

Asymmetry effect and jump effect on stock volatility -- Modeling and prediction based on GARCH-MIDAS

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Abstract. Based on GARCH-MIDAS model and its five extend models, this paper studies the effect of asymmetric effect and jump effect on stock volatility. According to research on NASDAQ, we find that asymmetric effect and jump effect can be beneficial to in-sample fitting and out-of-sample forecasting, especially short-term asymmetric effect and long-term jump effect. Regulators and investors can use this model to forecast stock volatility.

Keywords: GRACH-MIDAS, Asymmetric effect, Jump effect.

1. Introduction

Stock volatility refers to the fluctuation range of stock return rate along with the change of time in the mean. It can be used to measure the deviation of return rate from the mean. It is commonly used to measure the uncertainty of return rate, reflecting the severity of return on assets volatility. The fluctuation of the return rate of the stock market is of great importance to the supervision of financial risk by the regulatory authorities and to the investment portfolio management and risk management of investors. Therefore, the prediction of volatility is of great practical significance.

For volatility prediction, GARCH family models are often used, but GARCH family models can only use single high-frequency data and cannot consider data of multiple frequencies together. Therefore, the influence of long-term fluctuations on future fluctuations cannot be considered. Ghysels [1] et al. proposed mixed-frequency data sampling (MIDAS) in 2006, so that data of different frequencies can be studied in the same model. Engle [2] et al. introduced the MIDAS framework into the GARCH model in 2013 to form the GARCH-MIDAS model for the study of time-varying volatility.

In previous studies, many scholars have found that there is asymmetry between return rate and volatility, that is, the fluctuation caused by stock price decline is often greater than that caused by stock price rise of the same range. At the same time, the distribution of yield tends to be negative biased, which means that there is a large possibility of negative yield. Therefore, investors tend to pay too much attention to potential negative returns and have a great impact on volatility [3]. In addition, the income of assets often presents a certain continuity. However, in some special periods, such as when the macro-economic environment changes greatly, the return on assets may change greatly, which is the jump effect. The jump effect is significant, which often causes panic among investors and has a great impact on volatility.

Therefore, the research on asymmetric effect and jump effect can help the regulatory authorities to prevent in advance and help individual investors to control the risk of asset portfolio.

2. Model introduction

The GARCH-MIDAS model is selected in this paper to model and forecast volatility. On the basis of the standard GARCH-MIDAS model, the asymmetric effect and jump effect are added to study their impact on volatility. Therefore, two methods are used to decompose long-term fluctuations.

2.1 Standard GARCH-MIDAS Model

In the GARCH-MIDAS model proposed by Engle et al. in 2013, volatility is decomposed into long-term and short-term components. High-frequency data affect volatility through short-term components, while low-frequency volatility depends on long-term components. A standard GARCH-MIDAS model has the following form:

$$r_{i,t} = \mu + \sqrt{\tau_t g_{i,t}} \varepsilon_{i,t}, \quad \forall i = 1, \dots, N_t$$

$$\varepsilon_{i,t} | \Phi_{i-1,t} \sim N(0,1)$$
(1)

Where $r_{i,t}$ is the log return on day i in month t ; μ is the conditional expectation, given information up to time $(i - 1)$; $g_{i,t}$ is the short-term component, which accounts for daily volatility, τ_t is the long-term component, which accounts for monthly volatility.

In the model above, the short-term component $g_{i,t}$ follows standard GARCH (1, 1) process:

$$g_{i,t} = (1 - \alpha - \beta) + \alpha \frac{(r_{i-1,t} - \mu)^2}{\tau_t} + \beta g_{i-1,t}$$
(2)

The long-term component τ_t is determined by the exogenous variable—Realized Volatility—RV:

$$\tau_t = m + \theta \sum_{k=1}^K \varphi_k(\omega_1, \omega_2) RV_{t-k}$$
(3)

Where $\varphi_k(\omega_1, \omega_2)$ is determined by the following weight function:

$$\varphi_k(\omega) = \frac{\binom{k}{K}^{\omega_1 - 1} \binom{1-k}{K}^{\omega_2 - 1}}{\sum_{j=1}^K \binom{j}{K}^{\omega_1 - 1} \binom{1-j}{K}^{\omega_2 - 1}}$$
(4)

To simplify calculations and give more weight to recent information, we set ω_1 equal to one. The maximum lag period(K) selected in this paper is 24.

2.2 GARCH-MIDAS model with the asymmetric effect

The logarithmic price of the asset p_t meets the condition that:

$$p_t = \int_0^t \mu_s ds + \int_0^t \sigma_s dW_s + J_t$$
(5)

Where μ is a locally bounded predictable drift process, σ is a positive càdlàg process, and J is a pure jump process.

The quadratic variation of this process is $[p, p]$:

$$[p, p] = \int_0^t \sigma_s^2 ds + \sum_{s=0}^t (\Delta p_s)^2$$
(6)

Where $\Delta p_s = P_s - P_{s-}$.

Anderson [4] et al. proposed to use the sum of squares of high-frequency returns as realized volatility—RV in 2001:

$$RV = \sum_{i=0}^n r_i^2 \xrightarrow{p} [p, p] = \int_0^t \sigma_s^2 ds + \sum_{s=0}^t (\Delta p_s)^2, n \rightarrow \infty$$
(7)

Barndorff-nielsen and Shephard [5] proposed in 2006 to extend simple fluctuation estimation to a broader dimension, including quadratic power variation--BV:

$$BV = \mu^{-2} \sum_{i=2}^n |r_i||r_{i-1}| \xrightarrow{p} \int_0^t \sigma_s^2 ds, n \rightarrow \infty \tag{8}$$

Where $\mu = \sqrt{\frac{2}{\pi}}$.

Compared with RV, BV only contains the component of the secondary change caused by the continuous change of price, namely:

$$RV - BV \xrightarrow{p} \sum_{s=0}^t (\Delta p_s)^2. \tag{9}$$

Barndorff-nielsen et al. [6] proposed the realized semi-variance in 2010, which represents the fluctuation caused by positive or negative returns:

$$RS_{i-k}^- = \sum_{j=1}^N r_{i-j}^2 1_{\{r_{i-1} < 0\}} \tag{10}$$

$$RS_{i-k}^+ = \sum_{j=1}^N r_{i-j}^2 1_{\{r_{i-1} > 0\}} \tag{11}$$

Where 1_{Ω} is an indicator function. If the subscript is satisfied, it is 1, and vice versa.

RV can be completely decomposed here, namely:

$$RV = RS^+ + RS^- \tag{12}$$

In addition, similar to the realized variance, the realized semi-variance can also be expressed in the form of limit:

$$RS^+ \xrightarrow{p} \frac{1}{2} \int_0^t \sigma_s^2 ds + \sum_{s=0}^t (\Delta p_s)^2 1_{\{\Delta p_s > 0\}} \tag{13}$$

$$RS^- \xrightarrow{p} \frac{1}{2} \int_0^t \sigma_s^2 ds + \sum_{s=0}^t (\Delta p_s)^2 1_{\{\Delta p_s < 0\}} \tag{14}$$

It can be seen that RS^+ and RS^- contain a continuous part and a jumping part respectively. It can be seen that RV contains three parts, namely, continuous fluctuation process, a positive jump and a negative jump. Therefore, we can eliminate the continuous process by subtracting RS^+ with RS^- to obtain the jumping process caused by the change of sign:

$$\Delta J^2 \equiv RS^+ - RS^- \xrightarrow{p} \sum_{s=0}^t (\Delta p_s)^2 1_{\{\Delta p_s > 0\}} - \sum_{s=0}^t (\Delta p_s)^2 1_{\{\Delta p_s < 0\}} \tag{15}$$

2.2.1 Short-term asymmetric effect

Here, we use the GJR (1,1) paradigm to describe the influence of short-term asymmetric effects on volatility:

$$g_{i,t} = (1 - \alpha - \beta - 0.5\gamma) + (\alpha + 1_{\{r_{i-1} < 0\}} \gamma) \times \frac{(r_{i,t} - \mu)^2}{\tau_t} + \beta g_{i-1,t} \tag{16}$$

In the short-term asymmetric effect, the effect of positive and negative returns on the volatility of the next day is distinguished.

2.2.2 Long-term asymmetric effect

In this paper, we study the simple asymmetric effect and the jump effect due to the change of sign respectively.

2.2.2.1 For simple asymmetric effects, semi-variance decomposition method is adopted in this paper.

In this method of fluctuation decomposition, the expression of long-term fluctuation is as follows:

$$\tau_i = m + \theta^- \sum_{k=1}^K \varphi_k(\omega)RS_{i-k}^- + \theta^+ \sum_{k=1}^K \varphi_k(\omega)RS_{i-k}^+ \tag{17}$$

Where:

$$RS_{i-k}^- = \sum_{j=1}^N r_{i-j}^2 1_{\{r_{i-1} < 0\}} \tag{18}$$

$$RS_{i-k}^+ = \sum_{j=1}^N r_{i-j}^2 1_{\{r_{i-1} > 0\}} \tag{19}$$

According to the positive and negative daily returns, the fluctuation is decomposed into realized positive half-variance and realized negative half-variance.

2.2.2.2 Considering the jumping process due to the change of sign

Because the jump effect usually exists in the long term, this paper mainly changes the long-term wave component τ_i in GARCH-MIDAS model, and the short-term wave component $g_{i,t}$ follows the above formula. The main consideration here is that the change of sign brings variance and quadratic variation.

Under this decomposition method, the expression of long-term fluctuations is as follows:

$$\tau_i = m + \theta_j^- \sum_{k=1}^K \varphi_k(\omega)\Delta J_{t-k}^{2-} + \theta_j^+ \sum_{k=1}^K \varphi_k(\omega)\Delta J_{t-k}^{2+} + \theta_{BV} \sum_{k=1}^K \varphi_k(\omega)BV_{t-k} \tag{20}$$

Where:

$$\Delta J_{t-k}^{2-} = (RS_{i-k}^+ - RS_{i-k}^-)1_{\{RS_{i-k}^+ - RS_{i-k}^- < 0\}} \tag{21}$$

$$\Delta J_{t-k}^{2+} = (RS_{i-k}^+ - RS_{i-k}^-)1_{\{RS_{i-k}^+ - RS_{i-k}^- > 0\}} \tag{22}$$

$$BV = \mu^{-2} \sum_{i=2}^n |r_i| |r_{i-1}| \tag{23}$$

Among them $\mu = \sqrt{\frac{2}{\pi}}$.

Unlike the semi-variance, ΔJ_{t-k}^{2-} and BV reflect the component of the quadratic change caused by the sign change of returns.

2.2.3 The model summary

Table 1. Model summary

Model Number	Short-term asymmetric effect	Long-term asymmetric effect	Long-term jump effect
Model 0			
Model 1	√		
Model 2		√	
Model 3	√	√	
Model 4			√
Model 5	√		√

In this paper, a total of the above six models are considered. Model 0 is a standard GARCH-MIDAS Model, and Model 1 considers the short-term asymmetric effect of GJR normal form. Model

2 decomposed long-term realized volatility into realized positive and negative semi-variance to study the influence of long-term volatility on volatility modeling and prediction. Model 3 considers both short-term and long-term asymmetric effects. Model 4 considers the long-term jump effect and decomposes the variance into sign changes to bring variance and quadratic power variation. Model 5 takes into account both short-term asymmetric effects and long-term jump effects.

3. Data selection

The data selected in this paper are the daily return data of NASDAQ Index from January 2011 to August 2020, a total of 2432 data. Among them, 1,510 trading days from January 2011 to December 2016 were selected as the in-sample data; A total of 922 trading days from January 2017 to August 2020 were selected as out-of-sample data.

The following table is the descriptive statistics of this data set:

Table 2. Descriptive Statistics

Mean	standard deviation	kurtosis	Skewness	JB statistic	ADF statistic
1.52	1.230	-0.83	14.81	14913***	-35.23***

4. In-sample estimation

Table 3. Estimation result

	Model 0	Model 1	Model 2	Model 3	Model 4	Model 5
μ	0.0749*** (0.0264)	0.04305** (0.0236)	0.0754*** (0.0265)	0.0435** (0.0242)	0.0753*** (0.0264)	0.0440** (0.0197)
α	0.1522*** (0.0328)	0.0010 (0.0882)	0.1514*** (0.0328)	0.0010 (0.0761)	0.1454*** (0.0311)	0.0010 (0.1346)
β	0.7100*** (0.0599)	0.7809*** (0.0763)	0.7135*** (0.0612)	0.7821*** (0.0651)	0.7177*** (0.0561)	0.7840*** (0.1223)
ω	8.3969 (7.0547)	4.9783* (3.7273)	8.1438 (6.2567)	5.6315* (3.5324)	1.6427 (1.7928)	1.0010*** (0.3811)
m	0.6643*** (0.2109)	0.5999*** (0.1488)	0.6298** (0.2742)	0.5005*** (0.2015)	1.0000*** (0.4624)	1.0000*** (0.1230)
γ		0.2350*** (0.0182)		0.2343*** (0.0273)		0.2305*** (0.0504)
θ	0.0999** (0.0507)	0.1043** (0.0488)				
θ^-			-0.0530 (0.3523)	-0.0000 (0.1474)		
θ^+			-0.1449 (0.1947)	-0.1800*** (0.0696)		
θ_j^-					-0.0726*** (0.0298)	0.0629** (0.0323)
θ_j^+					-0.0000 (0.0449)	0.0003 (0.1330)
θ_{BV}					0.0897 (0.1276)	0.056283 (0.0760)

It can be found from the above table that:

Basically, the parameters of Model 0 are significant and positive, indicating that a standard GARCH-MIDAS can better fit the returns of stocks and can be used to predict the returns of stocks. $\alpha + \beta$ is close to 1, indicating that the volatility of stock returns is persistent.

In Model 1, 3 and 5, the short-term asymmetric effect was taken into account, and their γ values were all significantly positive, indicating that the short-term asymmetric effect was significant, and γ values were all significantly positive, indicating that short-term bad news had a significantly greater impact on stock volatility than good news.

Model 2 and Model 3 consider the long-term asymmetric effect, but the significance of θ is not high, indicating that the long-term asymmetric effect does not have a great impact on stock volatility. Among the four long-term asymmetric parameters, only θ_+ in Model 3 is significantly negative, indicating that good news will bring a decline in volatility in the long run.

In Model 4 and 5, the long-term jumping effect is considered, and only θ_j^- is significant, indicating that the jumping process is caused by the change of sign. The $RS^+ - RS^- \sum_{s=0}^t (\Delta p_s)^2 1_{\{\Delta p_s > 0\}} - \sum_{s=0}^t (\Delta p_s)^2 1_{\{\Delta p_s < 0\}} < 0$, leap forward leap is less than the negative can bring great influence to the volatility.

5. Out-sample forecasting

In this paper, the rolling window is selected to predict the volatility out of sample, and the prediction period is 5 days. The prediction results of each model are as follows:

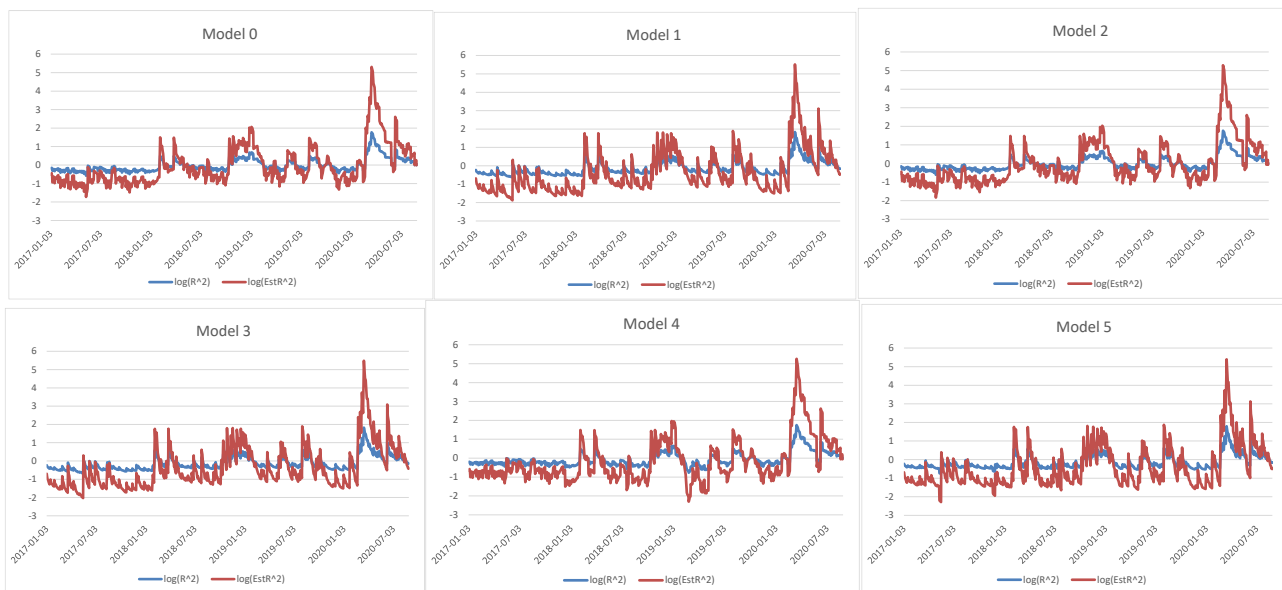


Figure 1. Dynamics of actual volatility and the forecasted volatility

In order to measure the accuracy of prediction, loss function MSE is adopted in this paper for calculation, and the formula of MSE is as follows:

$$MSE = \frac{1}{n} \sum_{t=1}^n (\sigma_t^2 - \hat{\sigma}_t^2)^2 \tag{24}$$

In addition, MCS test is also adopted in this paper to judge the performance of the model, and the results are as follows:

Table 4. Results of MCS test

Model Number	MSE	MCS Test
Model 0	4.24414	0.273
0.300Model 1	1.63070	0.300
Model 2	4.08169	0.273
Model 3	1.63611	0.300
Model 4	3.47917	0.258
Model 5	0.99118	1.000

It can be seen from the above results:

(1) from the point of asymmetric effect, only consider the short-term optimal asymmetric effect of model performance, compared with not considering asymmetric effect, consider the long-term effect of asymmetric can bring certain optimization to model, at the same time considering the effect of short-term long-term asymmetric model in whether to consider both short-term and long-term asymmetric effect of four models of the optimal performance;

(2) The model that considers only the long-term jump effect is better than the model that does not consider the short-term asymmetric effect. The model that considers both the short-term asymmetric effect and the long-term jump effect performs best and can well predict the future volatility.

6. Conclusion

This paper studies the volatility of stock market based on GARCH-MIDAS model, and proposes five extended models based on standard GARCH-MIDAS according to the asymmetric effect and the jump effect caused by sign change. Through the in-sample modeling and out-of-sample estimation of THE NASDAQ index, we find that the asymmetric effect and the long-term jump effect are conducive to the in-sample fitting and out-of-sample prediction, and the short-term asymmetric effect is much better than the standard model for volatility prediction. The long-term asymmetric effect can improve the forecasting ability of volatility, but it is limited. Long-term jump effect and short-term asymmetric effect have good effects on volatility. Regulators and investors can use this model to predict future volatility and provide certain theoretical basis for regulatory behavior and risk management.

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