Analysis of Corporation’s Financial Fraud and Independent Directors’ Decision Based on Evolutionary Game Theory

Ching Hung Cheng¹, *, †, Zhe Huang², †, Siqi Lin³, †, Hangyu Wu⁴, †

¹Department of Statistics and Actuarial Science, The University of Hong Kong, Hong Kong, China
²International College, Zhengzhou University, Zhengzhou, China
³College of Life Science and Technology, Huazhong University of Science and Technology, Wuhan, China
⁴Business School, Beijing International Studies University, Beijing, China

*Corresponding author: cchandy@connect.hku.hk
†These authors contributed equally.

Abstract. The case of Kangmei Pharmaceutical Company’s financial fraud reveals several important issues in the independent director system in China. It is valuable to study these issues separately from the corporation and the independent director’s perspective, as both wish to maximize their profitability. This paper applies evolutionary game theory to construct two games between the corporation and independent directors, and {honest, dishonest} is the strategy set for the corporation in both games, while {dutiful, undutiful} and {continue, resign} are the strategy sets for the independent directors in game 1 and game 2 respectively. The evolutionary stability strategies obtained from these two games and simulation results show that if the independent directors should be dutiful and not quit their positive while the corporation should never consider making any financial fraud, then the independent directors and the corporation will get the optimal benefit. The results suggest that the award mechanism and reputation mechanism for independent directors should be established and fully completed to improve the independent director system.

Keywords: corporate governance, independent director system, evolutionary game theory.

1. Introduction

1.1 Research Background

The independent director system has been part of corporate governance in China for many years. The purpose of an independent director system is to separate a company’s power and supervise the behaviour of a company so that it can prevent a company from taking any improper action, such as financial fraud [1]. It should be expected that the board of directors can improve the effectiveness and efficiency of corporate governance. However, according to the report, 54%, 53%, and 48% of the board of directors were subject to administrative penalties in 2019, 2020, and 2022 respectively. Thus, it reveals that there are still a lot of improvements for the independent director system in China [1-2]. The independent directors are similar to free-riders, and they do not make any contribution to their corporation, but they receive a salary from their corporation; at the same time, they have limited power, as they are manipulated by their manager, which shows that they are not independent indeed [2-5]. Undeniably, there are some serious problems in the independent director system in China. Fama and Jensen (1983) found that the salary of independent directors is quite low, but they have to be a scapegoat for their corporation when they commit improper action [6]. It implies that the independent director is in a risky position in a corporation to a certain extend.

Based on the case of Kangmei Pharmaceutical Company in 2021, 12 members of its board of directors were fined between 123 million RMB and 246 billion RMB for administrative penalties, while their annual salaries were between 37 thousand RMB and 164 thousand RMB [7]. After the exposure of this case, several listed Chinese companies suffered from the sudden resignation of the board of directors in November 2021 [7]. Hence, this phenomenon has a huge disastrous impact on
corporate governance. The main issue of these occurrences is that the salary of independent directors is not proportional to the risk they undertake.

Besides, the effect of salary awards should not be disregarded. Perry’s (2020) research mentioned that when there is enough salary encouragement for independent directors, they are more likely to find evidence of financial fraud by a dishonest corporation [8]. At the same time, Ryan and Wiggins (2004) further emphasized that salary encouragement can improve independent directors’ work efficiency and effectiveness [9]. Undeniably, supervision strictness by independent direction is affected by the salary award which is received from a corporation.

Concerning the impact of reputation awards, Fama and Jensen (1983) state that the reputation mechanism is essential for independent directors’ further careers, and it is also an important factor in determining the management ability of independent directors [6]. It is hard for independent directors who work in dishonest companies to be employed by other honest companies since the work experience in a dishonest corporation is much less credible than in an honest corporation [10-12]. Hence, the independent directors may not be able to make their profile competitive and outstanding if they work in dishonest companies, which may even put them in a disadvantaged stage.

1.2 Research purpose

The aforementioned information shows the seriousness of the issue between the corporation and the independent director system. Several approaches can address this problem. This research uses evolutionary game theory to examine two games between a corporation and independent directors. The strategy set for independent directors is different in each game, and the strategy set for a corporation is the same in both games. The construction of two models is based on the behavioural logic of corporation and independent directors. The mathematical method in ordinary differential equations will be used to obtain the evolutionary stability strategies of the two models. Meanwhile, the simulation method will also be used to emphasize the theoretical results from the two games. After obtaining and analyzing the theoretical results and the simulation results, some accessible suggestions will be provided to improve the independent director system and ensure the integrity of the corporation under the setting of model 1 and model 2.

2. Methodology

2.1 Models setup and assumptions

This paper uses evolutionary game theory to construct two models involving two participants, corporation and independent directors. The corporation has the same set of strategies, {dishonest, honest}, in both model 1 and model 2. More specifically, a dishonest corporation will consider making financial fraud, while an honest corporation will never consider financial fraud. The strategies set for independent directors in model 1 and model 2 is {be dutiful, be undutiful} and {continue, resign} respectively. More precisely, dutiful independent directors will be strict in supervision while undutiful independent directors will be quite lenient to the corporation. The independent directors in model 2 can consider continuing to be a member of the board of directors or resigning their current position.

The expression of model 1 is shown in Table 1, while the normal form of model 2 is illustrated in Table 2 The variables, which are used in model 1 and model 2, are listed in Table 3.

In model 1, it is assumed that the probability that independent directors are dutiful is $q_1$, and the probability for the corporation is honest is $p_1$. Then, the probability that independent directors are undutiful is $1 - q_1$, and the probability for the corporation is dishonest is $1 - p_1$.

In model 2, it is assumed that the probability for independent directors to continue working is $q_2$, and the probability for corporation to be honest is $p_2$. Then, the probability for independent directors to resign is $1 - q_2$, and the probability for corporation to be dishonest is $1 - p_2$.

In this research, the independent directors are assumed to be fully independent. Specifically, the independent directors will not consider conspiracy to cheat. The dutiful independent directors will
always hold adverse opinions. They will not expose a corporation’s financial fraud, since the procedure of supervision and exposure involve various costs. In model 2, it will be destructive to a reputation of a corporation, if any independent directors decide to resign. Likewise, a corporation must follow some rules to recruit an independent director. Thus, the transaction cost \( c_1 \) occurs during this process. In addition, it may be hard for a dishonest corporate to recruit a new independent director, since the phenomenon of the resignation of independent directors may be more significant. Hence, the transaction cost \( c_2 \) for a dishonest corporation’s recruitment will be higher. Furthermore, a dishonest corporation will try to reduce independent directors’ responsibility so that the company can enlarge its gain. Thus, it will increase the cost for independent directors to be dutiful.

The expression of model 1 is shown in Table 1, while the normal form of model 2 is illustrated in Table 2. The relevant variables in model 1 and model 2 are listed in Table 3.

<table>
<thead>
<tr>
<th>symbol</th>
<th>meaning</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \beta_1 )</td>
<td>The overall cost for independent directors to be dutiful ((\beta_1 &lt; S))</td>
</tr>
<tr>
<td>( \beta_2 )</td>
<td>Higher cost when a corporation tries to keep or recruit independent directors back ((\beta_1 &lt; \beta_2))</td>
</tr>
<tr>
<td>( l_1 )</td>
<td>The positive impact of reputation awards by independent directors</td>
</tr>
<tr>
<td>( l_2 )</td>
<td>The negative impact of reputation awards by independent directors ((l_1 &lt; l_2))</td>
</tr>
<tr>
<td>( g_1 )</td>
<td>The overall gain of a corporation from financial fraud ((g_2 \ll g_1))</td>
</tr>
<tr>
<td>( g_2 )</td>
<td>The overall gain of a corporation without any financial fraud ((g_1 \ll l))</td>
</tr>
<tr>
<td>( \theta_1 )</td>
<td>The probability that a dishonest corporation is supervised while independent directors are dutiful</td>
</tr>
<tr>
<td>( \theta_2 )</td>
<td>The probability that a dishonest corporation is supervised while independent directors are undutiful ((\theta_2 \ll \theta_1))</td>
</tr>
<tr>
<td>( \eta )</td>
<td>Fine imposed by a government on independent directors ((S, l_1, l_2 \ll \eta))</td>
</tr>
<tr>
<td>( S )</td>
<td>Independent directors’ salary</td>
</tr>
<tr>
<td>( c_1 )</td>
<td>The overall cost for a corporation recruiting new independent directors</td>
</tr>
<tr>
<td>( c_2 )</td>
<td>A higher cost for independent directors when quitting their position in a dishonest corporation</td>
</tr>
</tbody>
</table>

### 2.2 Simulation method

Based on the previous setting of the parameters of the two models and the data provided by Kangmei Pharmaceutical Company, this research uses the simulation method to examine the evolutionary process of the behavior of an independent director and a corporation in model 1 and model 2. The simulation also involves various probabilities of a corporation and an independent director choosing their strategy. By the end of the simulation, the results will be visualized by the
evolutionary trajectories, and these trajectories can be used to analyze the decision of two participants. In addition, whether the encouragements, such as salary and reputation awards, will be a significant factor in improving the independent director system or not under the assumptions and setting of model 1 and model 2 can be tested by the simulation method. Lastly, the stability of the critical points can also be checked by the simulation method.

3. Evolutionary Stable Strategy from the Models

3.1 Model 1 Results

From Table 1 in the previous section, the payoff matrices for a corporation and independent directors in model 1 can be obtained individually as follow:

\[
\begin{bmatrix}
g_1 - l \cdot \theta_1 & g_1 - l \cdot \theta_2 \\
g_2 & g_2
\end{bmatrix}
\quad \text{and} \quad
\begin{bmatrix}
S - \beta_1 + l_1 - I_2 & S - \beta_1 + l_1 - I_2 \\
S - l_1 + I_2 - \eta \cdot \theta_2 & S - l_1 + I_2
\end{bmatrix}.
\] (1)

The expected value that a corporation is honest, \(E_{h1}\), and dishonest, \(E_{d1}\), can be separately obtained by the following matrix multiplication,

\[
\begin{bmatrix}
E_{h1} \\
E_{d1}
\end{bmatrix} =
\begin{bmatrix}
g_1 - l \cdot \theta_1 & g_1 - l \cdot \theta_2 \\
g_2 & g_2
\end{bmatrix}
\begin{bmatrix}
q_1 \\
1 - q_1
\end{bmatrix} =
\begin{bmatrix}
g_1 - l \theta_2 - q_1 l (\theta_1 - \theta_2) \\
g_2
\end{bmatrix}.
\] (2)

Then, the replicated dynamic equation for a corporation can be set up,

\[
p_1 = \frac{dp_1}{dt} = p_1 (1 - p_1) (E_{h1} - E_{d1}) = p_1 (1 - p_1) (g_1 - g_2 - l \theta_2 - q_1 l (\theta_1 - \theta_2)) [13].
\] (3)

The expected value for independent directors to be dutiful, \(E_{dutiful}\), and undutiful, \(E_{undutiful}\), can be obtained by the same method.

\[
\begin{bmatrix}
E_{dutiful} \\
E_{undutiful}
\end{bmatrix} =
\begin{bmatrix}
S - \beta_1 + l_1 - I_2 & S - \beta_1 + l_1 - I_2 \\
S - l_1 + I_2 - \eta \cdot \theta_2 & S - l_1 + I_2
\end{bmatrix}
\begin{bmatrix}
p_1 \\
1 - p_1
\end{bmatrix} =
\begin{bmatrix}
S - \beta_1 + l_1 - I_2 \\
S - l_1 + I_2 - \eta \theta_2
\end{bmatrix}.
\] (4)

Then, the replicated dynamic equation for independent directors can be set up,

\[
q_1 = \frac{dq_1}{dt} = q_1 (1 - q_1) (E_{dutiful} - E_{undutiful}) = q_1 (1 - q_1) (- \beta_1 + 2 l_1 - 2 l_2 + p_1 \eta \theta_2).
\] (5)

Based on these results, the Jacobian matrix of the evolutionary game model for model 1 can be further derived as follow,

\[
J_1(p_1, q_1) =
\begin{bmatrix}
\frac{\partial p_1}{\partial p_1} & \frac{\partial p_1}{\partial q_1} \\
\frac{\partial q_1}{\partial p_1} & \frac{\partial q_1}{\partial q_1}
\end{bmatrix} =
\begin{bmatrix}
(1 - 2 p_1) (g_1 - g_2 - q_1 l (\theta_1 - \theta_2)) & -p_1 (1 - p_1) l (\theta_2 - \theta_1) \\
q_1 (1 - q_1) \eta \theta_2 & (1 - 2 q_1) (- \beta_1 + 2 l_1 - 2 l_2 + p_1 \eta \theta_2)
\end{bmatrix}.
\] (6)

The Jacobian matrix will be used to determine the stability of a critical point in the following parts.
As replicated dynamic equation is a type of ordinary differential equation, the equilibrium of model 1 is related to its replicated dynamic equations. This is the same as solving the homogeneous system of differential equations \( \begin{cases} p_1 = 0 \\ q_1 = 0 \end{cases} \), and the solutions to this system are the equilibrium points of the model [14-15]. By solving this system of equations, it implies that \((0,0), (0,1), (1,1), (1,0), \) and \((p_1^*, q_1^*) = \left( -\frac{\beta_1 + 2l_1 - 2l_2}{\eta \theta_2}, \frac{g_1 - g_2 - l \theta_2}{l(\theta_1 - \theta_2)} \right) \) are critical points for model 1.

Additionally, the way to judge the stability of a critical point, \((x, y)\), is the same as the method to judge the stability of a critical point in a system of ordinary differential equations. Equivalently, the stability of critical points in model 1, can be determined based on the trace and determinant of the Jacobian matrix at this particular point, \( J_1(x, y) \). The results for the critical points of the model are shown in Table 4.

### Table 4. Stability for model 1

<table>
<thead>
<tr>
<th>critical point</th>
<th>( \det J_1(p_1, q_1) )</th>
<th>( \text{tr} J_1(p_1, q_1) )</th>
<th>Stability</th>
</tr>
</thead>
<tbody>
<tr>
<td>((0,0))</td>
<td>uncertain</td>
<td>uncertain</td>
<td>Saddle point</td>
</tr>
<tr>
<td>((0,1))</td>
<td>+</td>
<td>uncertain</td>
<td>ESS</td>
</tr>
<tr>
<td>((1,1))</td>
<td>-</td>
<td>uncertain</td>
<td>Saddle point</td>
</tr>
<tr>
<td>((1,0))</td>
<td>-</td>
<td>uncertain</td>
<td>Saddle point</td>
</tr>
<tr>
<td>(p_1^<em>, q_1^</em>)</td>
<td>uncertain</td>
<td>uncertain</td>
<td>Not ESS</td>
</tr>
</tbody>
</table>

Table 4 indicates that if independent directors consider being dutiful, and the corporation should be honest, then model 1 will return an optimal result.

The following provides detailed proof of the stability of critical points which is shown in Table 4.

Proof of the stability of critical points:

At \((0,0)\), \( J_1(0,0) = \begin{bmatrix} g_1 - g_2 & 0 \\ 0 & -\beta_1 + 2l_1 - 2l_2 \end{bmatrix} \),
\[
\det J_1(0,0) = (g_1 - g_2)(-\beta_1 + 2l_1 - 2l_2) < 0, \quad \text{since} \quad g_2 \ll g_1 \quad \text{and} \quad l_1 < l_2.
\]

\[
\det J_1(0,0) = (g_1 - g_2)(-\beta_1 + 2l_1 - 2l_2) < 0, \quad \text{since} \quad g_2 \ll g_1 \quad \text{and} \quad l_1 < l_2.
\]

It implies that \((0,0)\) is a saddle point.

At \((0,1)\), \( J_1(0,1) = \begin{bmatrix} g_1 - g_2 - l(\theta_1 - \theta_2) & 0 \\ 0 & -\beta_1 + 2l_1 - 2l_2 \end{bmatrix} \),
\[
\det J_1(0,1) = (g_1 - g_2 - l(\theta_1 - \theta_2))(-\beta_1 + 2l_1 - 2l_2) < 0, \quad \text{since} \quad g_2 \ll g_1 \ll l, \theta_2 << \theta_1 \quad \text{and} \quad l_1 < l_2 \quad \text{(by simulation setting,} \quad g_1 - g_2 - l(\theta_1 - \theta_2) < 0, \quad -\beta_1 + 2l_1 - 2l_2 < 0)\]

Since \( \text{tr} J_1(0,1) = g_1 - g_2 - l(\theta_1 - \theta_2) + -\beta_1 + 2l_1 - 2l_2 < 0 \),
The critical point \((0,1)\) is a stable equilibrium.

Hence, \((0,1)\) is an ESS.

At \((1,1)\), \( J_1(1,1) = \begin{bmatrix} -(g_1 - g_2 - l(\theta_1 - \theta_2)) & 0 \\ 0 & -(\beta_1 + 2l_1 - 2l_2 + \eta \theta_2) \end{bmatrix} \),
\[
\det J_1(1,1) = (g_1 - g_2 - l(\theta_1 - \theta_2))(-\beta_1 + 2l_1 - 2l_2 + \eta \theta_2) < 0, \quad \text{since} \quad g_2 \ll g_1 \ll l, \theta_2 << \theta_1 \quad \text{and} \quad l_1 < l_2, \quad \text{(by simulation setting} \quad -(\beta_1 + 2l_1 - 2l_2 + \eta \theta_2) > 0)\]

It shows that \((1,1)\) is a saddle point.
At \((1,0)\), \(J_1(1,0) = \begin{bmatrix} -(g_1 - g_2) & 0 \\ -\beta_1 + 2I_1 - 2I_2 + \eta \theta_2 \end{bmatrix}\) (10)

\[\text{det} J_1(1,0) = \left(-(g_1 - g_2) \right)\left(-\beta_1 + 2I_1 - 2I_2 + \eta \theta_2 \right) < 0, \text{since } g_2 \ll g_1 \ll l, \theta_2 \ll \theta_1 \text{ and } I_1 < I_2 \ll \eta. \text{by simulation setting }\]

\[\left(-\beta_1 + 2I_1 - 2I_2 + \eta \theta_2 \right) > 0.\]

It implies that \((1,0)\) is a saddle point.

At \((p_1^*, q_1^*) = \left(-\beta_1 + 2I_1 - 2I_2 + \eta \theta_2 \right)\), \(\text{det} J_1(p_1^*, q_1^*)\) cannot be determined, since the point \((p_1^*, q_1^*)\) involves several factors, and the value will be oscillating a between negative and positive number.

### 3.2 Model 2 Results

From Table 2 in the previous section, the payoff matrices for a corporation and independent directors in model 2 can be obtained individually as follow:

\[
\begin{bmatrix}
g_1 - l \cdot \theta_1 \\
g_2 - c_2
\end{bmatrix}
\begin{bmatrix}
g_1 - c_1 - l \cdot \theta_1 \\
g_2 - c_2
\end{bmatrix}
\begin{bmatrix}
l \theta_1 - (1 - q_2) c_1 \\
l \theta_2 - (1 - q_2)
\end{bmatrix}
\]

The expected value for corporation to be honest, \(E_{h2}\), and dishonest, \(E_{d2}\), can be derived by matrix multiplication as follow,

\[
\begin{bmatrix}
E_{h2} \\
E_{d2}
\end{bmatrix} = \begin{bmatrix}
g_1 - l \cdot \theta_1 \\
g_2 - c_2
\end{bmatrix} \begin{bmatrix}
g_1 - c_1 - l \cdot \theta_1 \\
g_2 - c_2
\end{bmatrix} \begin{bmatrix}
l \theta_1 - (1 - q_2) c_1 \\
l \theta_2 - (1 - q_2)
\end{bmatrix}
\]

Then, the replicated dynamic equation for corporation can be set up,

\[
p_2 = \frac{dp_2}{dt} = p_2 \cdot (1 - p_2) \cdot (E_{h2} - E_{d2}) = p_2 \cdot (1 - p_2) \cdot (g_1 - g_2 - l \theta_1)[13].
\]

Similarly, the expectation for independent directors to continue, \(E_{\text{continue}}\), and resign, \(E_{\text{resign}}\), can be obtained as follow,

\[
\begin{bmatrix}
E_{\text{continue}} \\
E_{\text{resign}}
\end{bmatrix} = \begin{bmatrix}
l \theta_1 - (1 - q_2) c_1 \\
l \theta_2 - (1 - q_2)
\end{bmatrix} \begin{bmatrix}
l \theta_1 - (1 - q_2) c_1 \\
l \theta_2 - (1 - q_2)
\end{bmatrix} = \begin{bmatrix}
l \theta_1 - (1 - q_2) c_1 \\
l \theta_2 - (1 - q_2)
\end{bmatrix}
\]

Then, the replicated dynamic equation for independent directors can be set up,

\[
q_2 = \frac{dq_2}{dt} = q_2 \cdot (1 - q_2) \cdot (E_{\text{continue}} - E_{\text{resign}}) = q_2 \cdot (1 - q_2) \cdot (S + l_1 - l_2 - p_2 \beta_2 - \beta_1 (1 - p) - c_2 p_2).
\]

Based on these results, the Jacobian matrix of the evolutionary game model for model 2 can be further derived as follow,

\[
J_2(p_2, q_2) = \begin{bmatrix}
\frac{\partial p_2}{\partial p_2} & \frac{\partial p_2}{\partial q_2} \\
\frac{\partial q_2}{\partial p_2} & \frac{\partial q_2}{\partial q_2}
\end{bmatrix} = \begin{bmatrix}
(1 - 2p_2)(g_1 - g_2 - l \theta_1) \\
(q_2 - q_2^2)(\beta_1 - \beta_2 - c_2) + (1 - 2q_2)(S + l_1 - l_2 - p_2 \beta_2 - \beta_1 (1 - p) - c_2 p_2)
\end{bmatrix}
\]

(16)
Similar to the analysis for model 1, the equilibrium of model 2 can be solved by considering:

\[ p_2 = 0 \quad q_2 = 0 \]

By solving this system of equations, it implies that (0,0), (0,1), (1,1), and (1,0) are critical points for model 2. Likewise, the Jacobian matrix of model 2, \( J_2 \), will be used to determine the stability of these critical points. The results are shown in Table 5.

**Table 5. Stability for model 2**

<table>
<thead>
<tr>
<th>critical point</th>
<th>( \text{det} J_2(p,q) )</th>
<th>( \text{tr} J_2(p,q) )</th>
<th>Stability</th>
</tr>
</thead>
<tbody>
<tr>
<td>(0,0)</td>
<td>-</td>
<td>uncertain</td>
<td>Saddle point</td>
</tr>
<tr>
<td>(0,1)</td>
<td>+</td>
<td>-</td>
<td>ESS</td>
</tr>
<tr>
<td>(1,1)</td>
<td>-</td>
<td>uncertain</td>
<td>Saddle point</td>
</tr>
<tr>
<td>(1,0)</td>
<td>+</td>
<td>+</td>
<td>Not ESS</td>
</tr>
</tbody>
</table>

Table 5 exhibits that the optimal outcome of model 2 is that independent directors should consider continuing their position in their corporation, while the corporation should consider being honest.

The following is the detailed proof of the stability of critical points which is shown in Table 5.

**Proof of the stability of critical points:**

\[
\text{At } (0,0), J_2(0,0) = \begin{bmatrix} g_1 - g_2 - l \theta_1 & 0 \\ 0 & S + I_1 - I_2 - \beta_1 \end{bmatrix},
\]

\[
\text{det} J_2(0,0) = (g_1 - g_2 - l \theta_1)(S + I_1 - I_2 - \beta_1) < 0
\]

Under the model and simulation setting \( g_1 - g_2 - l \theta_1 > 0, S + I_1 - I_2 - \beta_1 < 0 \).

\[
\text{At } (0,1), J_2(0,1) = \begin{bmatrix} g_1 - g_2 - l \theta_1 & 0 \\ 0 & -(S + I_1 - I_2 - \beta_1) \end{bmatrix}
\]

By similar reasons as above,

\[
\text{det} J_2(0,1) = (g_1 - g_2 - l \theta_1)(-(S + I_1 - I_2 - \beta_1)) > 0 \text{ and } \text{tr} J_2(0,1) < 0
\]

Since \( \text{tr} J_2(0,1) < 0 \), then the critical point will be a stable equilibrium. Hence, (0,1) is an ESS.

\[
\text{At } (1,1), J_2(1,1) = \begin{bmatrix} -(g_1 - g_2 - l \theta_1) & 0 \\ 0 & -(S + I_1 - I_2 - \beta_2 - c_2) \end{bmatrix}
\]

Since \( \text{det} J_2(1,1) = (g_1 - g_2 - l \theta_1)(S + I_1 - I_2 - \beta_2 - c_2) < 0 \),

(1,1) is a saddle point

\[
\text{At } (1,0), J_2(1,0) = \begin{bmatrix} -(g_1 - g_2 - l \theta_1) & 0 \\ 0 & S + I_1 - I_2 - \beta_2 - c_2 \end{bmatrix}
\]

By the setting, it is obvious that \( \text{det} J_2(1,0) > 0 \) and \( \text{tr} J_2(0,1) > 0 \). Then, the equilibrium of the critical point (1,0) is unstable. Hence, it is not an ESS.

### 3.3 Simulation Results

The evolutionary trajectories illustrate the evolutionary trend and process under the different beginning values of probability. If there exists a stable equilibrium in the evolutionary game, then the trajectories will gradually achieve the point of stable equilibrium; meanwhile, the trajectory starting from a saddle point will evolve to the point of stable equilibrium [14]. Hence, the stability of a critical point can be determined by the behavior of the trajectory.

The simulation result for model 1 is shown in the following figure.
In this figure, the horizontal axis means the probability $p_1$ of a corporation that takes \{dishonest\} strategy, and the vertical axis refers to the probability $q_1$ of independent directors who is dutiful. It is equivalent that \{dishonest, dutiful\} is an evolutionary stable strategy. All the lines are the trajectory of this dynamic system with a certain initial value $(p_1, q_1)$.

Thus, it illustrates that a reasonable amount of encouragement, such as salary awards, will stimulate independent directors to be dutiful while the corporation will not have any incentive to consider financial fraud. Reputation award can also encourage independent directors to be dutiful, since good reputation will help independent directors in their long-term career.

The simulation result for model 2 is shown in the following figure.

In this figure, the abscissa axis means the probability $q_2$ of a corporation that takes \{dishonest\} strategy, and the vertical axis refers to the probability $q_2$ of independent directors who continue their jobs. All the lines are trajectories of this dynamic system with certain initial values $(p_2, q_2)$.

Thus, it shows that encouragement, such as salary awards, can prevent independent directors from resigning, and it also helps to prevent financial fraud. The excellent reputation of a corporation can help independent directors keep working in their company.
4. Conclusion

4.1 Key findings

Based on the results generated by the models and simulation results, it suggests two ways to improve the independent director system in the followings. Firstly, setting up the mechanism of awards in a corporation for independent directors is indispensable. Salary awards and stock awards can encourage independent directors to be dutiful and not to quit their position. Then, the possibility of a corporation committing financial fraud will also be reduced effectively. For instance, if the corporation does not provide any salary award to independent directors, it will likely commit financial fraud, since the independent directors have low motivation for being dutiful and may even quit their position. Then when they are fined for their dishonesty, the amount they need to pay is much higher than the salary award. Hence, a corporation’s award mechanism is beneficial to the independent directors and constructive to the corporation. Secondly, completing the reputation mechanism for independent directors is necessary. The securities supervision commission should set up a database for recording independent directors’ information. Dutiful independent directors should be awarded publicly, while undutiful independent directors should be exposed to the public. Then, it will be easier for the corporation to recruit high-quality independent directors.

Moreover, the reputation mechanism can ensure the overall qualification of the independent directors since corporations prefer prestigious independent directors, which will encourage independent directors to be dutiful. As dutiful independent directors will be strict in supervision, the corporation will be less likely to commit financial fraud. Therefore, the reputation mechanism is valuable to both corporations and independent directors.

4.2 Limitations and Further Studies

Although the two models in this research can suggest some solutions to the issue between corporate governance and the independent director system under some constraints, real-world problems are often more complicated and non-linear, and it even involves some random factors and uncontrollable variable. It is logical to assume that the stochastic game has a higher probability of giving more realistic and long-term solutions to the problem. It is worth solving the problem between a corporation and an independent director from the aspect of a stochastic game, and the method to obtain stable strategies is almost the same as solving a system of stochastic differential equations. To further improve the independent director system and address the corporation’s financial fraud, several stochastic variables need to be included, such as the strictness of government’s supervision and risk factors from financial market. The third participant, government or financial market, may also need to be included so that the model will be more realistic to current situation.

References