

Market share forecast analysis of three major operators based on Markov chain

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Abstract. Reasonable forecast products market share is the enterprise marketing decisions, one of the important factors which should be considered on the basis of three carriers in China mobile users, with the help of the theory of markov prediction model of market share, the change trend in maintaining market stability, the change of the future market share, the results show that the The results show that the market share of mobile users of China Mobile, China Unicom and China Telecom will be stable at 60.0%,24.2% and 15.8% in future.

Keywords: Market share; Markov chain; Market forecast.

1. Introduction

In recent years, China's communication industry has developed rapidly. As of May 2018, the number of mobile users in China has reached about 1.478 billion, among which China Mobile accounts for about 902 million, China Unicom about 300 million, and China Telecom about 276 million. However, in the upcoming 5G era, it is very important for each operator to predict the market share of their products.

There are few stability analyses on the market share of the three major carriers, but the application of Markov chain theory has been very mature, so this paper will use Markov model to explore the stability of the market share of the three major carriers. Markov chain is often used in the frontier research in China. Markov chains have been used in education, economics, medicine, natural disaster prediction, genetics and algorithm effectiveness. Markov chain is often used to evaluate the teaching effect of public teachers in the field of education. Markov chain is used in the economic field for stock price regression, which can be used to analyze and predict the stock price [1]. Markov chain is used in medicine to predict B encephalitis. Markov chain has been used to predict drought in south Ningxia during natural disasters and so on. In this paper, Markov chain is used to predict the market share of China's three major operators.

2. The basic theory of Markov model prediction

Markov chains are discrete-time stochastic processes with Markov properties. In a Markov chain, the system can either stay in its current state or transition from one state to another according to a probability distribution [2]. Suppose that the discrete state space of a random sequence $\{X(n), n = 1, 2, \dots\}$ is D . For any m nonnegative integers $n_1, n_2, \dots, n_m$ and the natural number K , and any j satisfies $i_1, i_2, \dots, i_m \in D$

$$P\{X(n_m + K) = j | (n_1) = i_1, X(n_2) = i_2, \dots, X(n_m) = i_m\} \quad (1)$$

$$P\{X(n_m + K) = j | X(n_m) = i_m\} \quad (2)$$

Then the random sequence $\{X(n), n = 1, 2, \dots\}$ is a Markov chain. The initial distribution and transition probability matrix need to be used in the process of Markov model prediction.

2.1 Generation principle of transition probability matrix

In the change process of a period of events, the possibility of starting from a state at the beginning of the period and transferring to other states at the end of the period is called the state transition probability [3]. According to the definition of conditional probability, the transition probability $P()$ from a state to another state is the conditional probability P , as shown in following.

$$p(s_i \rightarrow s_j) = p(s_j | s_i) = p_{ij} \quad (3)$$

Since the probability P_1 is nonnegative and the process must be transferred to some state, there is,

$$\begin{cases} 0 \leq p_{ij} \leq 1 & (i, j = 1, 2, \dots, n) \\ \sum_{j=1}^n p_{ij} = 1 & (i = 1, 2, \dots, n) \end{cases} \quad (4)$$

In order to remember the matrix of one-step transition probabilities, thus:

$$P = \begin{bmatrix} P_{00} & \cdots & P_{0n} \\ \vdots & \ddots & \vdots \\ P_{n0} & \cdots & P_{nn} \end{bmatrix} \quad (5)$$

One-step transition probability matrix, abbreviated as transition probability matrix.

2.2 Solution method of transition probability matrix

2.2.1 Theoretical solution method of transition probability matrix

Markov prediction model is used to predict the state transition probability of system objects by studying it[4]. Therefore, the determination of state transition probability becomes the key to the prediction of Markov model. More accurate calculation of Markov state transition probability matrix can improve the accuracy of Markov prediction[5]. At present, many scholars have done a lot of research on the solution method of the state transition probability matrix, aiming at applying the Markov prediction model in various aspects through effective methods, or improving the prediction accuracy of the Markov prediction model. Through literature review, it is found that the current methods of solving the state transition probability matrix can be divided into the following three categories: first, using statistical methods to estimate the state transition matrix[6]; Second, the state transition matrix is estimated by linear equations; Third, the state transition matrix is estimated by quadratic programming method.

2.2.2 Actual solution method of transition probability matrix

The establishment steps of transition probability matrix are as follows: First, conduct investigation and research to obtain recent data[7]. Second, the establishment of the market (customer) gain and loss of the transition probability matrix.

In this paper, the practical solution method of transition probability matrix is used to solve it.

3. Market share forecast and analysis theory

When Markov chains are used for market share forecasting, it is possible to predict its short-term and long-term market share.

3.1 Short term market share forecast steps

Assume the state space, $I = \{1, 2, \dots, N\}$, and let the system have N incompatible states and the initial state vector of the system be

$$P(0) = [P_1(0), P_2(0), \dots, P_j(0), \dots, P_s(0)] \quad (6)$$

Where $P_s(0)$ is the initial probability of the system in state j .

Assuming that the probability that the system is in state j after k transitions is $P_k(j)P_k(j)\{X_n = j\}$, then at time K , the probability of each state is

$$P(k) = (p_k(1), p_k(2), \dots, p_k(N)) \quad (7)$$

Where $P_k(j)$ is the probability that system k is in state j at time k ,

Thus, the Markov prediction model is: $P(k) = P(0) \cdot p_k$

Where, at time n , the probability of each state is equal to the product of the probability of its initial state and the n -step transition probability matrix. If the Markov chain is homogeneous, that is, the one-step transition probability p_1 of the Markov chain is independent of the initial time, then:

$$P(k) = P(0) \cdot p_1^k = P(0) \begin{bmatrix} p_{11} & \cdots & p_{1n} \\ \vdots & \ddots & \vdots \\ p_{n1} & \cdots & p_{nn} \end{bmatrix}^k \quad (8)$$

The above formula can be used when predicting the state vector at time K.

3.2 Stability analysis of market share

According to the ergodic theorem, the existence of stationary distribution of Markov chain is conditional on the Markov chain being ergodic. Only when the stationary distribution of Markov chain is known can the market share, that is, the long-term market share, be analyzed.

3.2.1 Ergodicity of Markov chain

From the point of view of quantitative relations, the development of a physical system can be approximated (under certain conditions) as a stochastic process. When there is no significant change in the causes affecting the development of the system, a physical system always reaches some stationary state after a period of time. In other words, the probability of a system being in a certain state is independent of its situation in the distant past [8]. Using mathematical theory to reveal the inner law of this phenomenon is of profound significance. This law is called "ergodic property" in stochastic process theory [9]. The ergodic property of Markov chains, that is, an important topic in Markov chain theory is to discuss the limit property of n-step transition probabilities when $n \rightarrow \infty$. The definition of ergodicity is $p_{ij}(n)$, let the state space of Markov chain $\{X_n, n \geq 0\}$ be I, if for all $i, j \in I$, there exists a constant that does not depend on i such that:

$$\lim_{n \rightarrow \infty} p_{ij}^{(n)} = \pi(j) \quad (9)$$

Then the Markov chain is said to have ergodicity, where $p_{ij}^{(n)}$ is the n-step transition probability of the Markov chain. The ergodicity of the Markov chain shows that the probability of transition to state j is close to the normal number $\pi(j)$ when the number of transition steps n is sufficiently large, no matter which state i starts from.

3.2.2 Stationary distribution of Markov chains

In theory, Gerschgorin's disk theorem is often used. Gerschgorin's disk theorem: All eigenvalues of the matrix $A = (a_{ij})_{n \times n} \in \mathbb{C}^{n \times n}$ are in the union of its N Gerschgorin circles. The matrix P satisfies $\sum_{j=1}^s p_{ij} = 1 (i = 1, 2, 3, \dots, s) \leq p_{ij} \leq 1$. We know that there must be an eigenroot $\lambda = 1$ and its corresponding nonnegative eigenvector for the matrix P, and λ is the positive eigenroot with the largest modulus. Other eigenroots of matrix P are all less than 1, which satisfy: $\lim_{t \rightarrow \infty} \lambda_i^t = 0, |\lambda_i| < 1$.

In practice, if every element of the K-step transition matrix P is greater than 0, then the Markov chain has ergodic property. If the transition probability matrix remains unchanged, the state probability will be in a stable state after multiple transitions, and the value of this stable state is independent of the initial state [10]. Let's say the limiting distribution is $(\pi(0), \pi(1), \dots, \pi(s))$. Then in this limiting distribution $\pi(j)$ is a system of equations $\pi(j) = \sum_{i=0}^s \pi(i) p_{ij}$ $j=0, 1, 2, \dots, N$, which satisfy the condition of $\pi(j) > 0, \sum_{j=0}^s \pi(j) = 1$. This conclusion gives a method to find the stationary distribution $\pi(j)$.

4. Analysis of the market share of mobile users of carriers

Operational data of the three major operators in December 2020. From the perspective of mobile service subscribers of the three major carriers, China Mobile is still far ahead, with a total of 942 million subscribers, while China Telecom has 351 million subscribers. And China Unicom, with 306 million subscribers, is still far behind China Mobile. Through the above data, the initial distribution can be obtained as $P(0) = (0.589, 0.220, 0.191)$.

In 2020, China Mobile had a cumulative net loss of 8.359 million subscribers, China Unicom had a cumulative net loss of 12.664 million subscribers, and China Telecom had a net gain of 15.45 million subscribers, becoming the only carrier to achieve positive growth in 2020. Overall, the total number of mobile subscribers is decreasing, which also indicates that the number of mobile subscribers is gradually saturated. The numbers for the three carriers combined represent 5.573 million fewer subscribers than in 2019. According to the data, some (about 73.5 percent) of consumers who used China Mobile and China Unicom switched to China Telecom in 2020. Assuming a proportional distribution, about 6.143 million customers switched from China Mobile to China Telecom, and about 9.308 million customers switched from China Unicom to China Telecom. The one-step transition probability matrix can be obtained by reducing the amount and transfer amount.

$$P_1 = \begin{bmatrix} 0.991 & 0 & 0.009 \\ 0 & 0.973 & 0.027 \\ 0 & 0 & 1 \end{bmatrix}$$

According to the one-step transition probability matrix, China Unicom has the lowest viscosity, while China Telecom has the highest viscosity.

4.1 Short-term forecast of market share

The initial distribution and one-step transition probability matrix are used to forecast the market share in 2021 with the formula $P(1)=P(0)P_1$. Putting the data into the formula, $P(1)=(0.584, 0.214, 0.202)$

The initial distribution and one-step transition probability matrix are used to predict the market share in 2023. If the Markov chain is homogeneous, $P(3)=P(0)$ is used to predict the market share. $P_1^3(3)=P(0.573, 0.203, 0.224)$.

According to the short-term forecast of market share, China Mobile has the highest market share among the three e-commerce companies, while China Unicom has improved its market share.

4.2 Long-term forecast of market share

Due to the small amount of information in the data, the transition probability matrix is always unable to satisfy that all elements are non-zero elements. This means that the market share cannot be predicted in the long term. According to the existing literature data and adjust the data appropriately, the following data can be obtained: of the 100 people who used China Mobile, 89 continue to use mobile communication, 5 switch to Unicom users, and 6 switch to telecom users; Among the 100 people who used China Unicom Communications, 81 people continue to use China Unicom Communications, 14 people transfer mobile users, and 5 people convert to telecom users; Among the 100 people who used to use telecom communication, 70 people continued to use telecom communication, 20 people converted to mobile users, and 10 people converted to Unicom users.

The one-step transition probability matrix is

$$P = \begin{bmatrix} 0.89 & 0.05 & 0.06 \\ 0.14 & 0.81 & 0.05 \\ 0.20 & 0.10 & 0.70 \end{bmatrix}$$

Every element of the one-step transition matrix P is greater than 0, so this Markov chain has ergodic property. If the transition probability matrix remains unchanged, the state probability will be in a stable state after multiple transitions, and the value of this stable state is independent of the initial state. Assume that the limit distribution is (π_1, π_2, π_3)

$$\begin{cases} \pi_j = \sum_{i=1}^3 \pi_i P_{ij} \\ \sum_{j=1}^3 \pi_j = 1 (i, j = 1, 2, 3) \end{cases} \quad (10)$$

That is:

$$\begin{cases} 0.89\pi_1 + 0.14\pi_2 + 0.20\pi_3 = \pi_1 \\ 0.05\pi_1 + 0.81\pi_2 + 0.10\pi_3 = \pi_2 \\ 0.06\pi_1 + 0.05\pi_2 + 0.70\pi_3 = \pi_3 \\ \pi_1 + \pi_2 + \pi_3 = 1 \end{cases} \quad (11)$$

$$\begin{cases} \pi_1 = 0.600 \\ \pi_2 = 0.242 \\ \pi_3 = 0.158 \end{cases}$$

It can be concluded that under the condition that the transition probability matrix remains unchanged, the market share of mobile users of the three carriers is stable at (0.600,0.242,0.158) after multiple selection.

5. Evaluation of Markov analysis method

From the initial distribution, transition probability matrix, short-term prediction, long-term prediction, the evaluation of Markov method for prediction is carried out.

When solving the initial distribution, the mean of the ratio of users of each carrier in several years (for example, 2009-2018) can be considered as the carrier's base period market share. In this paper, only the data from 2020 are used to make short-term forecasts for 2021 and 2023 and long-term forecasts for the future. The accuracy of the short-term forecast and long-term forecast of market share may be affected due to the selection of data.

When solving the transition probability matrix, it is difficult to obtain the transition probability, so the practical method should be combined with the theoretical method to solve the transition probability matrix.

Short-term prediction is based on the premise that the one-step transition probability matrix or n-step transition probability matrix is invariant. However, in real life, if the Markov chain is assumed to be a homogeneous chain, consumers' tendency to choose operators is not only related to the first choice behavior, but also may be related to several previous choices P_1 . At the same time, if the formula " $P(n) = P(0) \cdot p_n$ " is used, the tendency of consumers to choose operators is not only related to the previous choice behavior.

Fourthly, the long-term prediction is based on the assumption that the state transition probability matrix of the system is constant. However, in real life, it is difficult to ensure the stability of transition probability matrix due to the impact of new products of major operators on consumers' purchase intention to a certain extent, which increases the error of forecasting range. When Markov chain is used for long-term prediction, it is necessary to modify the one-step transition probability matrix in time.

6. Conclusion

Analyze the results for the three carriers. In both short-term and long-term forecasts, China Mobile always has the highest share. It can be concluded from the above examples that, under the condition that the transition probability matrix of the market change trend remains unchanged, no matter how the market share changes in the initial state, the market will reach a stable state, that is, the market share tends to be stable. By increasing the one-step transition probability, the product market share can be increased. Therefore, operators can improve the transfer rate by strengthening various measures in operation and management, timely grasp the information of various demands of consumers, and provide various service products to meet the needs of different consumers from person to person, and improve customer satisfaction.

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