from September 13, 2021 to November 24, 2021. The total sample size of historical data is 46.



Fig 1. Historical settlement price trend (2021.09.13-2021.11.24)

According to the above historical data, the logarithmic yield of treasury bond futures can be obtained:

$$r = \sum_{i=1}^n \ln \frac{p_i}{p_{i-1}} = \sum_{i=1}^{46} \ln \frac{p_i}{p_{i-1}} = 0.169068\% \quad (17)$$

Given the logarithmic return rate, the daily average logarithmic return rate can be obtained:

$$\bar{r} = \frac{1}{n-1} \sum_{i=1}^n \ln \frac{p_i}{p_{i-1}} = \frac{1}{46-1} \sum_{i=1}^{46} \ln \frac{p_i}{p_{i-1}} = \frac{1}{45} \sum_{i=1}^{46} \ln \frac{p_i}{p_{i-1}} = 0.003757067\% \quad (18)$$

According to the results of the above process, it can be obtained:

$$\sum_{i=1}^n r_i = \sum_{i=1}^{46} r_i = 0.169068\% \quad (19)$$

$$\sum_{i=1}^n r_i^2 = \sum_{i=1}^{46} r_i^2 = 0.568720723\% \quad (20)$$

Further obtained:

$$S = \sqrt{\frac{1}{n-1} \sum_{i=1}^n (r_i - \bar{r})^2} = \sqrt{\frac{1}{n-1} \sum_{i=1}^n r_i^2 - \frac{1}{n(n-1)} (\sum_{i=1}^n r_i)^2} = 1.124193\% \quad (21)$$

Assuming that treasury bond futures have 250 trading days a year, the volatility of the underlying asset linked to the product can be obtained as:

$$\sigma = \frac{S}{\sqrt{T}} = \frac{S}{\sqrt{\frac{1}{D}}} = S \times \sqrt{D} = 1.124193\% \times \sqrt{250} = 17.7750526\% \quad (22)$$

This paper adopts 46 samples of historical settlement price, so the annual standard deviation of volatility can be estimated as:

$$\mu = \frac{\sigma}{\sqrt{2n}} = \frac{17.7750526\%}{\sqrt{2 \times 46}} = \frac{17.7750526\%}{9.591663047} = 1.853177339\% \quad (23)$$

Where r represents the logarithmic return rate of treasury bond futures; $\bar{r}$ represents the daily average logarithmic return rate of treasury bond futures; p_i represents the actual settlement price on the trading day i ; s represents the standard deviation of daily logarithmic return rate; D represents the effective investment time of one year, and the one year effective investment time of treasury bond futures is calculated as 250 days; μ represents the annualized volatility.

3. Numerical Experiment

3.1 Monte Carlo Simulation Path

Foreign scholars Vladimir Anic & Mattin Wallmeier (2011) found that the issuers' disclosure of the probability distribution of structured products can improve investors' perception of products and influence investors' investment decisions [3]. Therefore, this paper intends to use the Monte Carlo simulation method to calculate the probability distribution of product revenue. The specific process is as follows:

(1) Determine the step size of the simulation as 1. Extract the normally distributed random numbers $\varepsilon_1, \varepsilon_2, \dots, \varepsilon_n$. Set the closing price of the TF2206 treasury bond futures on September 13, 2021 as the opening price, namely $S_0 = 100.400$. The initial fixed price can be calculated:

$$P_0 = \frac{1}{n} \sum_{i=1}^5 X_i = \frac{1}{5} \times (100.400 + 100.400 + 100.400 + 100.400 + 100.495) = 100.419 \quad (24)$$

(2) It is known that the risk-free rate of return is equal to the annualized rate of return of 2.1576% of the 2021 book-entry discounted (fifty-six issues) treasury bonds, and the Monte Carlo method can be used to simulate the 5-year treasury bond futures TF2206 in 124 trading days from November 19, 2021. The simulation equation is as follows:

$$S(t + \Delta t) = S(t) \exp[(r - \frac{\sigma^2}{2})\Delta t + \sigma \varepsilon \sqrt{\Delta t}] \quad (25)$$

(3) The observation days of each observation period are determined according to the product design requirements, namely the 43rd, 81st and 124th days. At the same time, the average performance of the underlying asset price on each observation day can be calculated. The calculation results are as follows:

$$\bar{P} = \frac{1}{n} \sum_{i=1}^3 P_i = \frac{1}{3} \times (101.2252 + 101.7783 + 101.1109) = 101.3715 \quad (26)$$

(4) Repeat step (2) for 10,000 times to determine the final return distribution of treasury bond futures TF2206. Based on this, 10,000 simulation paths can be used to execute the code to generate by using MATLAB, and the simulation path diagram is as follows:

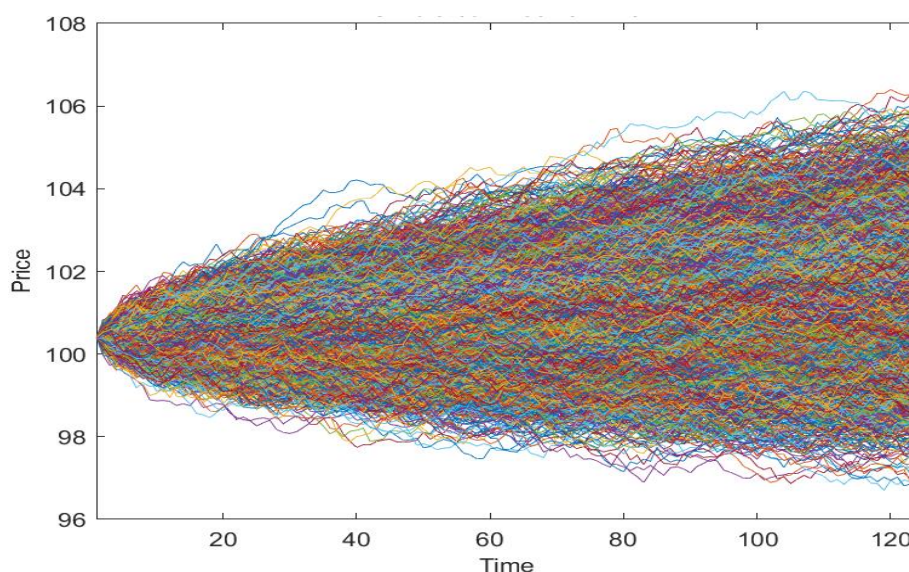


Fig 2. The price simulation path of treasury bond futures TF2206

3.2 Constructing Bear Market Call Option

Assume that S_T is the price of the underlying in the spread at the expiration of the option; A is the delivery of the call option (including the purchase of call and put options); X_A is the lower exercise price of the option in the transaction; C_A is the purchase option premium of call option; B is the delivery product for selling the call option (including selling the call option and put option); X_B is the higher strike price of the option in the transaction; C_B is the option premium for selling the call option, and the strategic maturity profitability is summarized as Table 1.

Table 1. Profits of Bear Market Call Option Strategies under Different Circumstances

Price Range	Option Long Return	Option Short Return	Total Revenue
$S_T \leq X_A$	$-C_B$	C_A	$C_A - C_B$
$X_A < S_T < X_B$	$-C_B$	$X_A - S_T + C_A$	$X_A - S_T + C_A - C_B$
$S_T \geq X_B$	$S_T - X_B - C_B$	$X_A - S_T + C_A$	$X_A - X_B + C_A - C_B$

Since the surplus and loss of the option contract will be affected by the price of the underlying on the expiration date of the option, this paper will simulate and calculate the option value corresponding to different strike prices under the bear market call option strategy. The results show that there is a negative correlation between the option exercise price and the option value. The specific data are shown in Table 2.

Table 2. Simulation Results of Exercise Price and Option Value of Bear Market Call Options

Exercise Price	Option Value	Exercise Price	Option Value
96.0	4.894	102.5	0.493
96.5	4.421	103.0	0.368
97.0	3.959	103.5	0.270
97.5	3.510	104.0	0.194
98.0	3.079	104.5	0.136
98.5	2.668	105.0	0.094
99.0	2.283	105.5	0.063
99.5	1.926	106.0	0.042
100.0	1.601	106.5	0.027
100.5	1.310	107.0	0.017
101.0	1.054	107.5	0.011
101.5	0.833	108.0	0.006

It is known that the treasury bond futures price simulation results of the structured financial product on the 43rd, 81st, and 124th observation days are 101.2252-yuan, 101.7783 yuan and 101.1109 yuan respectively. The theoretical price of the option expiration market calculated by the option expiration payment function is 101.3715 yuan. According to the analysis of bear market call option strategy, a call option should be purchased with an exercise price of 108 yuan and an option value of 0.006 yuan at the beginning of the period, and a call option should be simultaneously sold with an exercise price of 101.5 yuan and an option price of 0.833 yuan, so that the profit and loss of the bear market call option at different price levels can be calculated. The specific results are as follows:

Table 3. Profit and Loss of Bear Market Call Options

Price Range	Option Long Return	Option Short Return	Total Revenue
$S_T \leq 101.5$	—0.0060	0.8330	0.8270
$101.5 < S_T < 108$	—0.0060	$102.333 - S_T$	$102.327 - S_T$
$S_T \geq 108$	—6.6345	0.9615	—5.6730

It can be seen that the risks and benefits of bear market call option arbitrage are limited. The large loss is 5.6730 yuan, and the maximum return is 0.8270 yuan. At the same time, by comparing the expiration price of the option simulated by Monte Carlo with the lower exercise price, we can deduce that the total return of the option is 0.827 yuan.

3.3 Product Breakeven Probability Measure

Whether the inclusive pension financial product with embedded Asian options can maintain principal mainly depends on the fluctuation range of the maturity price of treasury bond futures. Through the above calculation, it is known that the maturity income of this product is 1.079 yuan. When the maturity price of treasury bond futures is less than or equal to 101.5 yuan, the option part can obtain a maximum income of 0.827 yuan, and the product can obtain a maximum income of 1.906 yuan. At this time, full breakeven can be achieved: when the maturity price of treasury bond futures is greater than 101.5 yuan but less than the option equilibrium price of 102.327 yuan, the income that can be obtained from the option part is the balance of the option equilibrium price minus the option expiration value. At this time, the income that can be obtained from product is the balance of the product equilibrium price of 103.406 yuan minus the option expiration value, and the product is still fully breakeven. However, when the maturity price of treasury bond futures is greater than the product equilibrium price of 103.406 yuan, the product will no longer be guaranteed, and investors need to bear the principal risk by themselves. It can be seen that whether the breakeven of this product depends on the price range of the maturity price of treasury bond futures.

Table 4. Profit and Loss of Inclusive Pension Financial Products with Embedded Asian Options

Price Range	Profit and Loss of Option Contract	Profit and Loss of National Debt	Total Profit and Loss	Breakeven or Not	Probability of Occurrence
$S_T \leq 101.5$	0.8270	1.079	1.906	Fully breakeven	74.3642%
$101.5 < S_T < 103.406$	$102.327 - S_T$	1.079	$103.406 - S_T$	Fully breakeven	23.8776%
$103.406 < S_T \leq 108$	$102.327 - S_T$	1.079	$103.406 - S_T$	Don't breakeven	1.7582%

4. Conclusion

This paper studies the pricing of embedded Asian options under Monte Carlo simulation. Different from standard American options, Asian options are a type of strong path-dependent options, and their average price is an independent variable in the whole pricing formula. In other words, the income of the embedded Asian option linked structured financial products will fluctuate with the fluctuation of the average price of all linked underlying assets during the financial management period. The pricing results and simulation paths under various parameter operations fully prove that the embedded Asian options can increase the breakeven probability of the product and have a good smoothing effect on the volatility of profit and loss at maturity. The design of inclusive pension financial products with embedded Asian options also fully reflects the advantages of embedded Asian option linked financial products in portfolio returns. Based on the progress of Asian option pricing, the hedging problem of Asian options under Monte Carlo simulation can be considered in the future.

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References

- [1] Harčariková, Monika, et al. Analysis of using options to the express certificates formation. *Economic Research-Ekonomska Istraživanja*, 2015, 28(1): 354-366.
- [2] Cupertino C M, Coelho R D A, et al. Cash flow, earnings, and dividends: A comparison between different valuation methods for Brazilian companies. *Economics Bulletin*, 2013, 33(1): 309-322.
- [3] Wallmeier M. Beyond payoff diagrams: how to present risk and return characteristics of structured products. *Financial Markets and Portfolio Management*, 2011, 25(3): 313-338.