Research on the Tendency Relationship between Individual Stock and Stock Index

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Abstract. This paper mainly centers on exploring the tendency relationship between individual stock and stock index using CAPM method. The research chooses the stock of Apple Inc. as an individual stock and S&P 500 as the stock index. Then CAPM is used to tests indicating that this model does not fit the rate of return on Apple very well. Therefore, this paper explores the possible reasons behind the phenomenon and refers to some constructive improvements in the evaluation of expected rate of return on individual stocks under the influence of the stock index. One reason is probably that AAPL is so powerful a corporation that it is extremely immune to anomalous volatility of the stock market, which implies that unsystematic risk dominates the trend of stock of Apple Inc. Therefore, this paper suggests that CAPM model should only be applied when the target company is a small-sized or medium-sized one that is prone to the fluctuation of the stock market.

Keywords: CAPM, S&P 500, AAPL, linear regression, beta, R-squared value.

1. Introduction

Capital asset pricing model (CAPM), an abbreviation of Capital Asset Pricing Model, was firstly introduced by Sharpe, Lintner, Treynor, and Mossin in 1964, and built on Markowitz's earlier work on diversification and modern portfolio theory. The model is simple yet powerful, proves to be useful not only in theory but also in practice over the past half century, which is recognized as the most widely used model [1, 2].

When time clocks back to 1950s, finance industry was less quantitative with more intuitive approaches. Pooled investments (like a Fund) had been around for decades and even relatively new concept such as diversification had been raised and well known to people. However, it wasn't until 1952, when Harry Markowitz's renowned article on portfolio selection was published, that the actual theory underlying this fundamental idea and its application to investments became apparent. The piece laid the groundwork for what is today referred to as modern portfolio theory (MPT) [3]. CAPM was a great innovation based on MPT, which was together with MPT, led the whole industry to a more scientific, quantitative and normative way. A great deal of concepts was introduced by MPT such as risk- free asset, systematic risk, nonsystematic risk, and efficient frontier etc. to describe the portfolio risk and return.

CAPM is a single factor/index model yet one of the most significant developments in MPT. The model provides a linear expected return–risk (beta) relationship that precisely relate the expected return from an asset to the risk of the return. The investors, given a settled risk level, prefer to choose investment portfolios which maximize return. And the more the risk, the larger the risk premium [4]. Typically, there are two aspects to the risk associated with an asset's return. Risks that affect the entire market or economy and cannot be mitigated are referred to as systemic risks. Nonsystematic risk can be reduced by diversifying away local or industry-specific risks by building up a sizable portfolio of various assets [5].

Over the decades, CAPM has been proved to have wide applications including but not limited to estimates of the expected return of an asset (such as a stock or portfolio), estimates of the expected return for capital budgeting, comparison of the actual return of a portfolio or portfolio manager with

CAPM return for performance appraisal, and the analysis of alternate return estimates and the CAPM returns as the basis for security selection [6]. Additionally, the relationship between systematic risk and the expected return in the market was used to predict returns on investments based on the level of risk exposure included into one's portfolio using the CAPM [7].

In following years since 1964, CAPM drawn the attention of scholars, for example, Fama and French then came up with famous three-factor model based on that [8]. As CAPM only uses market risk (beta) to represent systematic risk, Fama and French thus suggested factors that affect expected return on stocks should also include relative size of the company and relative book- to- market value of the company in addition to beta. Three-factor model contains three factors, namely market factor, scale factor, and value factor, to better explain excess return. Three-factor model proves to explain returns on stocks better than other models available at that time, most notably, the CAPM. Mark Carhart (1997) extended Fama and French's model by adding another factor-momentum, defined as relative past stock returns.

Given that investing in company stock is the main objective of investors, it makes sense that they would want to maximize their wealth through market stock return [9]. CAPM certainly can be used to model returns of a stock and to explore the relationship between the performance of the stock and the market. A particular stock usually has both systematic risk and nonsystematic risk, and as mentioned previously, only systematic risk is taken into account by CAPM, which is measured by β.

In other words, CAPM is not a very reliable indicator of the actual return for an individual stock [10]. This paper shall discuss and explore the applicability of CAPM regarding the estimation of return for individual stock, and also seek if there are any other tools to improve the prediction/regression results of CAPM.

2. Methodology

CAPM, as discussed in the Introduction part, mainly focuses on systematic risk. And the parameter beta is an indicator to measure such risk. By also taking into account risk-free asset, the CAPM formula is written as below:

$$E(Ri) = Rf + \beta i[E(Rm) - Rf] \quad (1)$$

Where Rf is the return on a risk-free investment that is usually approximated as the return on the state treasury bond, Rm represents the portfolio's performance across all possible investments, or the return on the entire market. Typically, it is represented as the return on an S&P 500-style stock index with broad diversification [11]. The beta (βi) is a parameter measuring the sensitivity of the asset's expected return ($E(Ri)$) to returns from the market. As mentioned previously, systematic risk of the asset is measured by βi. $E(Ri) - Rf$ or $E(Rm) - Rf$ is referred to as excess return. The slope that results from regressing the excess return on the asset over the risk-free rate versus the excess return on the market over the risk-free rate can be used to estimate the βi form historical data.

This paper picked Apple Incorporation (National Association of Securities Dealers Automated Quotations: AAPL) as the individual stock (its return as $E(Ri)$) for research on the applicability of CAPM. Meanwhile, this paper chose Standard & Poor's 500 Index (S&P 500) to approximate the market, denoted as $E(Rm)$ in formula (1). This paper also selected rate of return of the 10-year treasury bill of the United States to be the risk-free rate (Rf). For the purpose of the applicability of CAPM regarding the estimation of return for individual stock, the return rate as a constant for the treasury bill is determined as 2.92.

In order to take a sample that is large enough to have valid statistical significance, this paper considered the monthly rate of return of AAPL and S&P 500 over the past 30 years (from January 1992 to December 2021). This implies that there exist 360 observations for each of the 2 indicators. Firstly, the paper collects data from the Investing database and process the raw data. Next, the paper employed the formula of β_i that can be derived from formula (1):

$$\beta i = (E(Ri) - Rf)/(E(Rm) - Rf) \quad (2)$$

Then, the paper calculated the corresponding 360 betas respectively to acquire the monthly sensitivity of the expected return of AAPL to the expected return of the market. If the βi is larger than 1, then the volatility, or risk, of AAPL is greater than that of the stock market. If it is between 0 and 1, then the volatility of AAPL is less than that of the stock market. If the βi is negative, then there exists a negative correlation between the performance of AAPL and that of the stock market.

To figure out this kind of correlation, this paper can implement a scatter plot, with rate of return on the S&P 500 stock market on horizontal axis and rate of return on AAPL on vertical axis. Therefore, after all these 360 dots are projected on the scatter plot, a most fitted linear regression line can be gotten that reflects the least square tr with the minimum standard deviation. Undoubtedly, the steeper the slope of the linear regression line, the more sensitive the performance of AAPL to that of the stock market is. The smaller the least square error is, the more precise the linear regression model fits, which means it is more reasonable to employ the CAPM model to assess the rate of return of AAPL. Hence, under this circumstance, it is feasible to evaluate how appropriate it is to use the CAPM model. However, a drawback is that if there is an outlier, then it would impose great effect on the linear regression model, thus making the model less accurate.

3. Results and Discussion

Descriptive statistics is a study about how massive data can be summarized effectively and turned into useful information. In this section, this paper is going to summarize large sets of data to describe its key features and trends. Discussion regarding the correlation between returns of AAPL and S&P 500 follows the foregoing descriptive statistics.

3.1 Raw Data

As mentioned in Section 2, the data source used to approximately symbolize the market return is Standard & Poor's 500.The monthly return of recent 30 years (1992 to 2021) on the equally weighted index of the S&P 500 is used in this case.

The monthly return for the 30 years (360 observations) have been sorted in ascending order first. Then we can calculate the range of the date by using maximum value of return less minimum value. The data is further grouped into a number of intervals/sets say 14 intervals with interval width of 2%. The class limit of each interval can be determined by adding single 2% each time starting from the minimum value, and stopping until the maximum one is included in the interval. Count the number of observations (frequency) falling in each interval and finally construct a table showing the frequency for each interval. That process is what is known as constructing a frequency distribution and the distribution of the table is shown below:

Table 1 shows that the vast majority of observation data falls into intervals spanning from -2.94% to 4.94% with the percentage up to around 71%. A table is usually easier to read than plain text. The graphic presentation however is believed to be a better way to describe data as it can visualize key features and trends so quickly. Here a histogram is used as the graphical equivalent of Table 1, as Figure 1 shows.

Table 1. Frequency Distribution for S&P 500 Return 1992 to 2021

No.	Return Internal (%)	Absolute Frequency	Relative Frequency (%)
1	[-16.94, -14.94)	1	0.28
2	[-14.94, -12.94)	1	0.28
3	[-12.94, -10.94)	3	0.83
4	[-10.94, -8.94)	3	0.83
5	[-8.94, -6.94)	10	2.78
6	[-6.94, -4.94)	15	4.17
7	[-4.94, -2.94)	22	6.11
8	[-2.94, 0.94)	111	30.83
9	[0.94, 2.94)	89	24.72
10	[2.94, 4.94)	55	15.28
11	[4.94, 6.94)	31	8.61
12	[6.94, 8.94)	14	3.89
13	[8.94, 10.94)	4	1.11
14	[10.94, 12.94)	1	0.28

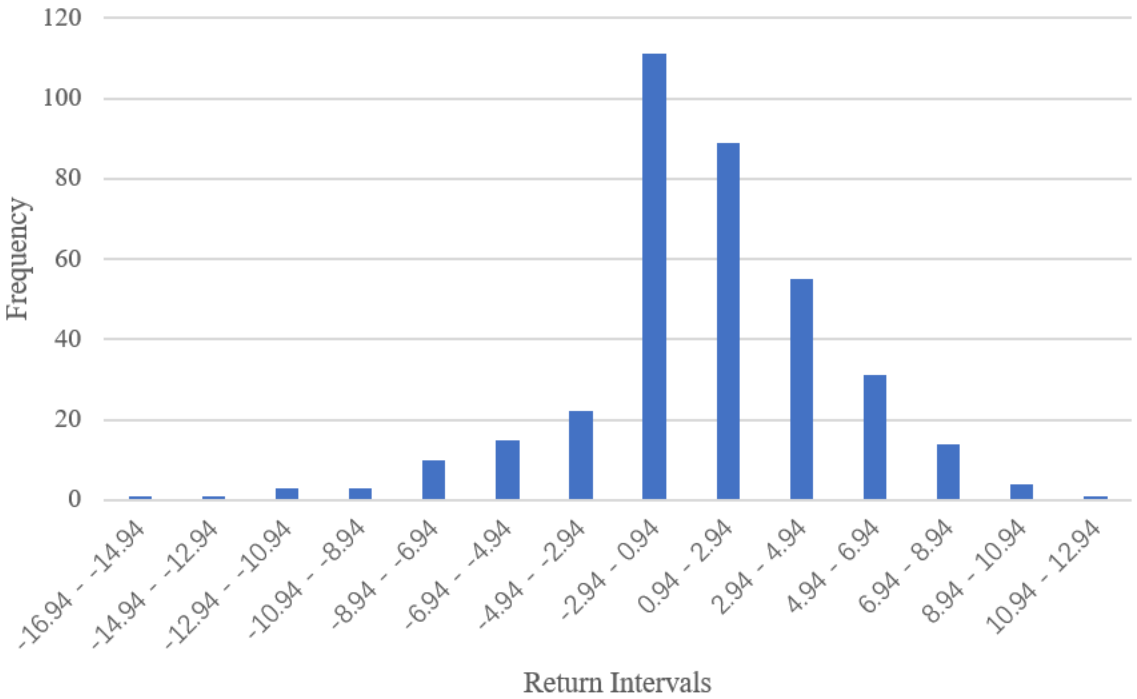

Fig 1 Histogram of S&P 500 Monthly Return 1992 to 2021

The great advantage of a histogram is that it can visually display the important characteristics promptly. For example, we can see at the first sight where most of the observations lie. The extreme values that either too low or too high are quite few with most observations approaching to the center. The arithmetic mean of the sample is 0.77% while the standard deviation of the sample is approximately 0.04.

Regarding the individual stock, the stock of apple group (AAPL)is adopted as our study object and the sample chosen is also monthly return spanning 1992 to 2021. Using the same procedure to construct frequency distribution table and histogram (as Table 2 and Figure 2). Here the parameters for the distribution are that range of 102.24%, interval width is determined as 5% with 21 intervals in total.

Table 2 Frequency Distribution for AAPL Return 1992 to 2021

No.	Return Internal (%)	Absolute Frequency	Relative Frequency(%)
1	[-57.80, -52.80)	1	0.28
2	[-52.80, -47.80)	0	0.00
3	[-47.80, -42.80)	0	0.00
4	[-42.80, -37.80)	0	0.00
5	[-37.80, -32.80)	1	0.28
6	[-32.80, -27.80)	4	1.11
7	[-27.80, -22.80)	3	0.83
8	[-22.80, -17.80)	6	1.67
9	[-17.80, -12.80)	17	4.72
10	[-12.80, -7.80)	28	7.78
11	[-7.80, -2.80)	48	13.33
12	[-2.80, 2.20)	65	18.06
13	[2.20, 7.20)	68	18.89
14	[7.20, 12.20)	57	15.83
15	[12.20, 17.20)	25	6.94
16	[17.20, 22.20)	21	5.83
17	[22.20, 27.20)	6	1.67
18	[27.20, 32.20)	7	1.94
19	[32.20, 37.20)	2	0.56
20	[37.30, 42.20)	0	0.00
21	[42.20, 47.20)	1	0.28

Table 2 shows the frequency distribution for AAPL which is more widespread in terms of rate of return. The arithmetic mean of the sample is 2.41% while the standard deviation of the sample is around 0.12. Comparing to that of S&P 500 return, return for AAPL has greater mean but also with greater standard deviation. That is in line with our expectation that individual stock tends to be riskier than the index as it is not only exposed to systematic risk but also vulnerable to nonsystematic one. The median for the sample is 2.87%.

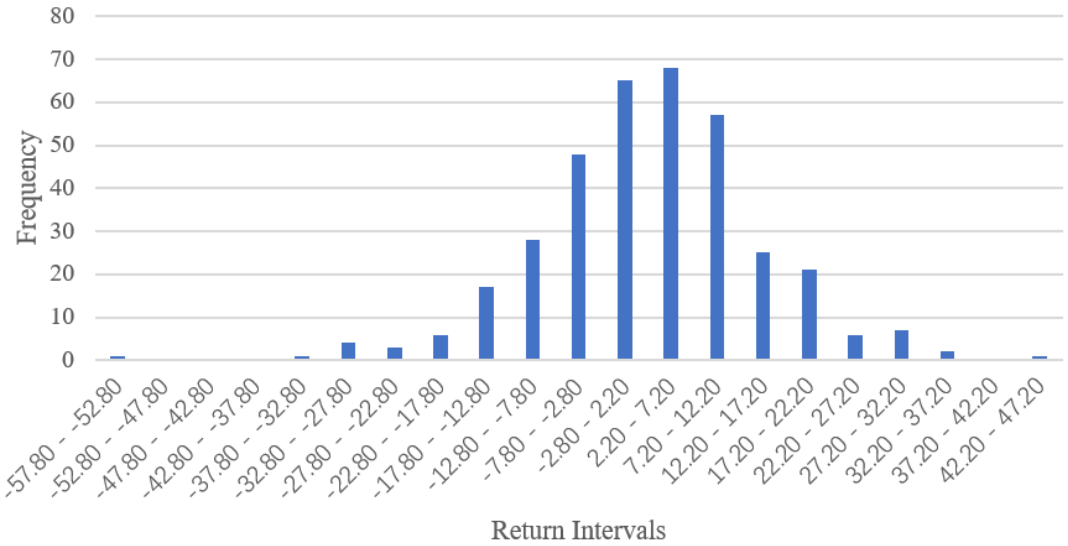

Fig 2 Histogram of AAPL Monthly Return 1992 to 2021

The trends and distribution seem similar to that of S&P 500 as vast majority of the observations lie in intervals ranging from -7.80 to 12.20 with proportion of around 66%. Comparing to that of S&P

500 return (Figure 1), return for AAPL is much more volatile with more dispersed observations falling in a much wider range.

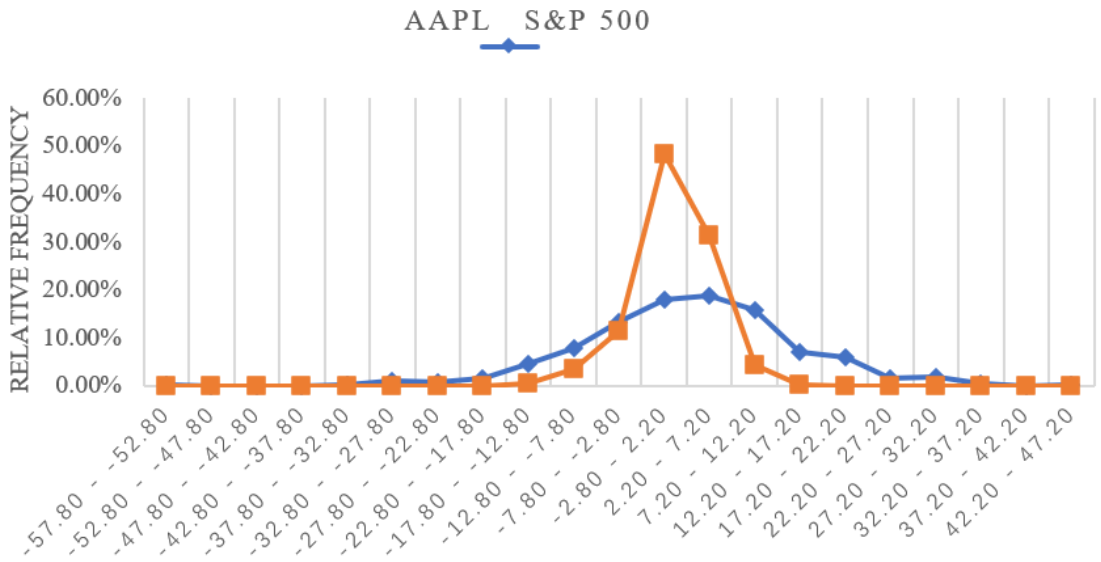

Fig 3 Monthly Return 1992 to 2021 AAPL VS S&P 500

After plotting the figures separately, a line chart combining S&P 500 and AAPL is created (Figure 3) to show the distribution of these two objects. Both two objects show different degrees of central tendency. Distribution for S&P 500 is more compact and focused around the mean as over 90% of the observations fall within three intervals (range from -7.8 to 7.2). In contrast, distribution for AAPL is widely spread with roughly 50% of the data falling into the three intervals and even considerable amount of data scattering over the extreme intervals. In order to see how the time series of the returns correlate and interact over the 360 months, a line chart for the monthly returns against the time is plotted as below:

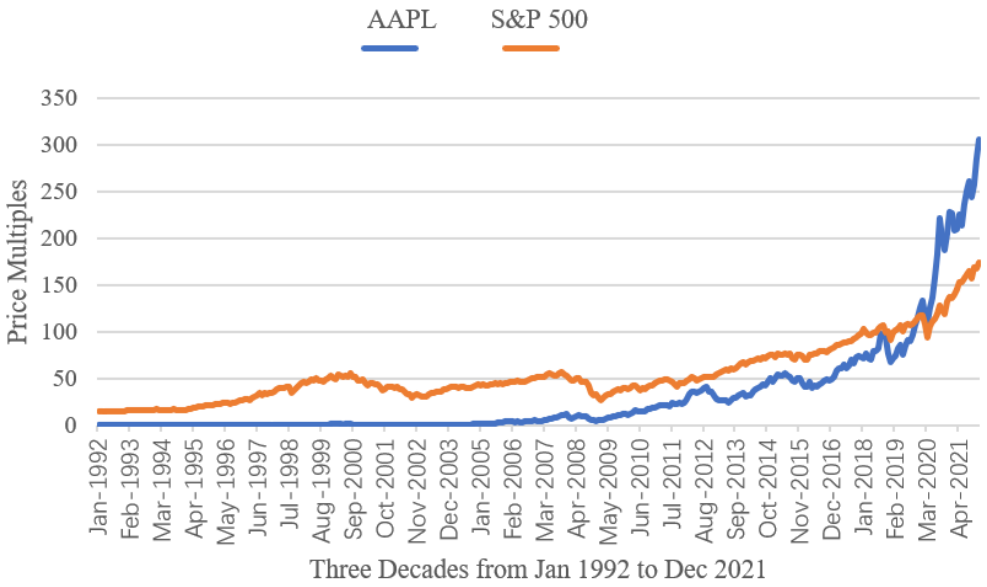

Fig 4 Price Multiples 1992 to 2021 AAPL VS S&P 50

By plotting price multiples against months, Figure 4 shows the trend and pattern of the stock price of AAPL and index amount of S&P 500. Generally, AAPL and S&P 500 was rising over the decades with the way and pattern of rises and falls is similar to some degree. For example, in 2008 of the worldwide financial crisis and 2020 of the Covid-19 pandemic, both AAPL and S&P 500 went down,

hit the bottom and then turned to go up again. AAPL's price however was quite flat in the first 15 years and boosted in the latest 10 years, which can be seen as a steep climb as compared to S&P 500.

3.2 Discussion

After plotting the scatter plot accordingly (Figure 5), this paper got a linear regression line about the correlation between the performance of the stock market (S&P500, on the horizontal line) and that of an individual stock (AAPL, on the vertical line). The slope of the linear regression line, or beta, measures how much the rate of return of AAPL changes as that of S&P500 changes one unit. The greater this ratio, the more sensitive the fluctuation of AAPL to that of S&P500, which means more volatility of the individual stock compared to the market performance. The intercept of the linear regression line, which is easier to understand, simply reflects the rate of return of AAPL when the stock index neither rises nor plunges. In addition, the R^2 value, usually a number between 0 and 1, measures the extent to what the performance of the stock market influences that of an individual stock. The higher the value, the more profound the influence. In other words, a high R-squared value implies great susceptibility of the performance of AAPL to that of S&P500. These principles can be applied to analyze the tendency relationship between S&P500 and AAPL.

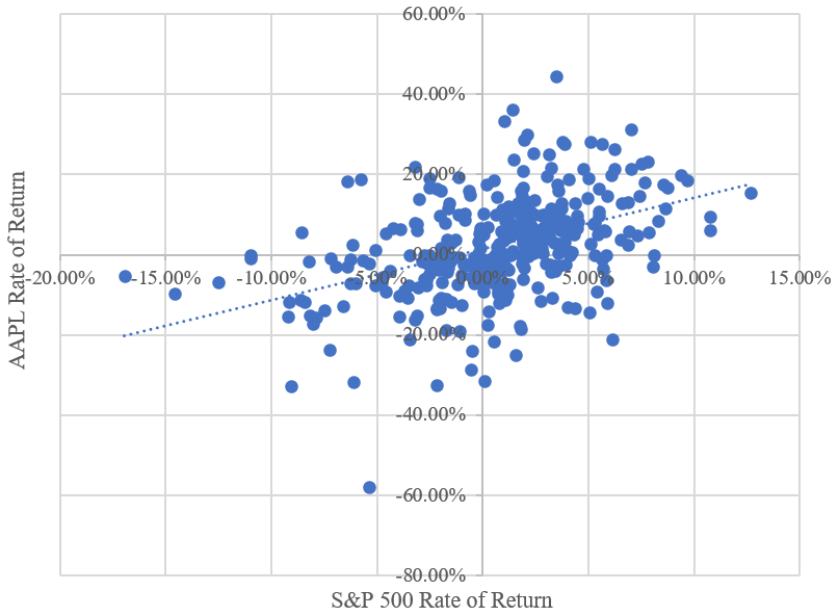

Fig 5 Scatter and Linear Fitting for S&P 500 and AAPL

From the scatter plot, this paper knows that the formula of the linear regression line is

$$y = 1.2785x + 0.0143 \tag{3}$$

With the $R2$ value being 0.1915. That is to say, on average, a 1% rise in the S&P500 Index is most likely to result in a 1.2785% increase in the stock price of AAPL. If the S&P500 Index neither soars nor falls, there is great likelihood that the stock price of AAPL goes up by 1.43%. 0.1915 means that the stock market risk, or more precisely the systematic risk, accounts for 19.15% of the overall risk. With the $R2$ value (0.1915) being relatively low, the effect of the behavior of S&P500 on that of AAPL is not very significant, which indicates a weak connection between the co-movement of the stock price of AAPL and the S&P500 Index. Therefore, the relatively low R-squared value signifies that the behavior of AAPL is influenced more heavily by nonsystematic risk than systematic risk. Conclusively, AAPL is less vulnerable to the fluctuation of the stock market than many other individual stocks. The reason why AAPL is more immune to the influence of S&P500 is that as a conglomerate with one of the highest values in market capital and cash flow, AAPL is more resistant

to abnormally high volatility of the S&P500 Index at some time. Since the performance of AAPL has not much to do with that of S&P500(systematic risk), this paper speculates that the application of CAPM model to assess the expected rate of return on AAPL is probably infeasible. As the slope of the regression line is 1.2785, this Beta Index purports greater volatility of AAPL than that of S&P500. It accounts for the phenomenon that the distribution of AAPL monthly rate of return enjoys a more spread dispersion than that of S&P500 monthly rate of return, which is identical with frequency distribution tables and histograms.

Nonetheless, it is obvious that the CAPM model is effective only when the systematic risk is a leading factor in terms of the trend of an individual stock, which is exactly opposite to the case of AAPL. It may explain why the trend of AAPL exhibits a higher volatility than that of the stock market, although AAPL has a stronger resistance to the impact of S&P500 than most other individual stocks do. Nonsystematic risk really counts in the assessment of AAPL stock performance, but its absence in CAPM model leads to an inaccurate result of prediction of the stock trend of AAPL.

To sum up, CAPM does not perform well in predicting tendency pattern of individual stocks. For large corporations that are not easily influenced by potential effects due to anomalous fluctuation of the stock market, especially in financial crises or during asset bubbles, nonsystematic risk is dominant in the trend of individual stocks. Under this circumstance, the importance of the tendency relationship between an individual stock and the stock market is trivial in terms of CAPM due to the fact that the relationship cannot clearly mirror the individual stock's own trend. By contrast, when it comes to most medium-sized or small-sized enterprises, their stock prices are usually extremely sensitive to changes in the stock market, say S&P500. As such, CAPM would play a pivotal role in the estimation of performance of an individual stock, hence getting a satisfying and accurate verdict. In this situation, the tendency relationship between the individual stock and the stock index is quite close. Above all, the immunity of an individual company to the volatility of the stock market really matters when taking advantage of CAPM to speculate the expected rate of return on an individual stock. The former can also greatly affect the tendency relationship between individual stocks and the stock market (S&P500, for instance). Therefore, this paper suggests that investors employ CAPM in the assessment of companies whose trend of stocks are primarily influenced by stock market rather than news of its own businesses. For the opposite ones, CAPM method is not recommended for its boundedness as it does not take into account the huge influence of nonsystematic risk. This condition often applies to robust companies whose stocks are impervious to volatile fluctuation of the macro stock market.

4. Conclusion

In conclusion, a linear regression model was built up in this paper and then R^2 value was derived and tested. It is fair to say, based on the previous observations and results, that CAPM's applicability is not convincing in explaining and predicting the tendency of individual stocks. Subject to length of the paper, only single one stock (namely AAPL in this case) was selected to explore such tendency relationship with the market. If a set of individual stocks are chosen to implement the same study, the results could be comparable to indicate the degree to which CAPM would estimate return of different individual stocks. There should be some selected stocks of which R-squared value is greater suggesting relatively strong correlation with the market. Exploring the differentiation of the results might discover more valuable clues. Moreover, this paper did not adopt measures or tools to revise CAPM model, nor was the observation data dealt with preprocessing. For instance, AAPL's weight in the constituents of S&P 500 kept rising over the sampling years (1992 to 2021), which would be an unwanted variable contributing uncertainty to the results. All these mentioned factors deserve to be further examined.

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