Model Comparison between CAPM and APT: With focus on application of Factor Models

Ruijun Wu*

HKU Business School, University of Hong Kong, Hong Kong, 999077, China

*Corresponding author. Email: wuruijun@connect.hku.hk

Abstract. Asset pricing theory has been one of the most rapidly growing research fields of western financial theory in the past few decades and one of the most significant theoretical tools extensively used in the financial industry, which attempts to interpret the price or value of assets paid in the future under uncertain circumstances. With increasingly more research on comparison between financial factor models, it is sometimes hard for scholars to have an explicit comprehension of various study of a certain topic. This paper aims to collect and analyse past research on comparing CAPM and APT, specifically focusing on the application aspect. The paper found that the CAPM needs careful time horizon screening while APT is more flexible. However, APT requires more data analysis for factor selection and sensitivity measurement. Both models prefer markets with many assets. The results of the paper provides useful information and insights to scholars who intend to do empirical study on the two models in hands on data collection preparation and model fitting process, which could potentially help them save time from try-and-error.

Keywords: Factor Model Comparison, CAPM and APT, Application of Financial Models.

1. Introduction

1.1 Research Background and Motivation

When financial decision involving evaluation of assets needs to be made by individual investors and corporation managers, the choice among pricing strategies is a crucial step. In general, factors that are associated with the expected return of a certain asset or an overall portfolio could be diverse, among which the most discussed one is risk. The trade-off between risk and expected return is an intensively explored topic in the finance field. Generally speaking, under the primary assumption that rational investors tend to be risk-averse while they seek for the highest return possible, it is agreed by many scholars that investors would expect higher return for undertaking higher risk from making the very investment decision. This idea is intuitively reasonable since people tend to avoid risks unless it comes with potential desired return. The very first explicit financial model that tries to explain the linear relationship between return and risk is the Capital Asset Pricing Model (abbreviated as CAPM), followed by many other factor models that add additional risk factors while retaining the linear structure, including the Fama and French Three Factor Pricing Model and Five Factor Pricing Model, as well as Arbitrage Pricing Model [1-3].

Specifically, the CAPM could be presented with the mathematical formula:

$$\mathrm{E}(\mathrm{R}_i) = R_f + \beta_i(E(R_m) - R_f) \quad (1)$$

$\mathrm{R_f}$ denotes the risk-free rate, the return of risk-free assets, which is a premise of the CAPM. β_i represents the undiversifiable risk of the i asset, which is numerically calculated as the ratio of covariance of the expected return of the i asset and the market return to the variance of the market return. The difference between $E(R_m)$, which is the expected return of the market, and risk-free rate, also known as the market premium, which demonstrates the amount of additional return required for undertaking additional risk in the investment.

The APT essentially says that the return of assets could be generated by a multi-factor model in the form:

$$E(R_i) = r + \sum_{j=1}^{k} \lambda_j \beta_{ij} \quad (2)$$

In the model, r is the constant term and equals to the risk-free rate when risk-free asset exists, and λ is a vector representing risk premium of each risk factor. Similar to the CAPM, β_{ij} measures how sensitive the *i* asset is to the *j* risk factor.

Instead of including specific risk factors, the model essentially allows scholars to add whatever factors they would like to consider, each multiplied with distinctive sensitivity of different assets to the factor β, also known as factor loadings. The expected return is just equal to the summation of these products plus a constant term and an error term ε, which represents idiosyncratic risk of each asset.

1.2 Literature Review

Ever since Ross' first proposal of Arbitrage Pricing Theory as an alternative of the CAPM, there have been various discussion and comparison between these two famous financial models analytically and empirically in different context[4-5]. The CAPM is raised by Treynor, William F., Litner and Mossin separately in 1960s, which is a theoretical financial model deals with risk-return trade-off when market is in equilibrium [6-8]. This model basically states that, in perfect market where borrow and lend rate are equal, the expected return of a certain asset or a portfolio of financial asset is linearly related to the systematic risk. Idiosyncratic risk is not discussed in the model, for it is based on the assumption that non-systematic risk could be diversified, and only the correlation, or covariance between assets plays a significant role when considering risk management of a well-diversified portfolio.

The CAPM has been widely known and used by scholars in academia and managers in the industry, also criticized by a lot of scholars because the model only involves market risk. APT, which was proposed by Ross and Roll, is a very comprehensive multi-factor linear model that also deals with risk-return trade-off. It is well acknowledged that APT gives scholars and managers enough freedom to be as inclusive as they intend to be in terms of risk consideration. But this model, without well-defined factors and clearly specified model fitting strategy, also requires scholars and managers to have much more additional investigation before they start to use the model to evaluate risky assets or make investment decision.

1.3 Research Significance and Framework

Given that both these two models are based on many assumptions, the application of the models requires scholars and investors to find suitable conditions, desired market environment and make reasonable estimation of parameters. Thus, it is significant to research on the application details of the two models and the difference between the two, which could benefit scholars and investors in that with the research results in this paper they could save much time before making future exploration or doing empirical test. This paper includes comparison between the CAPM and the APT in three aspects in application: limitation based on assumption, model structure and parameter estimation, and utilization of predicted return.

2. Methodology

2.1 Model Comparison

As mentioned above, there have been various research topics involving the CAPM and the APT, due to the fact that APT was often identified as an alternative to the CAPM with remarkable advancement in terms of risk measurement, and that the two models are comparable in assumptions, linear structure of the models and desirable conditions required in application. However, the chronological order of these two models does not necessarily justify APT as a better choice, and some scholars like Shankan have also elaborated their favor of the CAPM against APT, who was

subsequently reputed by Dybvig and Ross by showing the hidden misuse of APT in a situation that is not desirable based on the given assumptions [9-10]. There are also scholars like Robbert Jarrow who researched on conditions under which the CAPM and the APT could be equivalent and a condition that makes the two models differ [11].

Given the abundance of publications on this topic, the paper reviews comparison research of the CAPM and the APT from top journals from various period and analyse the comparison within the predetermined structure.

2.1.1. Commonalities and Discrepancies in Assumptions of the CAPM and APT

Both the CAPM and the APT assume. First, investors do not need to worry about the idiosyncratic risk by holding diversified portfolios. Second, investors are allowed to borrow and lend money at the risk-free rate. Third, a perfect capital market, which means the value of assets, is accurately reflected by the price of them. Forth, investors are naturally risk-averse and have same expected return for a specific level of risk. Fifth, that expected return of assets and the risk factors are linearly correlated. The assumptions of CAPM and APT are different. As far as systemic risk is concerned, the CAPM assumes that systematic risk could be simply measured by market risk, yet the APT assumes that various risk factors contribute to systematic risk. As far as risk-free asset is concerned, the CAPM assumes that there are risk-free assets in the financial market, while the APT does not.

2.1.2. Assumption Limitation in Application

Since the CAPM model assumes that risk-free assets and perfect market exist, the market should subsequently have an expected return to be higher than, or at least equal to the risk-free rate, which happens when the market premium is zero. Essentially, negative market return does not make any sense for the CAPM, because the expected return for any asset is at least the same as the risk-free rate, otherwise investors would just lend money at the risk-free rate as proposed assumption states. This means that in real world where there is a negative market return, usually when economic crisis happens due to various reasons like wars or pandemics and so on, the CAPM losses its validity at all. APT addresses this limitation of the CAPM by not defining what the constant and the factors could be. So, if market risk premium is negative, there might be other kind of measurable risk premium available for the model to use, like liquidity, inflation, interest rate and so on. In application, this difference makes APT much more flexible than the CAPM, since it does not require the whole market to have a positive return in a certain period. Apart from the number of factors included in the models, another difference between the two models is that the APT includes the idiosyncratic risk as noise term in the model. The authors who proposed APT claimed that while idiosyncratic risks could be well diversified, although regarded as negligible in empirical tests, they could still influence the pricing of assets theoretically.

2.2 Model Structure and Parameter Estimation of the CAPM and the APT

Both the CAPM and the APT have linear structure: with expected return as the response variable, risk premium as the explanatory variable(s), and a constant, which is risk-free rate in the CAPM. The major structural difference is yet obvious: different number of risk factors involved. Many scholars stated that the CAPM is just another APT model with single factor, while some others claimed the APT is a multi-beta version of the CAPM, both of which make sense given the similarity of the two models. In addition, the CAPM and the APT are static models, which means when used to make estimation in real financial market, they make use of previous data to only predict the expected return within the next transaction period. Since the risk factors are constantly changing, and new data are available every workday, the measurement of risk and the expected return will also update in a fast pace.

In the light of the structural similarity, the methodology can also be similar between the two models in terms of fitting the models with empirical data: data should be collected from markets that satisfy or at least approximate the market presumed in the assumption; time series models are used to estimate returns based on the past time series data. The estimation of parameter for the CAPM could

be quite straightforward. Risk-free rate can be estimated based on the past government bond yield or interest rate (not applicable for negative interest), since government issued securities, although not definitely safe, are viewed as "risk-free" in many empirical study scenarios. Market variance and expected return of market could be calculated from the major indices, which are believed to be representative of the corresponding stock markets.

Yet, the APT involves more complex risk measurement and factor choosing research and data collection process, since factors are not specified in the model itself. One of the major challenges in applying the APT to any stock market is the measurement of sensitivity of an asset to certain factor. However, Roll and Ross claimed that it is the unanticipated part of the risk factors that would contribute to the sensitivity, since the anticipated part is already reflected in the expected return. Regarding this concern, Chen, Roll and Ross provided four relevant factors, including unexpected variation in: inflation, industrial production, risk premium and the slope of the term structure of the interest rate[12]. As a result, among all of the uncountable factors that are related to assets, it is reasonable to sort out several primaries factors and test the significance before applying the model to a certain set of assets.

3. Conclusion

The paper compares the CAPM and the APT and their application to find out the conditions under which these two models could be used to make estimation properly. The CAPM has more restriction in its assumptions than the APT, which requires both positive market premium and positive risk-free rate. This means the time horizon needs to be carefully chosen and CAPM is simply inapplicable when the available market premium is continuously negative. The APT has more freedom in terms of time horizon selection since positive market premium and risk-free rate are not prerequisites. Although provided the four factors previously, factor selection is still a challenge that needs further research. This is obvious because if only the four factors are primary contributors, it is hard to understand why Roll and Ross did not ultimately change their APT to a four-factor model. As a result, it is important to review the past empirical tests before conducting further exploration on the comparison between the CAPM and the APT. For scholars who intend to do related empirical research, it is advised to find a satisfactory time horizon and a satisfactory market for both CAPM and APT, and have a careful exploratory data analysis beforehand.

References

[1] W.F., Sharpe. CAPITAL ASSET PRICES: A THEORY OF MARKET EQUILIBRIUM UNDER CONDITIONS OF RISK*. The Journal of Finance, 19: (1964),425-442.

[2] R., Roll, & S. A., Ross. The Arbitrage Pricing Theory Approach to Strategic Portfolio Planning. Financial Analysis Journal, Vol. 51(1): (1995)122-131.

[3] F.F., Eugene, R. F., Kenneth, Common risk factors in the returns on stocks and bonds, Journal of Financial Economics, Volume 33, Issue 1, 1993, Pages 3-56, ISSN 0304-405X, https://doi.org/10.1016/0304-405X(93)90023-5.

[4] N., Soufian. "Empirical Content of capital asset pricing model (CAPM) and arbitrage pricing theory (APT) across time." Manchester Metropolitan University. https://e-space.mmu.ac.uk/id/eprint/1462

[5] B., Alejandro, B., Beatriz, and B., Raquel. CAPM and APT-like models with risk measures, Journal of Banking & Finance, Volume 34, Issue 6, 2010, 1166-1174,DOI: 10.1016/j.jbankfin.2009.11.013.

[6] C. W., French. "The Treynor Capital Asset Pricing Model". Journal of Investment Management. 1 (2): (2003)60–72. SSRN 447580.

[7] J., Lintner. "The valuation of risk assets and the selection of risky investments in stock portfolios and capital budgets". Review of Economics and Statistics. 47 (1): (1965) 13–37. doi:10.2307/1924119. JSTOR 1924119

[8] J., Mossin, Jan. "Equilibrium in a Capital Asset Market". Econometrica. 34 (4): (1966)768–783. doi:10.2307/1910098. JSTOR 1910098.

[9] J., Shanken. Multi-Beta CAPM or Equilibrium-APT?: A Reply. The Journal of Finance, 40(4), (1985). 1189–1196. DOI: https://doi.org/10.2307/2328402

[10] [10] P.H., DYBVIG, and S.A., ROSS, (1985), Yes, The APT Is Testable. The Journal of Finance, 40: 1173-1188.DOI: 10.1111/j.1540-6261.1985.tb02370.x

[11] J., Robert, R., Andrew. A comparison of the APT and CAPM a note. Journal of Banking & Finance Volume 7, Issue 2, 1983, Pages 295-303, ISSN 0378-4266. DOI: https://doi.org/10.1016/0378-4266(83)90039-0

[12] N.F., Chen, R., Roll, & S. A., Ross. Economic Forces and the Stock Market. The Journal of Business, 59(3), (1986)383–403. http://www.jstor.org/stable/2352710