

Option pricing and risk hedging for Visa

Runze Yan^{1,*}

¹School of Finance, Tianjin University of Finance & Economics, Tianjin, China

*Corresponding author: runze622@hbut.edu.cn

Abstract. As the core of the option transaction, the option price changing with the supply and demand in the market is a variable which affects the profit and loss of both trading sides directly. In the 20th century, multitudinous econometric pricing models proposed lacked universal recognition until the Black Scholes Merton model came out. This paper focuses on the stocks and options from Visa Inc. to do the article consisting of calibration, option pricing and hedging using fundamental Black Scholes Merton model and the extensive jump model mainly under the seldom used method. The article demonstrates that calibrated parameters in Black Scholes Merton model perform better than that of the jump diffusion model with the same method, and the hedging portfolio based on the Black Scholes Merton model do keeps the profit and loss at a steady level though it should not be a preference at the certain circumstance. The results in this paper are beneficial for investors to forecast the price of option with the optimal model and describes the nature for option selection.

Keywords: Black Scholes Merton model; Merton's jump diffusion model; calibration; option pricing; hedging.

1. Introduction

Financial derivatives play an irreplaceable role in risk management of capital market with the enhancement of the mechanism of transaction and the dynamic trading frequency. An option, featuring non-linear derivative to be traded in a specified stock at a fixed price at any time on or before a given date, has been widely used as a financial instrument and enjoyed an expansion in the capital market [1]. The theoretical price of option, an initial issue in option transaction, provides investors with reference of the movements of the market and approximate solution on professional investment portfolio. And appropriate hedging strategy is established for the sake of reducing the risk of marketing volatility.

Under certain assumptions, Black Scholes Merton model was presented in 1970s. The relevant theories have been widely studied these years, and numerous complements on option pricing model are proposed. To illustrate, Göncü et. al. focused on the stochastic volatility model considering opposite factors [2], for which Fouque et al. provide a first order asymptotic approximation formula in terms of the homogenized Black-Scholes solution [3]. Hor et al. took into account the lognormal price process as well as the discrete dividend payments on the stock [4]. Besides, Kouritzin et al. provided some branching algorithms to enhance the performance of the pricing process despite of the existence of less suitable cases [5]. Moreover, Qin et al. utilized Bayesian nonparametric approach to match the implied volatility surface and made a forecast with observed market data [6].

Since the jump risk should not be ignored in the market portfolio, the Merton jump diffusion model has been perceived more realistic. Using jump-diffusion process, the estimation of the expected payoff with least squares can be verified to be employed in path-dependent and multifactor scenarios varying from the conventional techniques [7]. In addition, Hout et al. provided a simple operator splitting solution for the sake of ensuing linear complementarity problems for American options [8]. In terms of hedging, Iqbal carried out a thorough study of the ability of gold to hedge against unfavorable changes in stock prices, inflation, and exchange rates for India, Pakistan, and the United States [9]. And Batten et al. focused on the viability of the hedging stocks with oil [10]. Though there are already empirical phenomena in financial markets, researchers rarely discuss on a certain company with percentage of the square error method. Meanwhile, Visa Inc. offers global commerce which counts. Thus, the paper will make a sequence of operations on options and stocks of the company and make comparisons.

In this article, calibration, option pricing, and hedging are main components, utilizing Black Scholes Merton model and Merton's jump diffusion model with the data collected from Visa Inc. As for calibration process, both two models need parametric optimization respectively under both sum of percentage square errors and least square method with the help of programming. Then, in order to survey on the effectiveness of the calibration, prices of the chosen options can be computed with the optimal parameters and contrast between that of the real. Considering its complicated procedure containing the jump to achieve no-arbitrage operation, only Black Scholes Merton model is used to hedge on stocks with one chosen option.

The organized bulk of the paper is as follows: Section 2 introduces the data and methods. Section 3 refers to the results and discussion. Section 4 makes a conclusion.

2. Data and Methods

2.1 Data

All the data chosen for the article are from Visa Inc. in Yahoo Finance (<https://ca.finance.yahoo.com>). And the reference current stock price of Visa Inc. is selected at the time of viewing the data, which is 205.15 on July 11th, 2022. All chosen options and arguments are premised on the assumptions of European options because it is one of conditions for the relevant Black Scholes Merton model.

In the calibration process, ten historical data and ten options are needed. For the historical data, fluctuating open price from June 27th, 2022, to July 8th, 2022, on Visa Inc. are selected as reference stock prices. There is a series of data processing for preparations, including the added difference between the chosen stock price and the price one day before for the following simulated implied volatility. As for ten options, considering the features that implied volatilities of the reference stocks are around the middle, and strike prices approximately equal to the initial stock price, five call options and five put options are collected, which share the same maturity on July 15th, 2022, the hypothetical market price of each option is the midpoint of bid and ask. To be more accurate, the risk-free interest rate is referenced to the U.S. 10-year Treasury yield about 3%. In terms of details, the ten options contracts are shown in the table below.

Table 1. The 10 options selected for calibration

Call options selected for calibration				
V220715C00200	V220715C00202	V220715C00205	V220715C00207	V220715C00210
000	500	000	500	000
Put options selected for calibration				
V220715P00202	V220715P00205	V220715P00207	V220715P00210	V220715P00212
500	000	500	000	500

In the pricing process, a call option and a put option are chosen with the same maturity on July 22nd, 2022, for pricing using calibrated implied volatility to compare to the real market price. And for the sake of variables control, strike prices approaching to the S_0 are also kept consistence, which is 205. The selected option contracts are shown in the following table.

Table 2. The 2 options selected for pricing

Call option selected for pricing	Put option selected for pricing
V220722C00205000	V220722P00205000

In the hedging process, twenty-eight historical stock prices from May 31st, 2022, to July 11th, 2022, are collected and one call option whose strike price and maturity are 170 and July 22nd, 2022, respectively are chosen. Considering that the stock isn't up to expiration date, rough estimation on its

profit or loss can just be made on July 11th. Moreover, the risk-free interest rate may not change rapidly in twenty-eight days so that it is set to be 0. The selected option contract is as below.

Table 3. The 1 option selected for hedging

Call options selected for hedging
V220722C00207500

Furthermore, the fluctuating trends of stock prices on Visa Inc. are shown in the line chart below.

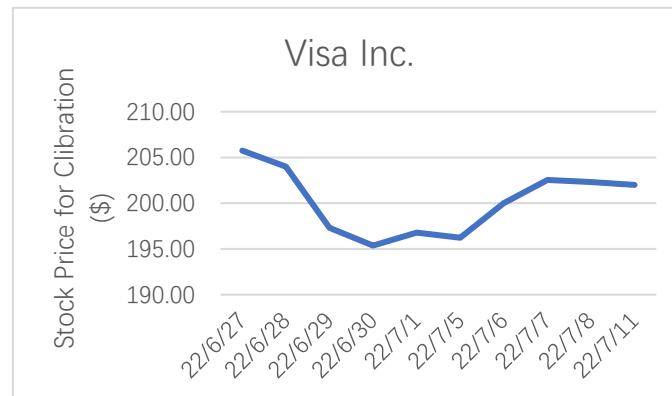


Fig. 1 The stock price trend of VISA Inc.

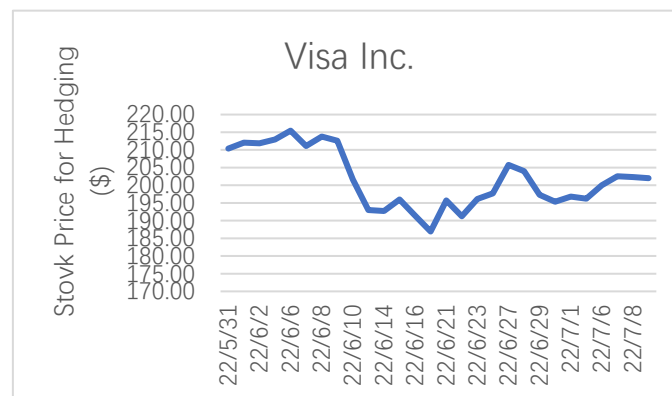


Fig. 2 The stock price trend of VISA Inc.

As two charts illustrated above, both trends decrease rapidly at first, and then perform a stable increase, with embedded volatility. The descriptive statistics of the rate of return of the stocks in calibration process from June 27th, 2022 to July 11th, 2022 and the stocks for hedging are illustrated below.

Table 4. The descriptive statistics

Visa Inc.	Mean	Standard Error	Median	Standard Deviation	Sample Variance	Kurtosis
Calibration	0.0249	0.0058	0.0207	0.0185	0.0003	-1.5387
Hedging	0.0020	0.0149	0.0025	0.0028	0.0002	0.9494
Visa Inc.	Skewness	Range	Minimum	Maximum	Sum	Count
Calibration	0.0246	0.0530	-0.0029	0.0501	0.2486	10
Hedging	-0.0534	0.0698	-0.0305	0.0393	0.0546	28

As the table seen, the sample stocks of Visa Inc. for hedging have a relatively smaller mean and median rate of return, but similar sample variance, demonstrating that chosen stocks have the well-performed rate of return with little errors during the validity periods. Nevertheless, stocks for hedging have a larger size range, which possibly make visible results. Compared to the kurtosis and skewness, stocks for calibration may have a strong dispersion on the left side of the average and a steeper distribution.

2.2 Methods

The Black Scholes Merton (BSM) model and Merton’s jump-diffusion model are used for the three processes above. Both mathematical models used for option pricing are well-known in the modern financial market, while the Merton’s jump-diffusion model is an expansion of BSM model on the basis. For BSM model, it takes into account the impacts of the prices, time, and risk factors. Nevertheless, it is still not applicable to the real market directly because of the following major assumptions. First of all, the market is assumed to be efficient while the movements are not predictable in practice. Then, the option selected should be European option which can only be exercised at expiration. Furthermore, there are no dividends paid out in the option period and no transaction costs for option buying. In addition, it is assumed that the risk-free rate and volatility of the underlying asset are constant and known, and the returns are normally distributed. However, in reality, the graph of the distribution reveals heavy left tail and negative skewness commonly.

In terms of implied volatility, it accounts for the forecast of the market’s potential movements on prices of stocks and options. Therefore, both two models are needed to calibrate on the same underlying asset respectively. For BSM model, the procedure contains three steps. First of all, five call options and five put options should be calculated with the initial four formulas, where five parameters σ , t , r , K , S_t represent implied volatility, the time to maturity, risk-free interest rate, strike price of the option, and price of the stock respectively. The function N in the equation (3) and equation (4) means the normal distribution of d_1 and d_2 . Then, optimization procedures, both optimal sum of percentage square errors (SSE) and least square method (LS), are applied to calibrate implied volatility, and the consequences are purposed to make an observation. The minimization formulas below are equation (5) and equation (6), whose p denotes price of the option calculated by BSM formula and p^m denotes the option price in the real market. Thirdly, aimed at verifying whether the calibration is effective, the simulated volatility using standard deviation approach by ten stock prices ought to be a contrast. The approach is corresponding to the equation (7) and equation (8). Both “Solver-Parameters” in Excel and Python codes can be used to compute results.

$$d_1 = \frac{1}{\sigma\sqrt{t}} \left[\log \frac{S_t}{K} + \left(r + \frac{\sigma^2}{2} t \right) \right] \quad (1)$$

$$d_2 = \frac{1}{\sigma\sqrt{t}} \left[\log \frac{S_t}{K} + \left(r - \frac{\sigma^2}{2} t \right) \right] = d_1 - \sigma\sqrt{t} \quad (2)$$

$$C(t, S_t) = S_t N(d_1) - K e^{-rt} N(d_2) \quad (3)$$

$$P(t, S_t) = K e^{-rt} N(-d_2) - S_t N(-d_1) \quad (4)$$

$$SSE = \text{minimize}(\sigma) \sum_{i=1}^n \frac{(p_i(\sigma) - p_i^m(\sigma))^2}{p_i^m(\sigma)} \quad (5)$$

$$LE = \text{minimize}(\sigma) \sum_{i=1}^n (p_i(\sigma) - p_i^m(\sigma))^2 \quad (6)$$

$$D_t = \log S_{t+1} - \log S_t \quad (7)$$

$$\sigma_{simulated} = \sqrt{\frac{1}{n-1} \sum_{t=1}^n (D_t - \bar{D}_t)^2} \tag{8}$$

For Merton’s jump-diffusion model, due to the jumps governed by essential Poisson process, the model formula consists of four major parameters, which are log-normal distribution coefficient (λ), implied volatility (σ), risk-neutral mean ($E^*(X_i)$) and variance (v) of log-normal distribution of the jump of size (X). Of these, the first two parameters ought to be calibrated. The equation (9) corresponds to the Poisson distribution with the given number of jumps (N_t) during moment 0 to t. The stock price satisfies the dynamic equation (10), whose m is related to the mean jump size, W^* denotes the risk-neutral Brownian motion, and dJ denotes the actual jump size. And the option price is as equation (11) shown, where BSM_k is relatively the Black and Scholes formula where r and σ depend on k . The optimization also uses the SSE as the equation (12) below whose formula is a bit different from equation (5).

$$P(N_t = k) = e^{-\lambda t} \frac{(\lambda t)^k}{k!} \tag{9}$$

$$dS_t = S_t(r - \lambda m)dt + S_t\sigma dW_t^* + dJ_t, \tag{10}$$

$$C_0 = \sum_0^\infty e^{-\lambda(m+1)t} \frac{(\lambda(m+1)t)^k}{k!} BSM_k \tag{11}$$

$$SSE = \underset{m < 2, 0 < v < \infty}{\text{minimize}}(\sigma, \lambda, m, v) \sum_{i=1}^n \frac{(p_i^{MJ(\sigma, \lambda, m, v, j_{max})} - p_i^m)^2}{p_i^m}, \quad 0 < \sigma < \infty, 0 \leq \lambda < 5, 0 < \tag{12}$$

After finding the calibrated implied volatilities in both two models, they need to be substituted into the pricing equations above correspondingly with the chosen two options. Then, there are three comparisons to be observed. Firstly, the price of the option evaluated by simulated implied volatility can be compared to that using SSE. Secondly, the differences between the pricing results evaluated by BSM model using SSE and LE and market prices of options can be compared respectively. Thirdly, it can differentiate the pricing results between BSM model and Merton’s jump-diffusion model using the same SSE method.

In the last part of article, the hedging strategy is composed of twenty-eight historical stock price and a specific call option. Started with the first day’s data, equation (3) is utilized to calculate initial call option price and initial delta. Next, update of the chosen data is used to compute delta, option price, and portfolio value as the following formulas. In the BSM model, the delta is likely expressed as the equation (13).

$$\text{delta}(t) = \frac{\Delta C}{\Delta S} \tag{13}$$

The First Day:

$$\text{Portfolio value}(0) = C(0, S_0) \tag{14}$$

Following Days:

$$\text{Portfolio value}(t) = \text{portfolio value}(t - 1) + \text{delta}(t)(S_t - S_{t-1}), t > 1 \tag{15}$$

$$\text{Loss without hedging}(t) = S_t - K - C(0, S_0) \tag{16}$$

$$\text{Loss with hedging}(t) = S_t - K - \text{portfolio value}(t) \quad (17)$$

As the profit or loss of the portfolio with or without hedging is estimated, it is possible to illustrate the character and effectiveness of the chosen option on the July 11th, 2022.

3. Results and Discussion

3.1 3.1 Results

As to the three comparisons mentioned, there are several consequential data. In the first pair of comparison, it's evident that the simulated implied volatility estimated by ten chosen stocks is about 0.3048 while the estimation for BSM model in SSE is about 0.2716, so that the prices of options can be evaluated in the following table, demonstrating that calibration makes BSM model more in line with the reality.

Table 5. The comparison on prices of options by BSM model in two methods

	SSE	Simulation	Market price	The smaller difference with the market price
Call option	4.3845	4.8965	4.35	0.0345
Put option	4.0150	4.5270	4.20	0.1850

Second pair, by calculation under LS method, the calibrated implied volatility is about 0.2762. As can be seen below, the prices yielded under SSE and LE method illustrate that the calibrated implied volatility under SSE makes the estimated call option similar to that of market but performs not as good as calibration under LS method for put option.

Table 6. The comparison options prices by BSM model in two calibration methods

	SSE	LS	Market price	The smaller difference with the market price
Call option	4.3845	4.4543	4.35	0.0345
Put option	4.0150	4.0848	4.20	0.1152

In terms of Merton's jump-diffusion model, referring to the codes (codearmo.com and scipy.org), the volatility σ and intensity λ can be calibrated as shown below. However, these seem to be something wired on the result under SSE so that the added typical sequential least squares programming optimization algorithm (SLSQP) is considered here. Therefore, the comparative results can be seen, demonstrating that calibration under SSE on BSM model performs better. Only under SLSQ did the Merton's jump-diffusion model yield ideally for put option.

Table 7. The comparison option prices

Calibrated Parameters	σ	λ	m	v
SSE	0.1538	0.9	0.2	0.9
SLSQP	0.2404	2.0	0.9274	0.1472
Prices of options under SSE	BSM Model	Merton's jump-diffusion model	Market price	The smaller difference with the market price
Call option	4.3845	4.8938 4.6283 (Under SLSQ)	4.35	0.0345
Put option	4.0150	4.5243 4.2588 (Under SLSQ)	4.20	0.0588

As the graph on the hedging process illustrated below, the cumulative profit or loss of the portfolio in a span from May 31st, 2022, to July 11th, 2022, demonstrates that the hedging strategy makes it relatively stable and fluctuate frequently without huge losses. Nevertheless, if not chosen, it probably obtains much more profits around \$9.39 than that of hedging.

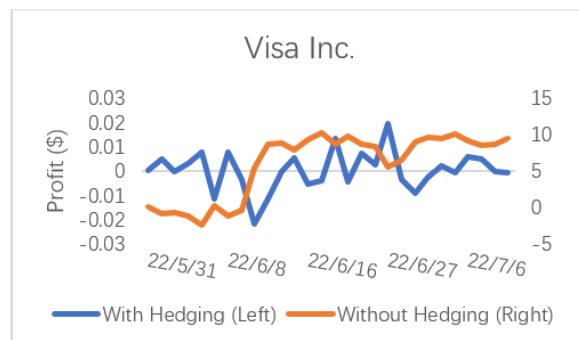


Fig. 3 The comparison of the profit or loss on Visa Inc.

3.2 Discussion

As the results above, calibration does make option pricing models more realistic, because the impact of time and corresponding risks on the option price, which makes sense, is considered in a close span. For Merton's jump-diffusion model, the deviation of parameters performed in the table 5 probably results from not only the 'algorithm risk' because of the commonly used SLSQP and the rough-generated SSE, but also the lack of samples leading to the accidental outcome. In terms of hedging, it can be initially verified that option is a sort of sophisticated tool for risk management [11] via observing the trend of the two lines in the chart III. The suitable option selected is of great help to prevent huge losses though it may not always useful to gain profit. The portfolio in the subject performs well as a result of reaching delta neutral. Referring to the equation (3) and equation (13), the change of price of the option keeps constant pace with that of the stock proportionally, meaning that \$1 upward for stock price results in \$ $N(d_1)$ upward for the price of call option approximately, ignoring the effect of $N(d_2)$ for its small coefficient. Therefore, in BSM model, delta can be viewed as $N(d_1)$ for call option and $N(-d_1)$ for put option similarly. Regardless of the well-performed strategy, not until the option arrived the maturity did it can be judged casually.

4. Conclusion

The paper is comprised of three main processes, which are calibration and calculation on both BSM model and Merton's jump-diffusion model and hedging only on BSM model. Despite the researchers have studied on both two models, the article in this paper have not been discussed before, utilizing different approaches and selected data from Visa Inc. Consequently, the calibrated parameters do make the option pricing models more suitable on recent circumstances, helping to attach importance to the optimization. Also, the last process is of great assistance to certify the insurance function of option.

There are still some defects. Known that the implied volatility is modified frequently as the macroeconomy changes, the sum of percentage square errors can't precisely generate ideal parameters with historical data, which can't be applicable to make any forecast. Moreover, the calibration on Merton's jump-diffusion model under SSE shows discrepancy with that under SLSQP. Besides, the hedging chosen is not an entire chain with zero interest rate. Therefore, the article still deserves further investigation in the future.

References

- [1] Cox, John C. et al. Option pricing: A simplified approach. *Journal of Financial Economics*, 1979, 7: 229-263.
- [2] Ahmet Göncü; Giray Ökten. Efficient Simulation of A Multi-factor Stochastic Volatility Model, *J. COMPUT. APPL. MATH.*, 2014.
- [3] Fouque J.P., Papanicolaou G., Sircar K.R., Solna K. Multiscale stochastic volatility asymptotics *SIAM Journal on Multiscale Modeling and Simulation*, 2004, 2: 22-42
- [4] Hor R. X., Ng W. S., Tan W. K., Cheong H. T. Valuation of American Option with Discrete Dividend Payments. *PROCEEDINGS OF THE INTERNATIONAL CONFERENCE ON MATHEMATICAL SCIENCES AND TECHNOLOGY 2018 (MATHTECH2018): INNOVATIVE TECHNOLOGIES FOR MATHEMATICS & MATHEMATICS FOR TECHNOLOGICAL INNOVATION*, 2019.
- [5] Michael A. Kouritzin; Anne MacKay. Branching Particle Pricers with Heston Examples, *ARXIV*, 2019.
- [6] Zhang Q, Caio A. A Bayesian Nonparametric Approach to Option Pricing, *BRAZILIAN REVIEW OF FINANCE*, 2020.
- [7] Longstaff, Francis A., Eduardo S. Schwartz. Valuing American Options by Simulation: A Simple Least-Squares Approach. *The Finance*, 2001
- [8] Karel in't Hout. Jari Toivanen. Application Of Operator Splitting Methods In Finance, *ARXIV*, 2015.
- [9] Javed I. Does Gold Hedge Stock Market, Inflation and Exchange Rate Risks? An Econometric Investigation, *INTERNATIONAL REVIEW OF ECONOMICS & FINANCE*, 2017.
- [10] Jonathan A. Batten, Harald K, Peter G. Szilagyi, Niklas Wagner. Hedging Stocks with Oil. *ENERGY ECONOMICS*, 2019.
- [11] Hopkin P. Fundamentals of risk management: understanding, evaluating and implementing effective risk management. Kogan Page Publishers, 2018.