Chooser Option Pricing based on Black-Scholes Model and Monte-Carlo Simulations

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Abstract. Since February 2022, the intensification of the Ukrainian conflict has triggered significant impacts on financial market. Huge changes in supply and demand have led to a spike in stock volatility, which resulted in the growth in risk aversion among investors. The gold is regarded as a safe haven asset and has hedge function in the stock markets. In this study, a Monte-Carlo pricing simulation based on Black-Scholes model for chooser option was investigated using the underlying asset of gold. The gold price and volatility index were sourced from Yahoo Finance website and the Chicago Board Options Exchange (CBOE). According to the analysis, the present value of the average return for one-month chooser option is simulated as $58.91, using the initial gold price of $1965.10 and the strike price of $1977.3. The present value of the average return for one-month chooser option is significantly higher than traditional call or put options. According to the sensitivity analysis, the return of chooser option under the pricing model adopted is not sensitive to the changes in volatility and time, which indicates that this pricing model is able to resist to risks and play a better role in a high-volatility market. These results shed light on the optimization of investment portfolio under complex situations. The utilization of a new pricing method paved a path for the research of exotic options and the state-of-art applications of them in the financial innovation procedures.

Keywords: Chooser option; Monte-Carlo simulation; Black-Scholes Model; Option pricing; Ukrainian conflict.

1. Introduction

Options trading has originated in the American and European markets in the late eighteenth century, which has been standardized with the Chicago Board Options Exchange. Option refers to a financial instrument based on the futures. It is a kind of right to decide whether option holders will buy or sell the underlying assets in a certain time with a pre-agreed price. An option contract provides security for this right. To be more specific, there is no need for the holders to buy or sell the asset if they decide against it. Besides, the specific expiration date by which the holder must exercise their option as well as the strike price are also given in the contracts [1].

According to the different underlying assets, trade direction, and exercise time, options can be divided into different categories. On the basis of trade direction, there are Call Options and Put Options. Call Options give holders the right to buy underlying assets while put options allow holders to sell certain goods within the period of validity. In addition, exercise time is a criterion to differentiate options. In detail, American options can be exercised at any time within the period of validity; European Options can just be exercised at the maturity date; and Bermuda Options is seen as the combination of the American Options and European Options and can be exercised within a specified period of time before maturity date. In addition, options can be classified as share options, real options and so on by various underlying assets [2].

Contemporarily, with the development of the options market, different kinds of options are combined to meet the personalized demands and to generate better yield. Exotic Options, as the derivative, are created to meet the needs of the market. Exotic Options, such as Packages, Compound Options, Barrier Options and Asian Options, depend on more index and can mix the characteristics of different types of options to resist risks and get more returns. Chooser option, as one of the usual Exotic Options, will be discussed in detail as follows [3].

Chooser Option is defined as options which are paid for in the present but which at some prespecified future date, which refers to the choice date, are chosen to be either a put or a call. From its definition, chooser option is special due to its dual nature, i.e., it is purchased at present and gives its holder the right to decide whether the option will finally be a put or a call before maturity. The chooser option can be divided into two categories, the simple chooser option, which refers to the same strike price and time to maturity. The other one is called the complex chooser option, which the strikes price or even expiry dates are not the same. Based on the characteristics above, it is assumed that chooser option is suitable for investors who expect strong volatility of the underlying asset but who are not uncertain about direction of the change. Chooser options enable investors to hedge against possible future events. Moreover, the straddle is really similar to the chooser option, which can also choose put or call according to the future direction of the underlying assets. However, it has to pay for both options immediately while chooser option do not have to pay for the both options entirely and is flexible to decide which option to buy later, which makes it a better choice.

Option pricing is regarded as one of the most important achievements in modern financial market investigations, for which it includes the core concepts of modeling and simulations. The pricing model for options has witnessed several innovations before forming into diversified methods nowadays. The prototype of option pricing model emerged in 1900, when French mathematician Louis Bachelier first came up with the arithmetic Brownian motion to simulate the change of stock price. Unfortunately, the theory announced by Bachelier didn't arouse much approval during that time.

The first widely accepted option pricing model was the Black-Scholes Model developed by Fischer Black and Myron Scholes in 1973 [4]. It is assumed that the short-term interest rate is known and constant, no dividends and transaction costs occurred with free right to borrow any fraction of a security, and no penalties to short selling. On this occasion, a set of equations was generated for a European call option:

$$-\frac{\left(\frac{1}{2}\omega_{11}\upsilon^2x^2+\omega_2\right)\Delta t}{\omega_1}=\left(x-\frac{\omega}{\omega_1}\right)\gamma\Delta t \tag{1}$$

Where

$$\begin{cases}\omega(x,t*)=x-c, x\geq c\\ \omega(x,t*)=0, x<c\end{cases} \tag{2}$$

To solve the equations, they pointed out the formular for European call options:

$$\omega(x,t)=xN(d_1)-ce^{r(t-t^*)}N(d_2) \tag{3}$$

Here,

$$d_1=\frac{\frac{ln\,x}{c}+\left(r+\frac{1}{2}\upsilon^2\right)(t^*-t)}{\upsilon\sqrt{t^*-t}} \tag{4}$$

$$d_2=\frac{\frac{ln\,x}{c}+\left(r-\frac{1}{2}\upsilon^2\right)(t^*-t)}{\upsilon\sqrt{t^*-t}} \tag{5}$$

Where x referred to the stock price, t referred to purchase date and t* referred to exercise date, the subscriptions on ω referred to partial derivatives, υ^2 was the variance rate of return on the stock. N(d) was the cumulative normal density function. Similarly, the formula for European put option was:

$$u(x,t)=-xN(-d_1)+ce^{-rt^*}N(-d_2) \tag{6}$$

Where d_1 and d_2 were the same as call option [4].

Later, Merton also extended the BS model to American options and identified the relationship between the formula and the stock price [5]. In 1979, Cox, Ross and Rubinstein developed a simplified option pricing model based on elementary mathematics. They developed the framework which followed a multiplicative binomial process over discrete periods. The value of the option could be calculated as the expectation of its discounted future value in a risk-neutral world [6].

With the development of information technology, computers took part in simulating the option price. The Monte-Carlo method was proposed to randomly generate the distribution of the sample to simulate the unpredictable changes in the stock market each time. With the help of underlying asset pricing models, the ultimate value of options can be predicted. Although the Monte-Carlo method has been issued for a long history, it is still regarded as the only feasible way to simulate high dimensional problems encountered in many areas. Zhang, et al. developed an approach to price Asian and basket option under Black-Scholes model using a modified Monte-Carlo method, during which a variance reduction technique was adopted to accelerate the speed of convergence [7].

Other pricing models also continued in improving the previous models while adapting to today's financial innovation, including finite difference method, arbitrage method, martingale methods etc. According to Liu & Wu [8], Black-Scholes model could provide a clear quantitative conclusion for the trading strategy of underlying assets, and the equation itself has no error. However, it is claimed that the BS model was only able to give out the price for European options and failed to simulate in complicated situations where options varied its path. Monte-Carlo method was advanced in utilizing the massive data simulation with high efficiency to calculated the European option price. The binary tree method offered by Cox et.al made it possible to simulation American option price, but was impeded by large volume of calculation and low efficiency.

All of above are the option pricing methods for specific underlying assets, and the underlying asset in this study was gold. Gold is traditionally known as one of the important safe haven assets and has hedge function in the stock markets. This has been certified by many researches. In 2010, Baur and McDermott proved gold's role as a safe haven in the United States and major European stock markets by analyzing the stock return series of 13 countries [9]. Baur and Lucey also showed evidence that gold is a safe haven for the United States, the United Kingdom, and German stock markets [10]. Therefore, gold can be a good underlying asset to do options-trading under the unstable market situation.

Contemporarily, the stock market is not peaceful. This mainly because of the intensification of the Ukrainian conflict, with the sanctions against Russia by the U. S. and the E. U., the risk aversion is soaring. Gold, as one of the safe-haven assets is popular in the current market. Its price has risen sharply after the conflict. In addition, regional conflict leads to the tight supply of energy and part of the goods, which heats up inflation. This also cause the rise of the gold price. However, in order to restrain inflation, the Federal Reserve begins to gradually tighten monetary policy, by raising interest rates in March. This may control the rise of the gold price, even decrease it. To sum up, in the next month, there is not a clear direction of the gold price. Gold prices will continue to fluctuate in the near future. Therefore, chooser option is suitable in this situation [11].

Based on the previous study above, most of the researches were conducted under normal situation when significant fluctuations didn't occur on stock prices. Thus, the simulation process didn't highlight its data volatility. Since the dual nature of chooser option, changes in the price of the underlying asset are likely to affect the investors' decisions on whether to call or put. Therefore, this research is conducted under the background of high volatility for underlying asset prices to investigate and improve the previous pricing model. As an exotic option, chooser option can promote financial innovation and its pricing model can provide ideas for market valuation in complex situations.

The rest part of the paper is organized as follows. The Sec. II will focus on the methodology of this study, including data selection, modeling process, related theory and detailed descriptions. The Sec III will present the pricing outcomes and the interpretation of the results. Implication is also involved in this section to evaluate the model and point out its limitations. Eventually, a brief summary and expectation will be given in Sec IV.

2. Methodology

2.1 Data

The data involved in this study was sourced from official financial data sites. The current and historical gold prices were sourced from Yahoo Finance (https://www.yahoo.com) and the Gold ETF Volatility Index was quoted from the Cboe Gold Markets (www.cboe.com). All data was summarized into the Excel spread sheet and formed the original data set for this study.

The historical gold prices were selected between 1st October, 2021 and 4th Marth, 2022 in order to calculated the volatility for gold. The sampling frequency of the gold price was defined as the daily adjusted closing price. The changes in daily adjusted close price is shown in the Figure 1.

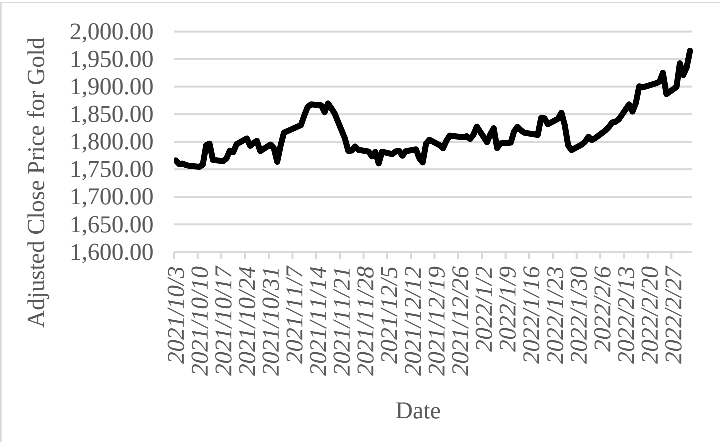

Fig. 1 Adjusted Close Price for gold between 1st October, 2021 and 4th Marth, 2022 ($)

2.2 Method

In this study, the Monte-Carlo method was adopted to simulate the uncertainty of the option market. The pricing model chosen in this study was the Black-Scholes model in order to minimize the potential error.

Based on Fischer Black and Myron Scholes analysis presented above, the future stock prices follow a log-normal distribution, and they can be represented as a function of "z", which was used in this study:

$$S_T = S_0 e^{\left(\alpha - \frac{1}{2}\sigma^2\right)T + z\sigma\sqrt{T}} \tag{7}$$

The basic data processing software adopted in this study was Excel. A set of random number was generated to make the simulation normally distributed between zero and one, using the formula in Excel NORMSINV(RAND( )). The formula was iterated twenty thousand times, generating twenty thousand random numbers for further calculation. The random number was divided into two groups, defined as z_1 and z_2. z_1 was used for simulating the gold price on choice date while z_2 was used to simulation the price on execution date.

In order to use the Black-Scholes pricing model, the following assumptions are made:

- During the life of the option, the underlying stock of the call option will not pay dividends.
- There are no transaction costs for options-trading.
- The short-term risk-free rate is known and remains constant over the life of the option.
- Any purchaser of securities can borrow any amount of money at a short-term risk-free rate.
- Short selling is allowed, and the short seller will immediately receive funds for the day's price of the short-sold stock.
- The call option can only be exercised on the maturity date.
- All securities are traded in a continuous fashion, with stock prices walking randomly.

The explanation of the variables in the Black-Scholes model are as follows.

- S_0 refers to the initial price of the underlying assets. In this study, S_0 is set as the price of gold in March 4th, 2022.
- Σ represents the volatility of the underlying assets.
- T is the period of validity of the option.
- A can be calculated using the formula below:

$$\alpha = r - \delta \tag{8}$$

Here, r refers to the risk-free rate and δ refers to the dividends paid. As to the parameter setting, for the choice date, according to Raimonda's research, when choice date is closer to the maturity date, the value will be higher [8]. Thus, one week before the maturity date is set as the choice date T_1 and the maturity time T_2 is one month. For volatility, annual volatility from 3rd March, 2021 to 4th March, 2022 was computed and used here. To compute strike price, according to the Ukrainian conflict in 2014, the gold price after one month was simulated by analogy, which was set as $1977.3 approximately. Interest rates on US 10-year treasury bonds is seen as the risk-free rate. Assume that no dividends are paid during this period.

After collecting the data and setting the parameters, the price of the gold at the choice date S_1 is calculated by the Black-Scholes pricing model using the first set of the random numbers Z_1. It is seen as the symbol of the trend of the future gold price. Then the formula below is used to compare initial price and the price at the choice date, and decide to exercise call option or put option. If $(S_0 > S_1, \text{"put"}, \text{"call"})$, the second set of random numbers Z_2 are used in the Black-Scholes pricing model to compute S_2 which is the gold price at the maturity date. The S_0 in the formula here will be S_1. Subsequently, the formulae below are used to calculate the return of the call choice option and the put choice option respectively.

$$\text{Call: } \mathrm{Max}(0, S_2 - \text{strike price}) \tag{9}$$

$$\text{Put: } \mathrm{Max}(0, \text{strike price} - S_2) \tag{10}$$

After the simulation of chooser option return, the average value and present value of the return is calculated for further comparisons. To further compare the yield generated by chooser option and traditional options, the series of simulations based on traditional put or call options are conducted. The results are presented as the average return of traditional options and their present value. The present values are compared to identify the rate of return between two types of options. As for the sensitivity analysis, the performance indicator is the present value. Besides, T_1, T_2, S_0, risk-free rate, strike price and volatility are indeterminacy factors whose variation range is from -10% to 10%. This study investigates the effect of the different indeterminacy factors by calculating the final revenue with the single variable.

3. Results & Discussion

Based on the methodology presented above, ten thousand random numbers were generated. The initial gold price (S_0) was quoted as $1965.10, σ represents the volatility of the underlying assets, which was calculated as 13.93%. With the provided risk-free rate, α was calculated as 0.01872. With the usage of the Black-Scholes model, the gold price on the choice date can be simulated as S_1, combined with the decision of investors on whether to call or put the option.

The choice above was made based on the comparison between S_1 and S_0. If S_1 was higher than S_0, it is assumed that investors would choose a call option. If S_1 was lower than S_0, the investors would choose a put option instead.

The simulation for the gold price on maturity date was conducted by re-performing the random number generation and the price simulation. The value of the option is calculated based on the comparison between S_1 and the strike price.

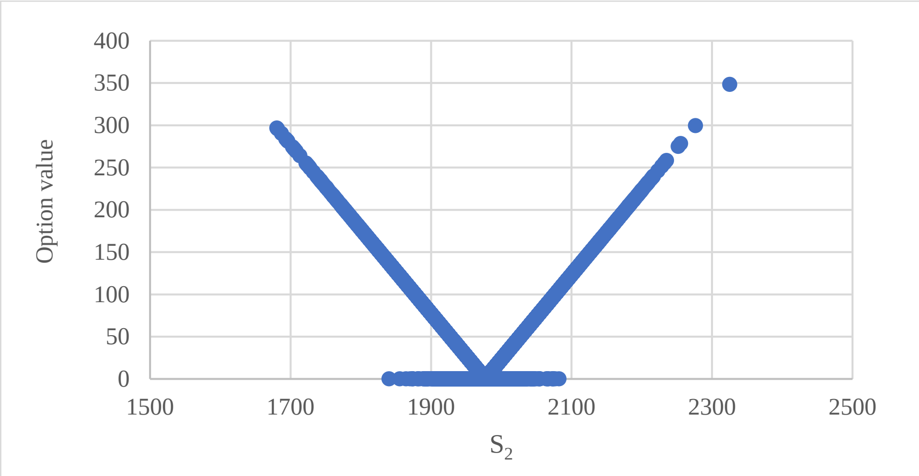

Fig. 2 Relationship between S_2 and return value for chooser option

Based on the obtained value, the average value of the chooser option was calculated as $59.01 with the present value of $58.91. The relationship between S_2 and return value is presented in Fig. 2.

For traditional options, the value for the call option and the put option based on the simulation of S_1 are also calculated. The average value and the present value of the call option was calculated as 27.65 and 27.61, respectively, while the average value and the present value of the put option was 35.76 and 35.71.

The results for the simulation of the traditional option using S_0 are also derived. According to the simulation, the call option has the average value of 28.08 and the present value of 28.04. The put option has the average value of 35.67 and the present value of 35.62. These results are shown in the Fig. 3.

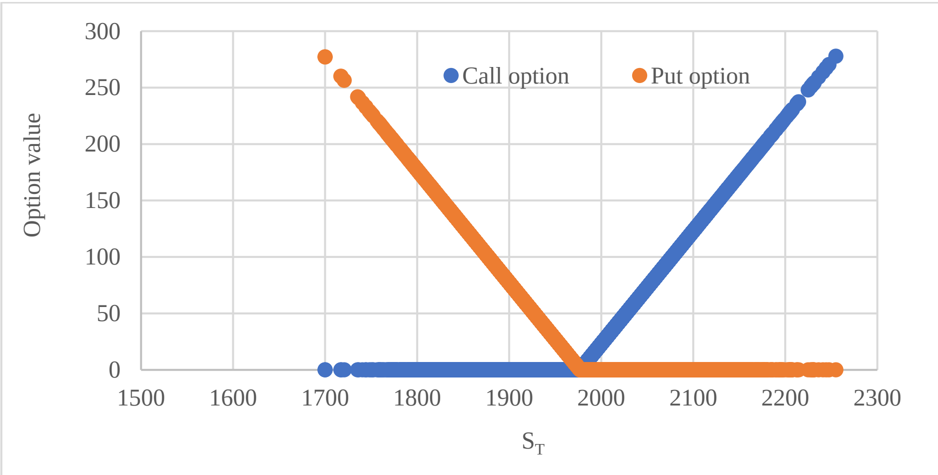

Fig. 3 Relationship between S_T and return value for traditional option

In comparison to the traditional option, the chooser option was presented to reach a higher average value and present value, indicating that investors who invested in the chooser option tended to earn a higher return than normal option buyers.

The result of the sensitivity analysis is shown in the Fig. 4 and Fig. 5. T_1, T_2, S_0, risk-free rate, strike price and volatility are regarded as the indeterminacy factors which changed within the scope of -10%~10% spaced by 1%. The corresponding present value of the option return and the rate of change are listed in Table I and Table II, respectively.

Table 1. The present value of the average option return with the changing variables

	VOLATILITY	STRIKE PRICE	T1	T2	S0	RISK-FREE RATE
-10.00%	53.04587	122.7927	57.59928	57.2434	126.2923	58.941
-9.00%	53.62929	113.0664	57.73592	57.41585	116.8939	58.9341
-8.00%	54.22071	103.5152	57.87168	57.58688	107.6086	58.93822
-7.00%	54.80519	94.24866	58.00639	57.75679	98.51752	58.93304
-6.00%	55.39072	85.48088	58.13993	57.92551	89.7599	58.93703
-5.00%	55.98232	77.48784	58.26674	58.09292	81.5823	58.93108
-4.00%	56.57132	70.57145	58.39852	58.25891	74.25687	58.92974
-3.00%	57.15704	65.06905	58.52958	58.42361	68.05392	58.93266
-2.00%	57.74279	61.16119	58.65605	58.58776	63.23346	58.92657
-1.00%	58.3301	59.05114	58.78588	58.75164	60.13187	58.91991
0.00%	58.915	58.915	58.915	58.915	58.915	58.915
1.00%	59.50987	60.71149	59.04019	59.07745	59.63043	58.91466
2.00%	60.10067	64.34098	59.16816	59.23899	62.26415	58.91338
3.00%	60.68229	69.60103	59.29991	59.40005	66.60261	58.91061
4.00%	61.273	76.14802	59.42711	59.5602	72.41194	58.90935
5.00%	61.86114	83.73794	59.55347	59.71936	79.51162	58.9081
6.00%	62.43969	92.11675	59.6785	59.87788	87.63371	58.90685
7.00%	63.02898	101.0705	59.80284	60.03533	96.46456	58.90561
8.00%	63.61973	110.3613	59.92621	60.192	105.8166	58.90101
9.00%	64.21645	119.8776	60.0489	60.34779	115.4827	58.8948
10.00%	64.81212	129.519	60.16729	60.50249	125.3391	58.89244

Table 2. The rate of change of the present value of the average option return with the changing variables

	VOLATILITY	STRIKE PRICE	T1	T2	S0	RISK-FREE RATE
-10.00%	-9.96%	108.42%	-2.23%	-2.84%	114.36%	0.04%
-9.00%	-8.97%	91.91%	-2.00%	-2.54%	98.41%	0.03%
-8.00%	-7.97%	75.70%	-1.77%	-2.25%	82.65%	0.04%
-7.00%	-6.98%	59.97%	-1.54%	-1.97%	67.22%	0.03%
-6.00%	-5.98%	45.09%	-1.32%	-1.68%	52.35%	0.04%
-5.00%	-4.98%	31.52%	-1.10%	-1.40%	38.47%	0.03%
-4.00%	-3.98%	19.79%	-0.88%	-1.11%	26.04%	0.03%
-3.00%	-2.98%	10.45%	-0.65%	-0.83%	15.51%	0.03%
-2.00%	-1.99%	3.81%	-0.44%	-0.56%	7.33%	0.02%
-1.00%	-0.99%	0.23%	-0.22%	-0.28%	2.07%	0.01%
0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
1.00%	1.01%	3.05%	0.21%	0.28%	1.21%	0.00%
2.00%	2.01%	9.21%	0.43%	0.55%	5.68%	0.00%
3.00%	3.00%	18.14%	0.65%	0.82%	13.05%	-0.01%
4.00%	4.00%	29.25%	0.87%	1.10%	22.91%	-0.01%
5.00%	5.00%	42.13%	1.08%	1.37%	34.96%	-0.01%
6.00%	5.98%	56.36%	1.30%	1.63%	48.75%	-0.01%
7.00%	6.98%	71.55%	1.51%	1.90%	63.74%	-0.02%
8.00%	7.99%	87.32%	1.72%	2.17%	79.61%	-0.02%
9.00%	9.00%	103.48%	1.92%	2.43%	96.02%	-0.03%
10.00%	10.01%	119.84%	2.13%	2.69%	112.75%	-0.04%

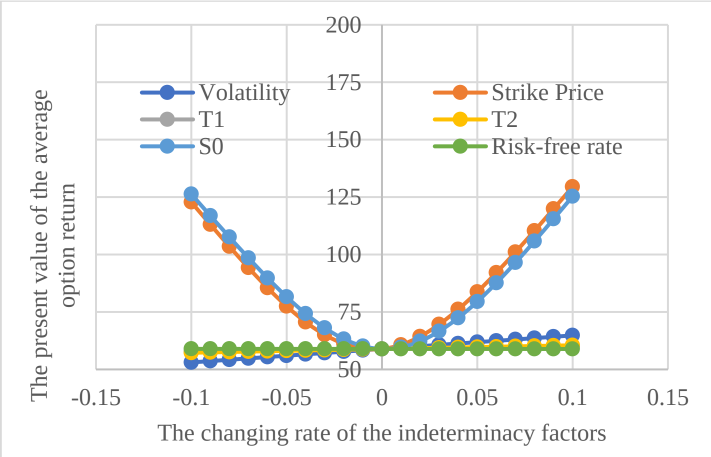

Fig. 4 The relationship between the indeterminacy factors and the present value of the average option return.

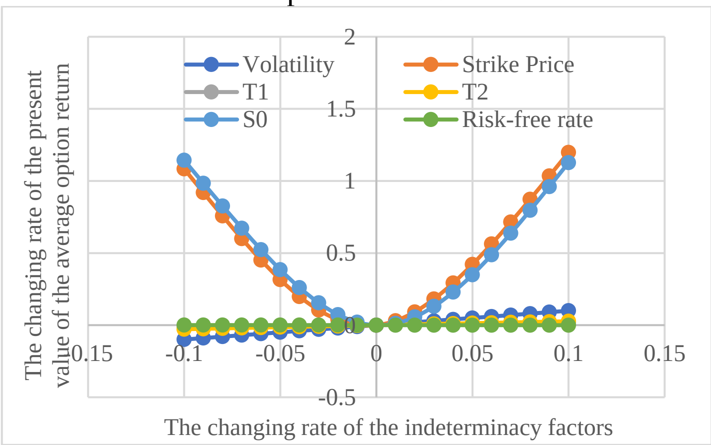

Fig. 5 The rate of change of the present value of the average option return with the change of the indeterminacy factors.

The result of the sensitivity analysis shows that the strike price and the initial price has the greatest influence on the option return, and the yield changes more than 100 percent when then strike price and the initial price change 10 percent. Moreover, T_1, T_2 and Volatility show a positive correlation with the return. This indicates that option holders can gain more return with longer maturity time and greater volatility which makes the market situation more unstable and have bigger risk. Additionally, when the change of volatility reaches 10 percent, the change of the return is just slightly lower than 10 percent, which has little impact on the result. Therefore, according to the relationship shown, the model of Chooser Option can resist to risks and have the ability to play a better role in a high-volatility market. However, Risk-free Rate is proportional to the option price, which may be because the higher the risk-free rate, the higher the time cost, and thus the lower option value.

One concern about the findings was that sample size is just ten thousand, which is too small to simulate the real market situation and predict the option price, which can be ascribed to the limitations in the instruments used to collect study data. The study is also limited by the forecast of the trend of the Ukrainian conflict. If there is radical change of the relationship between Russia and Ukraine, the background and underlying asset chosen for this study may be not able to exactly show the characteristics of chooser option. In addition, another major limitation of the present study is brought by Black-Scholes pricing model. Although this model can reduce errors to some extent and simplified research method, it ignores some variables such as dividends and option premium. This will lead to the bias between simulation and reality. These limitations should be noticed in the further research.

4. Conclusion

In summary, this paper investigates chooser option based on Monte-Carlo method using gold as the underlying assets under the background of the intensification of Ukrainian conflict. Specifically, this paper estimates the return of the chooser option in a specific situation by using the Black-Scholes pricing model twice with 10 thousand samples. Sensitivity analysis is also made to test the effect of the indeterminacy factors T_1, T_2, S_0, risk-free rate, strike price and volatility on performance indicator, the present value of the average option return. According to the analysis, chooser option is a better choice of investment compared with the simple call or put option, which can achieve higher yield, and make the holder gain more return. Moreover, chooser option is able to acquire stable profit when there is a high volatility in the market, which is known as its ability to withstand risk. In the future, considerably study ought to be carried out to address the issue mentioned above, i.e., using larger sample size with the consideration of all the related variables such as option premium to obtain more accurate results. Overall, these results offer a guideline for optimizing investment plan and pave a path for the research of the exotic option and the applying of it in the financial innovation.

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