Economic Significance of Arbitrage Transactions in the Tesla Industry

Yuqin Feng*

Department of Engineering, Virginia Polytechnic Institute and State University, Blacksburg, 24060 VA, United States

*Corresponding author: yuqin19@vt.edu

Abstract. Arbitrage transactions have played a significant role in allowing Tesla to exploit the minor differences in price between its similar and identical assets in two or more markets. In this case, as an arbitrage trader, Tesla undertakes the responsibility of purchasing assets in one market and selling them out at the same period to pocket any differences between the two prices. In this paper, there are two main parts of arbitrage transactions. The first part is three methods that can be utilized to analyze the options prices. These methods are linear regression and random Walk Theory, and Black Scholes. Their functionalities, influences, and theory are demonstrated in this paper. When using Black Scholes to calculate desired options prices and figure it in linear regression, assuming time to the expiration date, volatility, and risk-free rate are the same. The estimated trend and outcome are stated and described in this paper. The second part is about arbitrage transactions based on option prices. The circumstances of arbitrage transactions used by Tesla are analyzed and the utilized method called fundraising is demonstrated, and its functionality is analyzed with data and facts to prove the significance of arbitrage transactions. The paper also points out it is possible to use black scholes to estimate option prices in linear regression graphs and random walk theory. Call and put options prices are related to stock prices. They all shared a similar trend when time flows. The true value of this research is that people do not need to simply use a single method to estimate values, but to combine them, use them together to better understand the estimation of prices.

Keywords: Arbitrage transactions; Linear regression; Random walk theory.

1. Introduction

Tesla Inc. is an American multinational automotive and environmentally friendly company with its headquarters located in Austin, Texas. It primarily deals in designing and manufacturing electric vehicles, motorcycles, battery energy storage systems from home to grid-scale, solar panels and solar roof tiles, among other related products and services. It started as a single store, but currently, it has expanded to more than 160 stores in the United States alone and more than 70 stores across the globe with a total of over 110,000 employees. The company has been managed and executively governed by Elon Musk as its chief executive officer from 2008 to date. The revenue collected by the company in the first quarter (Q1) of the financial year 2022 is $18.76 billion, which indicates healthy financial operations within the organization. However, for the question how to estimate its different options prices could be difficult to answer due to its financial success. In order to construct an efficient arbitrage transaction, some values and methods need to be considered.

Changes in stock prices are held by random walk theory, and such changes are independent of each other and have the same distribution [1]. Trend of a stock price or market or the past movement cannot be taken advantage of making predictions of its future movement. It is believed that without assuming additional risk, it would be impossible to make the market outperformed. The process of gathering and seeking financial contributions by making contacts with people, companies, and industries is defined by fundraising which makes efforts to collect currency for non-profit organizations. Linear regression is a linear model which makes assumptions about a linear relationship between the one output variable and multiple input variables. It can make predictions to predict output values for inputs that are not present in the data set [2].

The goal of this paper is to combine random walk, linear regression, and black scholes to form possible options prices estimation to construct strong and bold arbitrage transactions method. The

very first step is simply understanding how black scholes compute its call and put option prices, and put it into linear regression model and random walk model to form two possible estimations. Definition of volatility and expiration date in black scholes need to be seriously considered. The second part is about the date or time when the data is collected. The time or day must be seven days. The number of days of times of seven needs to be analyzed, along with its selected stock prices. The first step is to demonstrate the separate outcomes of stock prices and option prices in random walk model, linear regression model, and black scholes. The emphasis is on the option prices based on black scholes separately on linear regression model and random walk model. Then, the comparison of the trends will be executed.

2. Methodology and Data

2.1 Methodology

2.1.1 Random walk theory

About this theory, Cheng and Deets state that there are two distinct hypotheses, one economic, the other statistical, consisting of such a theory [3]. Technically, these are two arguments. For the statistical argument, it makes an assertion that price changes are independent random variables for security. For the economic argument, it makes an assumption that there is no investor who can earn a return which is systematically superior in efficient markets which are often seen as security markets.

$$X_k = \pm 1, with\ P(X_k = 1) = p\ and\ P(X_k = -1) = 1 - p = q \tag{1}$$

2.1.2 Linear regression

There are some variables that have result relation and reason [4]. The relationship among the variables is estimated by using a statistical technique called regression analysis. If the relationship has a clear direction, there are positive associations or negative associations. The strength of a relationship is determined by how close the points in the scatterplot lie to a simple form such as a line. A regression line is a straight line that describes how a response variable y changes as an explanatory variable x changes. Analysis of variance summarizes information about the sources of variation in the data.

$$Y_i = f(X_i, \beta) + e_i \tag{2}$$

2.1.3 Fundraising in arbitrage transaction

Arbitrage transaction refers to the simultaneous sale and purchase of the same asset in different markets with the sole objective of gaining significant profits from the disparities in the listed prices of the investment. One of the top features of arbitrage transactions is that it is marked by the exploitation of the various short-lived variations in the prices of either identical or similar financial tools in diverse markets [5]. Essentially, such a situation plays a significant role in creating an ultimate opportunity for the trader to enjoy a risk-free profit. Arbitrage also plays a fundamental role in providing a framework that ensures that the prices of assets do not deviate from their fair value for extended periods. Consequently, technological advancements have made it extremely complex to incur significant amounts of profit from errors associated with pricing. As a result, many traders have resorted to computerizing their trading systems to monitor any potential fluctuations regarding similar financial instruments. And it makes traders to effectively and quickly act upon some of the most inefficient pricing set-ups.

2.2 Data

2.2.1 Random walk theory experiment data

The first is to simply understand how black scholes compute its call and put option prices, and put it into linear regression model and random walk model to form two possible estimations. Because

both models are related to time, when the time flows, the y-variable values will change. Definition of volatility and expiration date in black scholes need to be seriously considered. The time or day must be seven days. Due to the fact that it is seven days, it is necessary and important to do desirable computation and estimation every seven days. The number of days of times of seven needs to be analyzed, along with its selected stock prices. Due to the difference of those stock prices, the strike prices must be different due to one reason that strikes prices change as stock prices change. In this collected data in this paper, the strike prices will be determined by the round number of stock prices and minus it by fifty values. For example, 904.55 dollars as stock prices will be analyzed, which means that the strike prices will be 850 dollars. Then in every seven days of calculation, it is necessary to see the value of volatility and the value of time to expiration to be the same in every time for themselves. Based on these values of variables, the black scholes computation will be operated in linear regression model and random walk model. And the calculation of mu and sigma of call option and put option must be considered in random walk theory.

From the investor perspective, Tesla is a risky investment due to its uniqueness in the industry, which already has a set of established players that could pose stiff competition. However, based on its forecasted and predicted stock prices that have presented a steady increase in the past years, it is a gain worth investing in because, in the long run, Tesla Inc. is in the position of generating a substantial profit for the stockholders. This assertion is underpinned by the random walk theory, which states that the changes in stock prices have the same distribution and are independent of each other [6]. The random walk theory is based on the assumption that the past movements or trends in the stock prices or markets are not used while predicting their future movements or trends but rather use another approach that involves multiple forecasting models. Both the call option and put options present similar features. However, the interpretation of such is the opposite, depending on the nature of the stock price movement. Therefore, this theory proclaims that stock prices follow a random and unpredictable path that renders all other approaches to predicting stock prices futile in the long run [7]. Tesla's prices have been increasing over the past year, with the current value at $679.5 as of June 29th 2022. Based on the trends and Tesla Inc. stock price changes between 2021 and 2022, there is a constant distribution.

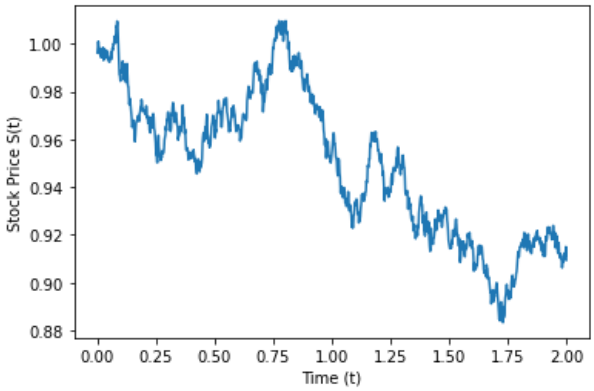

Figure 1. Geometric Brownian Motion for Tesla Stock Prices

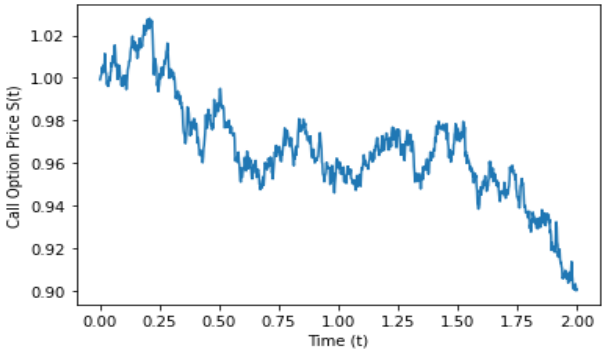

Figure 2. Geometric Brownian Motion for Call Option Prices

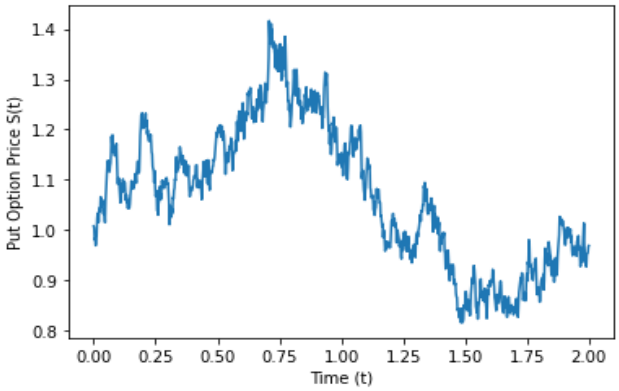

Figure 3. Geometric Brownian Motion for Put Option Prices

Based on Figure 1, Figure 2, and Figure 3, the trend of geometric brownian motion is quite obvious, which means that the trend of price overall is going down. In Figure 1, the trend is going down since the beginning. Based on Figure 2, the trend is increasing until the time is equal to about 0.85. After that, it goes down rapidly. From Figure 3, before the time is equal to about 1.25, the trend is up and down and unstable. After time 1.25, the trend decreases. Then, the overall motion of option price is decreasing along the time.

2.2.2 Linear regression method

On the other hand, using linear regression, the call option and the put options are presented. The values of options prior to expiration and vs. at expiration vary for the call option and the put option against the stock prices. Tesla Inc. presents the call option that is constant between the stock prices of $124 to $135, where it steadily increased to the call option of 10 against the stock price of $144. This is an indication that there is an increase in Tesla Inc.'s stock prices. The transition point shown in the linear regression graph is $135, where the stock prices started changing. The fluctuation of stock prices prior to expiry and at expiration shows a small margin of an average of $3.5. Such changes affect the overall stock prices of the company, thus affecting the investment decisions. A graphical presentation of the Call Option data: Value of Option Prior to Expiration vs. At Expiration ($135 Strike) is shown below:

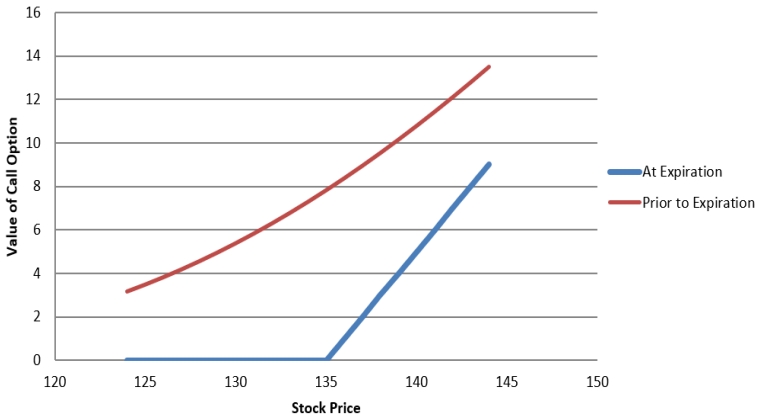

Figure 4. Call Option -- Value of Option Prior to Expiration vs. At Expiration ($135 Strike)

The call option presents a different output from that of the put option that presented the value of the option prior to expiration versus that of the same stock at expiration. Although both linear regression graphs presented a common transition at the stock price of $135, the entire results are different. In this context, there is a constant decrease in the values of the put option from 11 to 0, with

a subsequent increase in the stock prices from $124 to $144. At the stock price of $135, the graph flattens, indicating that the value of the put option is zero. Based on the findings obtained from both approaches, a common transition point for Tesla Inc.'s stock prices is $135, and therefore the future stock prices can be easily predicted using the extrapolation methods. Below is the graphical presentation of the Put Option data: Value of Option Prior to Expiration vs. At Expiration ($135 Strike).

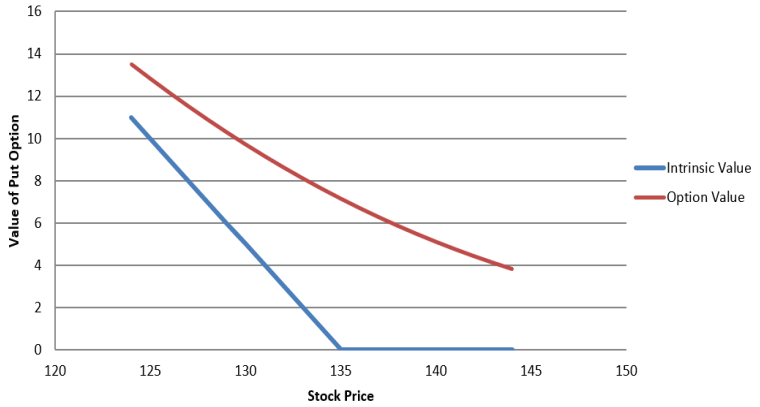

Figure 5. Put Option -- Value of Option Prior to Expiration vs. At Expiration ($135 Strike)

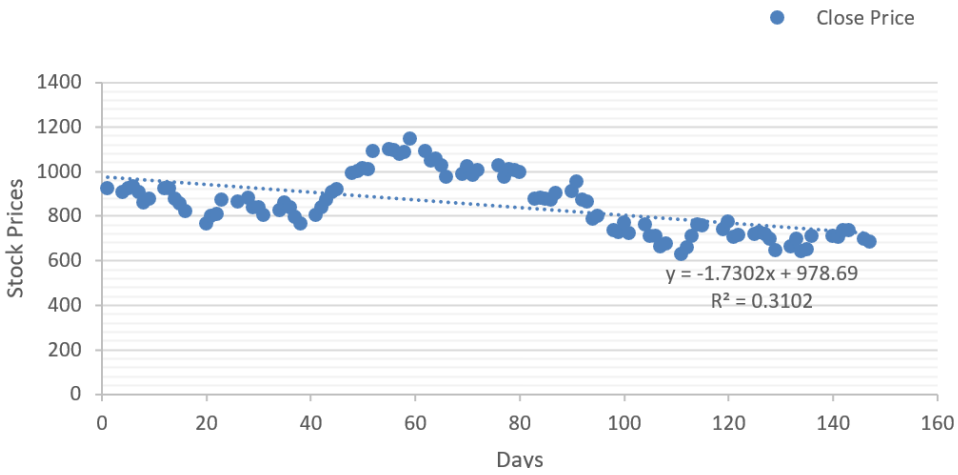

Figure 6. Tesla Stock Prices in 147 Days

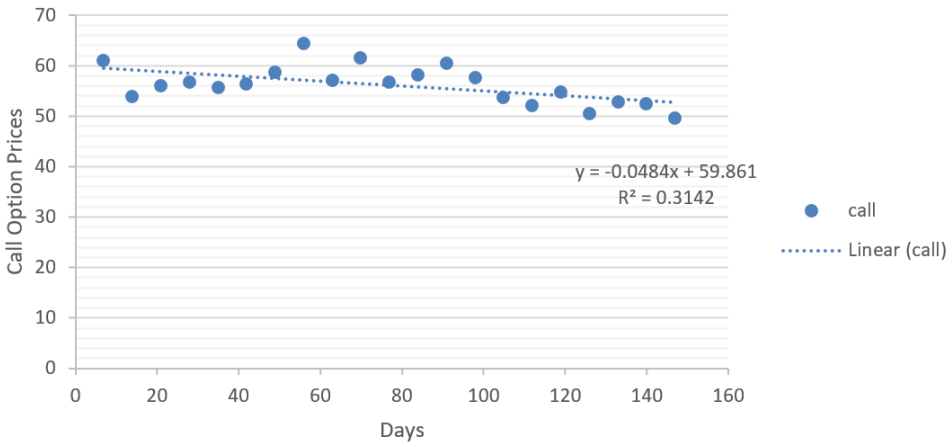

Figure 7. Call Option Prices in Black-Scholess

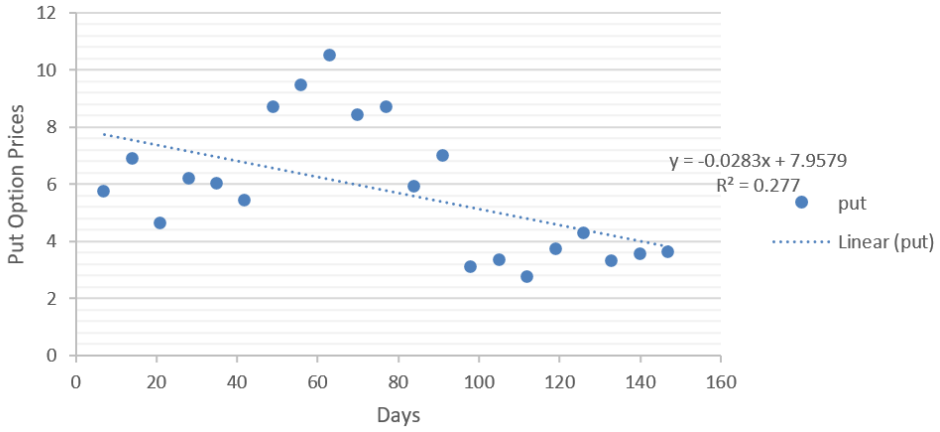

Figure 8. Put Option Prices in Black-Scholess

Figure 6 shows the overall trend of Tesla stock prices. Although there is a portion of trend going up, in overall, it mainly goes down in 147 days. The call option prices and put option prices are recorded every seven days. Figure 7 shows that the trend of call option prices goes down smoothly. Figure 8 shows the trend of put option prices goes down more than call option prices. When the value of strike price is being determined, it is possible to make expiration date, volatility, and risk free rate similar, even the same, which means that the strike price can be set as the stock price after rounding and minus 50.

2.2.3 Comparison of the two stock prices prediction approaches

Both approaches, such as the random walk theory and the use of linear regression for Tesla Inc.'s stock price prediction, displayed similar prediction features. The future stock prices of the company are easily predicted using the illustrations in both cases. For the linear regression approach, the call option and put option shows predictable findings that Tesla's future stock price behavior might take. Through this approach, it is easy to make an investment decision based on these findings. Similar to the random walk theory, which uses the behavior of the stock prices in the past, analyzes them and uses them to make predictions for future stock prices. Although this approach does not use the financial analysis option, it only hypothesizes the future stock prices based on the financial theory of the company by stating that stock market prices evolve according to a random walk and, therefore, cannot be predicted using analysis [8]. Therefore, both the random walk theory stock prices prediction method and the use of the linear regression approach present similar predictions that there will be a constant increase in the stock prices of the Tesla Company in future by using past stock prices behaviors (multiple models to predict the price) and the linear regression graph presentation of the movements of a single stock respectively.

2.2.4 The option price based on black scholes

The Black Scholes option pricing model facilitates the determination of fair stock prices or theoretical values for the call option or the put option based on six key variables such as volatility, type of option, underlying stock price, time, strike price, and the risk-free rate [9]. Tesla Company’s stock prices have proven to be steady despite external forces affecting the stock market. Below is a tabular presentation of the average prices of both the call option and the put option in table 1. The stock prices imply that it is possible to buy and sell the company stock shares with a substantial profit turnover.

Table 1. Average prices of both the call option and the put option of Tesla

First		Second	
Time to Expiration	0.038356164	Time to Expiration	0.038356164
Exercise Price	635	Exercise Price	650
Current Stock Price	681.79	Current Stock Price	681.79
Volatility	0.7094	Volatility	0.7094
Risk-Free Rate	0.04	Risk-Free Rate	0.04

From Table 1 above, Tesla Company's current stock price is $635, which is subject to the six critical variables of the Black Scholes. From both the first and second predictions shown in the table above, there are insignificant changes in the stock prices as both indicated the same risk-free rate, volatility, and the current stock prices within the same time to expiration. The only different variable noticed is the exercise price which presented 635 in the first prediction and 650 in the second. Therefore, the six variables of the Black Scholes pricing model have no significance in forecasting the stock prices of Tesla Company.

3. Tesla's Arbitrage Transaction

3.1 The Effect of Arbitrage

Tesla primarily employs convertible bond arbitrage. Convertible bond arbitrage is an approach centered on capitalizing on mispricing between the available convertible bonds and the underlying stock. One of the primary characteristics of this arbitrage strategy is that it is market neutral. In this respect, the arbitrageur primarily focuses on generating consistent returns with significantly reduced volatility regardless of the particular market's direction. Essentially, this arbitrage works through a combination of short and extended positions in convertible bonds and the underlying stock [10]. One of the key takeaways in this context is that there is a possibility of converting the convertible bonds into equity in a company at a given price at one time in the future. Another key takeaway is that a convertible bond can benefit the user because it is associated with a lower interest rate than a comparable bond without an embedded option. One of the main characteristics of this bond arbitrage type is its significantly reduced yield. However, this is often balanced by the framework that a convertible bond holder has the potential to convert the security into equity at a given discount based on the stock market's price. If the stock price is predicted to increase, the bondholder has the leverage of exercising their choice of converting bonds into equity. Essentially, it is worth mentioning that convertible arbitrage primarily involves embarking on extensive and short positions in a convertible bond and the associated underlying stock. In this case, the arbitrageur looks forward to profiting from any changes in the market by considering the most appropriate hedge between the long and short positions. The number of assets an arbitrageur purchases or sells largely depends on the relevant and most appropriate hedge ratio, which can quickly be evaluated using the delta.

Table 2. Convertible Bond Terms and Conditions

Equity	ISIN	Coupon	Issue Date	Maturity	Conversion		Issue Size
					Ratio	Price	
DISH-US	US25470MAA71	3.375%	8/3/2016	8/15/2026	15.34	65.18	3BN
	US25470MAB54						
DISH-US	US25470MAC38	2.375%	3/13/2017	3/15/2024	12.16	82.217	1BN
	US25470MAD11						
CHK-US	US165167CR64	5.5%	9/29/2016	9/15/2026	116.71	8.568	150MN
	US165167CY16						
VRNT-US	US92343XAA81	1.5%	6/13/2014	6/1/2021	15.51	64.463	350MN
BMRN_US	US09061GAH48	0.6%	8/11/2017	8/1/2024	8.02	124.7	450MN
LYV-US	US538034AM11	2.5%	3/15/2018	3/15/2023	14.7	68.025	550MN
	US538034AQ25						
ADS-AG	DE000A2LQRW5	0.05%	9/12/2018	9/12/2023	687.38	290.959	500MN
WDC-US	US958102AP07	1.5%	9/13/2019	2/1/2024	8.2	121.913	1BN
	US958102AN58						
TSLA-US	US88160RAB78	0.25%	3/5/2014	3/1/2019	2.78	359.87	800MN
TSLA-US	US88160RAC51	1.25%	3/5/2014	3/1/2021	2.78	359.87	1.2BN
TSLA-US	US88160RAD35	2.375%	3/22/2017	3/22/2022	3.05	327.51	850MN
TSLA-US	US88160RAG65	2%	5/7/2019	5/15/2024	3.23	309.83	1.6BN

Notes: This table is from Renewable electricity finance in the United States: A state-of-the-art review [11].

From Table 2, the company must find no insufficient demand for the $1.35 billion convertible bond it sells via returns. The company must ensure this by keeping in mind that the terms it issues seem inferior on paper. One of the main reasons behind these inferior terms is the company's volatile and underperforming stocks. Income and protections of low-yielding bonds characterize convertible securities. However, they offer an ultimate opportunity of profiting far from a jump in the stock prices that play a fundamental role in triggering repayment in shares instead of cash. In the latest deal that In this respect, the key takeaway point is that there is an escalated bar for taking advantage of the stock price. Given that the company's stock has been wobbling, it has a significantly minimized chance of attaining a rich equity conversion before the bonds reach maturity in the succeeding three years. With the new bond, the company will have five years until it starts receiving payoffs. These bonds need not be successful or hit at a home run. In this respect, they must remain in business to refinance the company within the stipulated five years. The arbitrage that Tesla engaged in involved fundraising, which had the potential of topping a total of $2 billion cumulatively. The fundraising

played a fundamental role in easing the concerns presented by Wall Street regarding Tesla, which has been on the record struggling to maintain its profitability and its potential to overcome a decline in sales and establish new product lines. The company's shares, which had declined by approximately 10% in the two years after selling the last convertible, ended over 4% higher during the same period.

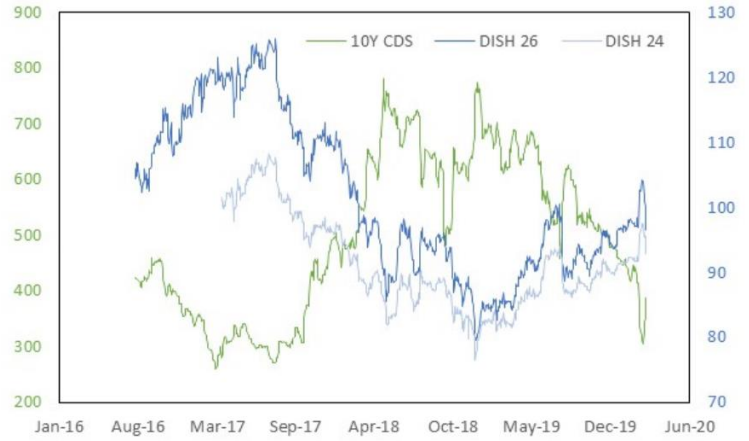

Figure 9. Dish Network Corp. 3.375% 2026 and Dish Network Corp. 2.375% 2024

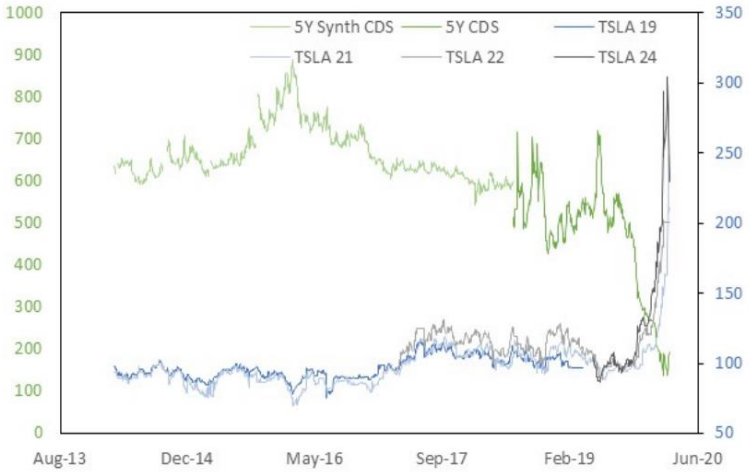

Figure 10. Tesla Inc. 0.25% 2019, Tesla Inc. 1.25% 2021, Tesla Inc. 2.375% 2022, Tesla Inc. 2% 2024.

From Figure 9 and Figure 10, the new convertible bond arbitrage carried a coupon that ranged between 1.5 to 2.0% with a conversion premium of 27.5-32.5% as far as Tesla share's reference price during the sale is concerned [11]. The above terms seem inferior for the investors compared to the 2.375% coupon based on the $977.5 million convertible bonds that had matured by March 2022 [11]. The conversion premium characterized by a midpoint of 30% compared less favorably with the 2022 bond's lowest conversion premium, which stood at 25%. However, during this time, the stock's underperformance eventually closed at $244.10, which implied that the holders of the 2022 bond required an approximately 34% increment in the stock to a price of $327.50 to facilitate a conversion [11]. According to a potential buyer for the new bonds, the convertible's terms seemed reasonable, and the company had no issue selling them. Another potential boost to demand that Tesla would have incurred could emanate from a team of short-selling Tesla shares who tend to purchase convertibles as a hedge, which is perceived as one of the most common arbitrages applied by hedge funds. The convertible bond arbitrage is also significant because it presents several opportunities for a company's operations to streamline. After Tesla had documented an unprecedented quarterly profit on October 13th, 2020, its share prices demonstrated an upward trajectory. Between November and November,

the company experienced a share increase of approximately 30%. On the other hand, the short interest was significantly reduced by about 23% [11]. The reaction from the shorts never came as a surprise to anyone, given that Tesla was already one of the leading shorted equities in the United States based on the dollar value and outstanding shares. Essentially, it is worth mentioning that this case left a relatively short seller to participate at a time when shares went substantially beyond the 2019 low that was witnessed in May. One of the surprising incidents that happened next was that the resiliency of the short sellers escalated the shares up to $780, or +136%, since the beginning of December.

The short interests as far as the Tesla shares are concerned stood at 24.9 million as of January 25th. Therefore, this indicates that Tesla's short position declined by approximately 1.3 million, representing 5%. On the other hand, the share prices rallied by 24% [11]. As of December, the share price increased by about 27%, while the short interests significantly reduced by 8%. Generally, this implied 13% declining shares short during 1.5 months besides a share price increment of 57%. Since the beginning of December, the prices of Tesla shares in 2021 convertible bonds increased by a whopping 96% through to February 3rd, compared to the 136% increment in the standard equity prices for Tesla [11]. In this case, one of the key takeaway points is that for a convertible arbitrageur, there is often a culture of increasing short positions as far as the common shares are concerned. In this case, the short position in the common shares until it reaches a share price of 1 implies that the price change for the convertible bond equates to the changes in the equity prices. In this case, the main consequence of the price transformation is that the convertible arbitrageurs significantly increased their shorts positions within the preceding three months. That may also imply that the various direction shorts have had the potential to significantly reduce their exposures by a greater amount than the suggested ones by evaluating the cumulative short interest. If the companies that own the convertible bonds have completely hedged shares that would subsequently allow their bonds to be converted, this would mean a 12.7 million shares short.

3.2 Leading Factor

A leading factor that has significantly influenced the share prices over the latest rally is short exposure to call options. When Tesla began selling its convertible bonds, it also ensured that calls at the conversion price were bought and warrants simultaneously sold at higher strike prices. The primary aim of these transactions was to minimize the implications of dilution in instances where the share price exceeded the conversion price at maturity. In this respect, Tesla had the upper hand of effectively purchasing back shares in the market besides the proceeds from the profitable call option they had previously bought. However, research demonstrated that this anti-dilutive approach would only hold up to the strike price of the warrants the company had sold out, intending to offset the costs attributed to the call option.

As far as Tesla's case is concerned, special considerations apply to convertible bond arbitrage. In this case, it would be imperative to acknowledge that the price of a convertible bond is quite sensitive to the transformations in interest rates, a user's credit rating, and the prices of the different underlying stocks [12]. Therefore, another form of convertible bond arbitrage is characterized by purchasing a convertible bond and hedging to gain exposure to an additional factor at a more suitable price. There are several requirements for convertible bond arbitrage. Here, a critical takeaway is that convertible bonds can sometimes be insufficiently priced concerning the underlying stock's price. The best approach to taking advantage of such price differentials is by giving the arbitrageurs the ultimate opportunity of utilizing a convertible bond arbitrage strategy, as in the case of Tesla. In such a scenario, there are two complex variations. However, these variations primarily depend on the identification of potential market deficiencies. Tesla has engaged in an arbitrage transaction in which the arbitrageurs transact on its behalf.

4. Summary

In this paper, black scholes, linear regression, random walk and fundraising are utilized in this paper to study. Using Tesla stock prices in 147 days as basic data to computing possible desired call and put option prices. First step is to collect data of stock prices, manage data to determine different values of variables. Second is to build black scholes computation in linear regression model and random walk model in stock prices, call options prices, and put option prices. Third step is to consider the strategy of the Tesla company to arbitrage transactions. The main result is that it is successful to use black scholes in linear regression model and random walk theory model, but assuming volatility and time to expiration date being the same is important. Also, stock price is related to call and put option prices. When the Tesla's prices are going either way, the price of option prices is predictable. Last, according to estimation in the second major part, the Tesla company has great financial success in the future due to its running structure and data to support it. This paper provides good circumstances for Tesla stock market users and investors to make an estimation for the future. This paper can only be utilized as a basis for Tesla stock market circumstances, which does not mean this paper can be helpful for other types of market. Based on this, more experiments to be done and more data to be tested by using the same methods like those methods in this paper. There is some recklessness in this research paper. One thing is about the correctness of the result. Observation of option prices and Tesla's business transactions work in the future need to be proved by calculating more data to strengthen its result.

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