Performance of Delta-Hedging Strategy on TQQQ Stock Options

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Abstract. This paper studies the performance of the same incremental hedging strategy of stock options. Especially when the stock market fluctuates greatly, this delta hedging strategy can always ensure investors' fund always keep in a relative safe level. Thus, it can help individuals and institutional investors to build their portfolios and choose hedging strategies. This paper use the data of ten options of TQQQ stock to calibrate the implied volatility of the stock. Then, by calculating the implied volatility, a hedge portfolio is formed, including a specific option unit of the company and the incremental shares of the underlying stock. Finally, this paper evaluate the performance of the hedged portfolio. The results show that, the hedging strategy of TQQQ stock option performs well. The results of this paper are helpful for individual and institutional investors to choose the most suitable hedging strategies according to the nature of the underlying assets.

Keywords: Hedging strategy; delta-hedging; Black Scholes model; options hedging.

1. Introduction

Hedging is a common investment tool used by investors to reduce the risk of market uncertainty in a series of transactions in order to maintain a stable return on the existing portfolio. Among these trading actions, part of the investment direction is the opposite, but the amount of capital with the opposite direction of the trading capital, so that in the market changes always maintain part of the profit and loss equivalent, another part of the unaffected can be normal gain. There are different hedging models for different transactions, including stock index futures hedging, commodity futures hedging and options hedging. The hedging method used in this article is option hedging.

In the last two years, more and more institutional and individual investors have been adding options hedging to their portfolios. The spread of the epidemic and its impact on society led to great volatility in stock and futures markets around the world, which caused the global stock market to shake violently, and the three major indices of the U.S. stock market suffered the biggest drop since the financial crisis in 2008, leading to a massive loss of capital and losses for countless institutions, financial giants and ordinary investors, including Warren Buffett. After this, options hedging was more widely used in investment portfolios. Option hedging strategies have been proven to be generally reasonable and reliable, and most large investment institutions all over the world always use it as a main hedging strategy. At the same time, option hedging strategies can be applied in different trading markets, such as futures, European options and American options. In a seminal paper from 1973, Fischer Black and Myron Scholes introduced what is known as Black-Scholes Model, and show that the Black-Scholes hedging strategy is near-optimal for the Investors [1], and Gurdip Bakshi, Charles Cao and Zhiwu Chen proved that option hedging model can always be used in any type of options, in either long term options and short term options it can help investors reduce risk [2]. Manamba Epaphra mentioned that how volatility is influnce the risk rate [3]. Halil Mete Soner, Steven Eugene Shreve and Cvitanic, Jaksa proved that, within the cost of proportional transaction, the best way to trade and donate a European option is to keep it to the maturity date [4]. Louis Ederington presented that how main Financial Derivatives such as Forward, Futures, Options, and Swaps could be applicated by hedging strategy [5]. John Y. Campbell had an explained that how different volatility is changing the stock return [6]. Stefan Spinler, Arnd Huchzermeier and Paul R. Kleindorfer explained all the existing risk of option hedging [7]. Kenneth R. French, G. William Schwert and Robert F. Stambaugh proved that how are options, stock returns and volatility are influenced by each other [8]. The options hedging strategy in this paper will focus more on how Delta

hedging prevent asset crisis and prevent extreme risks in the U.S. stock market. The selected U.S. stock representative in this paper is TQQQ (ProShares UltraPro QQQ) which is a NASDAQ long stock, chosen because NASDAQ is one of the largest indices in the U.S., and it is stable enough and convincing enough. This investment plan can be applied to most of the stocks included in the NASDAQ index.

In this study, the same delta hedging strategy using the Black-Scholes model is applied to TQQQ-specific options (10 call options and 10 put options as figure below) and they all in same maturity date. The hedging performance of the 10 paired options using the same hedging model. The results of this study show that the delta hedging strategy using the Black-Scholes model performs well in TQQQ and it can be concluded that the delta hedging strategy using the Black-Scholes model can be applied to most NASDAQ stocks.

The paper is organized as follows: Section 2 shows the methodology. Section 3 presents the results and discussion, and Section 4 concludes the paper.

2. Method

The method in this hedging is Black-Scholes model, which is also called as BS model. The Black-Scholes model can be said to be the most classic mathematical model used for option pricing and hedging. It was first proposed by Black and Scholes to price European options and was later modified by Merton to make it available in dividends (dividend) can also be used. The model assumes the stock of the option is following geometric Brownian motion and the model gives the option a unique price. in extra, it is used to derive Greeks of options to construct hedge portfolios to eliminate risk. In this article, this paper introduce the Black-Scholes model and conclusions related to asset pricing. The methodology is that this paper chose the stock price with the exercise price from 26 to 30. Wish same maturity date, the risk-free interest rate (federal fund interest rate) of 1.75 %and the actual transaction price of options, this paper determine the implied volatility of options by calibrate the model and trying various Sigma to find the minimum value of SSE. When finding the smallest SSE to complete the calculations and get the result. This paper tried various of different Sigma to ensure that our final calculated SSE is the smallest, and this paper need to use the risk-free rate in the calculation process. This paper calculate all options, stock prices, time, σ, actual call and put price and square error by this method. Thus, these minimum SSEs for different date are shown in the chart below. The main formula used in the calculation and the explanations of letters and symbols are listed below [9-10].

$$dS_t = \mu S_t dt + \sigma S_t dWt \tag{1}$$

$$D_1 = \frac{ln \times \frac{S}{L} + (r + 0.5 \times \sigma^2) \times T}{\sigma \times \sqrt{T}} \tag{2}$$

$$\mathrm{C} = \mathrm{S} \times \mathrm{N}(\mathrm{D}_1) - e^{-r \times T} \times \mathrm{L} \times \mathrm{N}(\mathrm{D}_2) \tag{3}$$

$$D_2 = D_1 - \sigma \times \sqrt{T} \tag{4}$$

$$C(S_o, t) = S_o N(d_1) - K e^{-r(T-t)} N(d_2) \tag{5}$$

In the above equations, S is the current stock price, r means the risk-free interest rate, t represents time to maturity, C refers to the call option price while K is the strike price. Besides, N means the normal distribution, and N(d1) and N(d2) are the cumulative distribution functions.

After that, next step is Delta Hedging strategy. In this hedging, The so-called positive gamma gains after delta hedging are actually delta gains, because gamma is positive. If the stock price rises, the total delta will be greater than 0. On this basis, if the stock price continues to rise, there will be Positive earnings, and since gamma is positive, the delta will become larger as the stock price rises, resulting

in faster earnings. If the stock price falls when the delta is positive, it will actually face a loss. Because the positive gamma will cause the delta to decrease when it falls, and then it will become a negative delta and profitable state. The gamma return given by the option formula is an instantaneous return (return in a very small period of time). When gamma is positive, the instantaneous return is indeed positive, in fact delta will not be 0 at any moment, so if the direction of the stock price change at the next moment is opposite to the positive or negative value of delta, it will bring positive returns.

3. Result

Based on the above-mentioned empirical process, the results in this paper can be summarized in the following Table 1 and 2, respectively.

Table 1. SSE for put and call options

Date	min SSE for puts	min SSE for calls
2022/6/13	0.018845	0.017748
2022/6/14	0.003018	0.028932
2022/6/15	0.140513	0.044036
2022/6/16	0.012312	0.078851
2022/6/17	0.004538	0.00298
2022/6/21	0.015447	0.003848
2022/6/22	0.006784	0.005264
2022/6/23	0.010728	0.007738
2022/6/24	0.018126	0.013157

Table 2. Portfolio performance

Day	Date	Stock Price	IV	delta	Call Price	Portfolio
1	2022/6/13	23.08	107.45%	0.254	0.5133	0.5133
2	2022/6/14	23.23	104.14%	0.2464	0.2862	0.5514
3	2022/6/15	24.9	102.08%	0.3377	0.385	0.9629
4	2022/6/16	21.86	103.95%	0.1527	0.0158	-0.0637
5	2022/6/17	22.67	100.38%	0.1756	0.0183	0.0599
6	2022/6/21	24.38	85.27%	0.2292	0.0391	0.3602
7	2022/6/22	24.29	84.80%	0.2106	0.0176	0.3396
8	2022/6/23	25.33	79.92%	0.2692	0.0527	0.5586
9	2022/6/24	27.95	74.17%	0.5218	0.4056	1.2639

In the calculations, this paper collected the data of ten options with different strike prices from $26 to $30, and then calculates their σ based on their time remaining until the maturity date from 6/13/2022 to 6/24/2022 and the stock's daily price at the price when market close for each day. Then their actual call and put prices are compared with their theoretical prices separately, and their square errors are finally calculated and derived based on this. This paper examined each square error that calculated by given data to ensure the difference between the expected value of all 20 call options and put options and the actual value of each of those options calculated is within an acceptable range. Based on this, the calculation results and square error data are brought into the calculation of subsequent option hedging.

In the result, all the results of min SSEs for put options and call options are listed in the chart above. In the trading, the portfolio value is 1.2639 and the total gain in option: 1.2639-0.5133=0.7506.

When the TQQQ stock price is moving from 23.08 to 27.95 from 2022/6/13 to 2022/6/24. All the calculated results of TQQQ hedging options are listed above. Investor can still make a profit without any risk under the hedging strategy. In the research and survey, it was found that when the stock price fluctuates, option hedging can indeed guarantee the maximum security of funds for investors, and only need to pay a relatively small price for this. In the case of a lot of bad news, investors can still get stable profits. The results show that hedging long positions on the Nasdaq successfully guarantees the safety of the principal, and the method is reliable. By the same token, the vast majority of publicly traded stocks included in the stable Nasdaq can use this hedging method to avoid risk. The reliability of this method is also closely related to the activity of options trading. Stocks with more options trading are usually more stable and reliable to hedge, such as Apple, Microsoft, Tesla, etc. On the other hand, smaller stocks with less option trading volume, or smaller stocks with no option trading, are generally not recommended or unable to use this hedging method to avoid risk. Research surveys show that most technology companies prefer investors to trade options contracts, so listed stocks of technology companies are more suitable for this hedging scheme than stocks of financial companies. Therefore, stocks included in the Nasdaq are generally better suited for this hedging scheme than stocks included in other indices.

4. Conclusion

This paper studies the application of the delta option hedging strategy in long stocks on the Nasdaq index. This paper uses the option hedging strategy to carry out the research on investment hedging in this stock. This paper calibrates 10 options data for this stock with different strike prices but the same exercise time. And by calculating how good a row of volatility is, a hedging investment portfolio is successfully formed. This portfolio also applies the risk-free yield of bonds in the United States this year. Therefore, all the data sources in this article are true and reliable, and the experimental research has proved that Reliability of delta option hedge views. In the study, Nasdaq performed this paper in long stocks and successfully proved the feasibility of option hedging. Due to the diversity of market risks, the study could not prove that all option hedging could be realized thereafter. And because the transaction costs such as taxes, handling fees, and income taxes are ignored in the experiment, the model does not simulate market risks completely and accurately, so it is worthy of more follow-up research by researchers. At the same time, the experiment can only prove TQQQ and speculate that most of the stocks of technology companies in the Nasdaq index can be hedged by perfect options, and the rest of the stocks that are not listed in the Nasdaq index may not be able to apply this hedging strategy.

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