Comparative Analysis of the Application of Monte Carlo Model and BSM Model in European Option Pricing

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Abstract. At present, the expansion of China's domestic options market brings positive factors and risks, and in order to avoid risks, it is crucial to choose a suitable model for option pricing. This article provides an example of an option for the underlying asset of the SSE 50 ETF. Using the BSM model and the Monte Carlo model for the selected option pricing, and comparing the actual option price. It is found that the pricing efficiency of the Monte Carlo model is higher than that of the BSM model when the number of simulations reaches 30,000 times in the call option, and there is little difference between the two in the put option pricing. It is recommended to prefer the Monte Carlo model when pricing call options and the more convenient BSM model when pricing put options.

Keywords: Monte Carlo model; BSM model; Option pricing.

1. Introduction

As a financial derivative product, an option is a contract that gives a holder the right to buy or sell an asset at a fixed price at any time before a specific date [1]. It can be divided into call options and put options. In 2015, China listed the first exchange-traded stock index futures, and the underlying object was the SSE 50ETF. The options market development report provided by the Shanghai Stock Exchange in 2018 showed that the 50ETF has become one of the world's major ETF options [2]. While the options market is driving the rapid development of China's economy, it also has risks and various problems, such as a single option variety, few options professionals, and high market access thresholds. The problem of pricing deviation caused by the relative singleness of option products is particularly significant [3].

In order to better utilize the benefits of options and avoid the risks brought about by them, it is more important to price options reasonably through known information [4]. There are several models for pricing options, for example, Cox Binomial Option Pricing Model [5] Guided Stochastic Differentiation, Black Scholes Model, Heston Model and Monte Carlo Model [6]. Various models provide reasonable methods for option pricing, However, the degree of deviation between the pricing results of different models and the actual price varies. At present, the BSM model has been widely used in the field of option pricing [7]. In 1977, Boyle applied the Monte Carlo simulation method to option pricing for the first time, and found that this method can perform well in pricing European options [8].

This paper takes the Monte Carlo model and the BSM model as the research objects, by actually selecting stocks and recording their options and comparing them with the predicted option prices. Through the degree of deviation analysis, it is found that in the pricing of call options, the pricing efficiency of the Monte Carlo model is significantly higher than that of the BSM model when the number of simulations is 30,000, while there is little difference between the two for put options.

2. Model Introduction

2.1 BSM Model

The full name of the BSM model is Black-Scholes-Merton Option Pricing Model [9]. This model is a generalization of the binary tree model in the case of continuous change. When applying the model, it should be noted that. Before the option is executed, no dividends are paid for the underlying asset; the market is frictionless, i.e. there are no taxes and transaction costs, and all securities are completely divisible; during the term of the option, the risk-free interest rate and the return on financial assets are constant; investors are able to borrow at risk-free rates; call options can only be exercised on the expiration date; securities trading is continuous, and the stock price walks randomly [10]. When the model is applied to European options, the option price can be obtained by the following formula:

$$\mathrm{C}\big(\mathrm{t},\mathrm{S}(\mathrm{t})\big)=\mathrm{S}(\mathrm{t})\mathrm{N}(\mathrm{d}_1)-\mathrm{K}e^{-\mathrm{r}(\mathrm{T}-\mathrm{t})}\mathrm{N}(\mathrm{d}_2) \tag{1}$$

Where

$$N(x)=P[Z\le x]=\frac{1}{\sqrt{2\pi}}\int_{\infty}^{x}e^{-\frac{y^2}{2}}dy \tag{2}$$

$$d_1=\frac{1}{\sigma\sqrt{T-t}}\left[log\left(\frac{S(t)}{K}\right)+\left(r+\frac{\sigma^2}{2}\right)(T-t)\right] \tag{3}$$

$$d_2=\frac{1}{\sigma\sqrt{T-t}}\left[log\left(\frac{S(t)}{K}\right)+\left(r-\frac{\sigma^2}{2}\right)(T-t)\right] \tag{4}$$

Table 1. Symbol Comparison table

Symbol	Meaning
C	call option price
P	put option price
S_0	The price of the underlying asset at initial time 0
K	strike price of the option
r	Continuously compounded risk-free rate
σ	Annualized Volatility of Underlying Asset Price Percentage (Yield)
T	Option contract duration (years)
$N(*)$	Probability Density of Cumulative Standard Normal Distribution

2.2 Monte Carlo Model

Monte Carlo simulation is one of the random simulation algorithms, and its essence is to use the sample mean of random sampling to approximate the overall expectation of the random distribution [11]. The basic idea is to simulate as much as possible the multiple paths of the bid asset price in the risk-neutral world, calculate the average option return under each path, and finally get the option price by discounting. Taking The European option as the research object, the random path of the underlying asset is sampled in the risk-neutral world, and an implementation of the underlying asset price path is given ; calculate the option return on this path; repeat the above two steps to get many sample results, that is, the option return value in a risk-neutral world; average these returns to get the expected option return value in a risk-neutral world; discount at the risk-free rate to get the estimated value of the option; in the specific operation, first define the time range and step size according to the due date to specify the total number of simulation paths; simulate the change path of stock price, because the change of stock price can be regarded as Brownian motion.

$$dS_t=b(t,S_t)dt+\sigma(t,S_t)dW_t \tag{5}$$

The parameters b and σ can be constants or specific functions. Combined with the time division method in the previous step, the stock price path can be obtained as

$$S_h^h\coloneqq S\big(t_h^h\big)=S_{n-1}^h+b\big(t_h^h,S_{n-1}^h\big)h+\sigma\big(t_h^h,S_{n-1}^h\big)\Delta W_n^h,\Delta W_n^h=\sqrt{h}Z_n^h,Z_n^h{\sim}N(0,1) \tag{6}$$

Since the research object is European options, the exercise price is K. If the stock price is higher than K, the buyer has the right to buy the stock at the price of K and sell it at the market price, and the income is $S-K$.

$$C_n^h=(S_n^h-K)^+ \tag{7}$$

The option price is the average value of the discounted earnings of all path options

$$C=\frac{1}{N}\sum_{i=1}^{M}e^{-rT}(S_i(T)-K)^+ \tag{8}$$

A total of M paths are simulated, and i is a different path.

3. Pricing Analysis

Select the options on SSE 50ETF. China's stock options currently include SSE 50ETF options (with 50ETF as the target), CSI 300 index options (with the CSI 300 index as the target), CSI 300 ETF options (the Shanghai Stock Exchange with Huatai Borui CSI 300 as the target, Shenzhen Stock Options with Harvest CSI 300 as the target). Among them, the SSE 50ETF option listed in China and is exercised on the expiration date. It is a European option and meets the requirements of the research object.

3.1 Estimated Call Option Price

3.1.1 Data

Select the call option with the option code 10004243 and the expiration date is December 28, 2022 and the strike price is 2800. According to data provided by Oriental Fortune.com. The volatility of the SSE 50ETF in the past year was 19.95%, and the stock price on July 5, 2022 was 3.047. The interest rate of China's short-term treasury bills is the risk-free rate. At present, the one-year interest rate of China's one-year government bond is 3.14%. Time is 0.487 years.

3.1.2 Methodology

Using Excel to calculate the BSM model

According to the prediction method of the BSM model above, the price of the call option calculated through the excel sheet is 0.34043 and the actual latest transaction price of the day is 0.3298. The price is lower than the actual price. It can be seen from the following equation calculations. [12]

$$\text{deviation}=\frac{|\text{actual price }-\text{ theoretical price}|}{\text{theoretical price}} \tag{9}$$

The eviation of the call option is calculated to be 3.123%. It may be due to the inaccurate volatility provided by the website, the unsatisfactory assumption of continuous stock price changes, etc [13].

Using Python to calculate the Monte Carlo model

The data is the same as above, and the price of the call option under different simulation times is obtained by simulating the stock price change path multiple times through python. The table below shows the specific values.

Table 2. Call Option price

Number of simulations	Call option price
1000	0.34443
5000	0.34278
10000	0.34000
15000	0.33135
20000	0.34339
25000	0.34426
30000	0.33473

Comparing with the actual price of 0.3298 and calculating the degree of deviation, the table below can be obtained.

Table 3. Deviation

Number of simulations	Deviation
1000	0.042476
5000	0.037867
10000	0.03
15000	0.004678
20000	0.039576
25000	0.042003
30000	0.014728

It can be clearly seen that with the increase of the number of simulations, the deviation of the predicted value decreases, and it can be concluded that the more the number of simulations, the more accurate the prediction of the Monte Carlo model.

3.2 Estimated put option price

3.2.1 Data

Select the put option with the option code 10004252 dated December 28, 2022 with an exercise price of 2800. The rest of the data are the same as those in 3.1.1.

3.2.2 Methodology

Using Excel to calculate the BSM model

The method is the same as above, and the put option price is calculated through the excel sheet as 0.052826 and the actual latest transaction price of the day is 0.0760. Compare with the actual price, the price is low. According to the formula, the deviation degree of the call option is calculated to be 30.49%. Compared with the call option, the degree of deviation increases significantly, and it can be concluded that the BSM model is better than the put option in predicting the call option.

Using Python to calculate the Monte Carlo model

The data is the same as above, and the price of the call option under different simulation times is obtained by simulating the stock price change path multiple times through python.

Table 4. Put Option price

Number of simulations	Put option price
1000	0.04748
5000	0.04890
10000	0.05063
15000	0.05104
20000	0.04938
25000	0.04951
30000	0.05124

Compared with the actual price of 0.0760, and calculating the degree of deviation, the table below can be obtained.

Table 5. Deviation

Number of simulations	Deviation
1000	0.60067
5000	0.55419
10000	0.50109
15000	0.48903
20000	0.53908
25000	0.53504
30000	0.48322

Similar to the call option, it can be clearly seen that with the increase of the number of simulations, the deviation of the predicted value decreases. It can be concluded that the more the number of simulations, the more accurate the prediction of the Monte Carlo model. But like the BSM model, the deviation is significantly higher compared to the Monte Carlo model predicting call options.

4. Conclusion

Through the practical application of the two option pricing models in the SSE 50ETF call and put options, combined with the theoretical differences between the two option pricing models, the following conclusions are drawn.

In the comparison of call options, it can be found that when the number of Monte Carlo simulations reaches 30,000, the degree of deviation of the option price is significantly smaller than that predicted by the BSM model. Through a large number of computer simulations, the unknown variables in the prediction model can be accurately estimated. Due to the way in which the unknown quantity is determined in the BSM model, the volatility accuracy of the model is limited and lacks consideration of people's trading psychology. These irrational factors cause the difference between the calculated data and the real data [14], which makes the Monte Carlo model more accurate.

In the comparison of put options, it can be found that although the Monte Carlo model has a downward trend in the degree of deviation when the number of simulations increases, the degree of deviation is greater than that of the option price under the BBSM model at the same level of simulation times as the call option. The degree of deviation of the two is significantly higher than that of the call option. It can be seen that the performance of the two in the call option pricing is better than that in the put option pricing.

According to the above analysis, in call options, when Monte Carlo simulated a large number of data, the pricing efficiency is higher than that of the BSM model. It is recommended that relevant practitioners choose the Monte Carlo model for option pricing or price prediction. For put options, since neither of the two pricing models showed obvious advantages in this study, it is recommended that practitioners choose the relatively simple BSM model.

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