

# Research on the Investment Strategy Based on Option Volatility Smiles

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**Abstract.** Under the Black-Scholes efficient market assumptions, the implied volatility of options should be constant when the option value state and the remaining maturity period change. However, in the real world, the implied volatility of options in different value states will show smile pattern with given remaining maturity period. Numerous empirical studies have found that implied volatility smile slopes have significant predictive function for stock market returns. Based on this, this paper selects the historical trading data of SSE 50 ETF options to verify the existence of option volatility smile in Chinese capital market. And uses the tilt degree of implied volatility as a stock risk premium factor to select stocks, as to obtain a portfolio with excess returns. This paper portrays the morphological characteristics of option implied volatility, as well as confirms the existence of option volatility smile in Chinese capital market. In addition, this paper demonstrates that the tilt of the smile has significant predictive function for the underlying market trend. Finally, this paper constructs an investment strategy using the tilt degree of the implied volatility smile as an indicator. The findings of this study will help investors to select stocks and construct portfolios based on implied volatility in order to obtain higher returns.

**Keywords:** Black-Scholes model; implied volatility; implied volatility slope; stock returns.

## 1. Introduction

The implied volatility is a function of the option's value and remaining maturity, which indicates investors' expectations of future volatility of the underlying asset. Under the assumptions of the Black-Scholes option pricing model, options on the same underlying asset have the same implied volatility.[1] However, many scholars have observed, in empirical studies, the existence of "volatility smile" and "volatility skew", and explored the reasons behind this phenomenon in terms of asset price process and market trading mechanism. The mainstream view is that the difference of implied volatility in different value states reflects investors' expectations and perceptions of the market, and reflects the tail risk of future market returns.

Theoretical study of implied volatility and the smile curve: The famous option pricing theory of Black and Scholes (1973) assumed that the volatility of any option is constant [1]. However, Platen and Schweizer (1998) observed that the volatility of certain kinds of options is higher than that of parity options, and that the prices of call option and put option are influenced by the existing level of volatility as the expiration date approaches [2]. Exponentially, there is a negative skewness in implied volatility, a phenomenon known as the implied volatility smile, which indicates that the volatility should not be considered as constant. Besides, since the curve that represents the relationship between the strike price of the underlying and the implied volatility resembles a smile, it is called the "smile curve". Merton (1976) demonstrated that the underlying price is consistent with a mixture of price outliers and diffusion [3]. Hull and White (1987) found that when the stochastic volatility of a stock call option price is not correlated with the stock price, the price obtained using the BS formula is not accurate: the price of a two-even option is overvalued, and the price of an out-of-the-money or in-the-money option is underpriced [4].

Empirical test of the B-S model and implied volatility smile: Zhang (2019) explored the relationship between option volatility and stock returns, using volatility skewness as a proxy variable for the volatility smile degree, and the results showed that its predictive function for future stock returns is significant [5]. Chen and Wu (2020) used the SSE 50 ETF call option data, calculated the implied volatility through the B-S Model and the CHEN & PALMON nonparametric option pricing

model, respectively, and found that the former curve showed a smile shape while the latter showed a smooth horizontal line, which was closer to the actual market volatility, and the fitting effect was superior [6]. Cao and Wang (2021) fitted the volatility smile curve to the panel data using 50 ETF option data on January 9, 2017, introduced the interaction term of time and strike price, and found that the best strategy for call options is a trigonometric function, and for put options is a quadratic function [7].

This paper is dedicated to verifying the existence of implied volatility smile in Chinese capital market and studying the predictive function of the skewed degree of implied volatility on stock returns. The paper selects data of SSE 50 ETF options, portrays the morphological characteristics of implied volatility, and confirms the existence of option volatility smile in the Chinese capital market. In addition, this paper demonstrates that the smile tilt has significant predictive function on the underlying market trend. Finally, this paper constructs an investment strategy using the tilt degree of the implied volatility smile as an indicator. These findings will help investors to select stocks and construct portfolios based on implied volatility in order to obtain higher returns.

## 2. Data and Method

### 2.1 Data source

This paper used the trading price data of all option contracts from February 9, 2015 to February 25, 2022, the weekly trading price data of SSE 50 ETF from January 4, 2019 to December 31, 2021, and the weekly trading price data of SSE 50 constituent stocks. The data source is Choice Financial Data Terminal.

### 2.2 Black-Scholes option pricing model

The B-S Model is one of the most commonly used option pricing models. Although the model's assumptions are not consistent with actual conditions, it provides a solid foundation to price various derivative financial instruments, including stocks, currencies, bond, and commodities.

#### 2.2.1 Assumptions of B-S Model

The B-S Model has 9 main assumptions, including:

- (1) The stock price follows a log-normal distribution.
- (2) The option is a European option, which only exercised on a specific date.
- (3) The market risk-free rate is a given constant over the term.
- (4) The stock does not pay any dividends over the term.
- (5) The market does not have any transaction taxes or transaction costs.
- (6) Both the stock and the option are infinitely divisible.
- (7) The investor is able to sell the stock short at a risk-free rate.
- (8) There is not any risk-free arbitrage opportunity.
- (9) The movement of stock prices conforms to geometric Brownian motion.

#### 2.2.2 Equation of B-S Model

The equation of B-S Model is as follows.

$$C(S, t) = S_0 N(d1) - Ke^{-r(T-t)} N(d2). \quad (1)$$

$$d1 = \frac{\ln\left(\frac{S_0}{K}\right) + \left(r + \frac{\sigma^2}{2}\right)(T-t)}{\sigma\sqrt{T-t}} \quad (2)$$

$$d2 = d1 - \sigma\sqrt{T-t} \quad (3)$$

Where  $N(d1)$ ,  $N(d2)$ : the normal cumulative distribution function of  $d1$  and  $d2$ ,  $t$ : the start time of the option contract,  $T$ : the expiration time of the option contract,  $S_0$ : the opening price of the stock,  $K$ : the strike price of the option,  $r$ : the risk-free rate, and  $\sigma$ : the volatility of the stock's return.

Use the B-S Model to obtain the implied volatility. For the current stock price, its distribution and its characteristic  $\sigma$  are unobservable. However, the implied volatility  $\sigma_{imp}$  can be obtained by replacing its theoretical price with the current market price of the option and inverting  $\sigma$  into the computational equation.

Because the assumptions do not correspond to the real market, the BS model has limitations and the volatility smile curve appears in the empirical evidence. The implied volatility is not a constant but a downward convex function with low middle and high sides about the strike price.

### 2.3 The smiling slope of implied volatility

This paper chooses two methods to measure the implied volatility smile slope.

$$\sigma_{OTM\ put\ option}^{imp} - \sigma_{ATM\ call\ option}^{imp} \tag{4}$$

The OTM put option is defined as a put option with a value state  $K/S$  in the interval  $[0.75, 0.95]$ , and  $\sigma_{ATM\ call\ option}^{imp}$  is the mean of its implied volatility. The ATM call option is defined as a call option with a value state in the interval  $[0.95, 1.1]$ , and  $\sigma_{ATM\ call\ option}^{imp}$  is the mean of its implied volatility.

$$\text{Linear fitting with } \sigma^{imp} = \alpha + \beta \log\left(\frac{K}{S}\right) \tag{5}$$

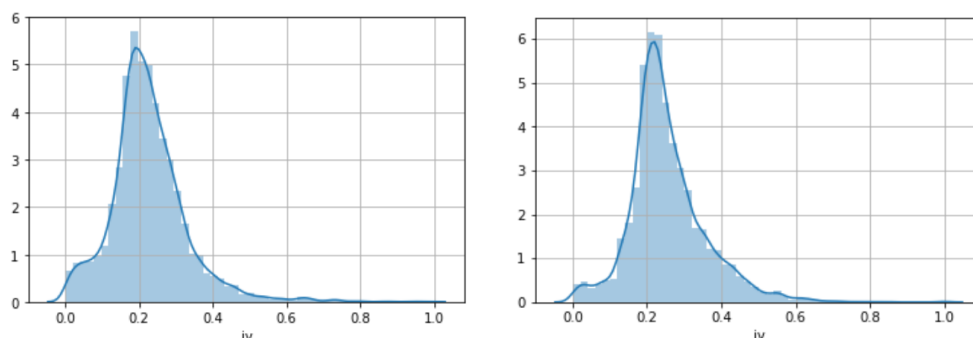
The regression coefficient  $\beta$  can be used as a second indicator to measure implied volatility. By definition, the results of the above two indicators will be negatively correlated. If the predictive effect of implied volatility slope on stock returns holds, the above indicators will predict future returns in opposite direction.

## 3. Empirical tests and analysis of results

### 3.1 Calculate the implied volatility

#### 3.1.1 The ditribution of implied volatility

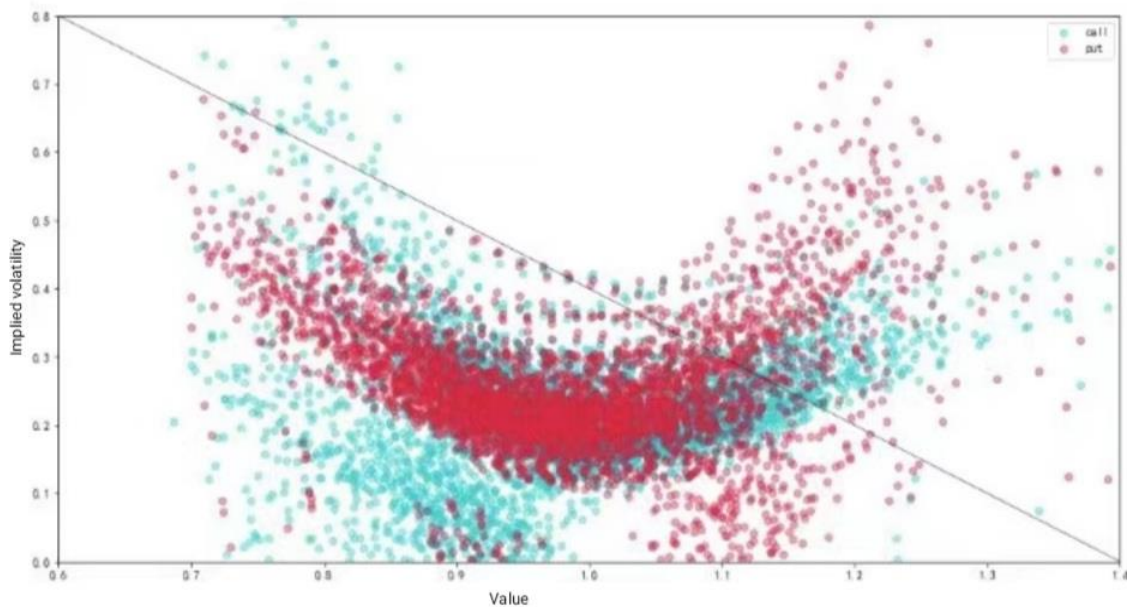
Use the B-S Model to calculate the theoretical prices of call option and put option separately, and then calculate the implied volatility of each option at each point in time. In order to avoid the impact of excessive option price volatility on the estimation accuracy, this paper selects option contracts with durations between 10-60 days for further study, and presents a histogram of the normal distribution of implied volatility for call option and put option (Fig.1).



**Fig. 1** Histogram of normal distribution of implied volatility for 10-60 day call option and put option

### 3.1.2 The smiling pattern of implied volatility

In order to examine the shape of implied volatility, this paper uses  $K/S$  to measure the option's value state, where  $K$  means the strike price and  $S$  means the current price. The option contracts are categorized by type, value state and remaining maturity, and then calculate the mean of implied volatility of sample options in each category. Then, this paper makes a scatter plot to show the fitted image of the implied volatility with respect to the value state (Fig.2). In the figure, the blue scatter points represent call options and the red scatter points represent put options. It can be seen that the smile pattern of implied volatility of SSE 50 ETF options is very obvious, which verifies the existence of the volatility "smile" in the Chinese market.

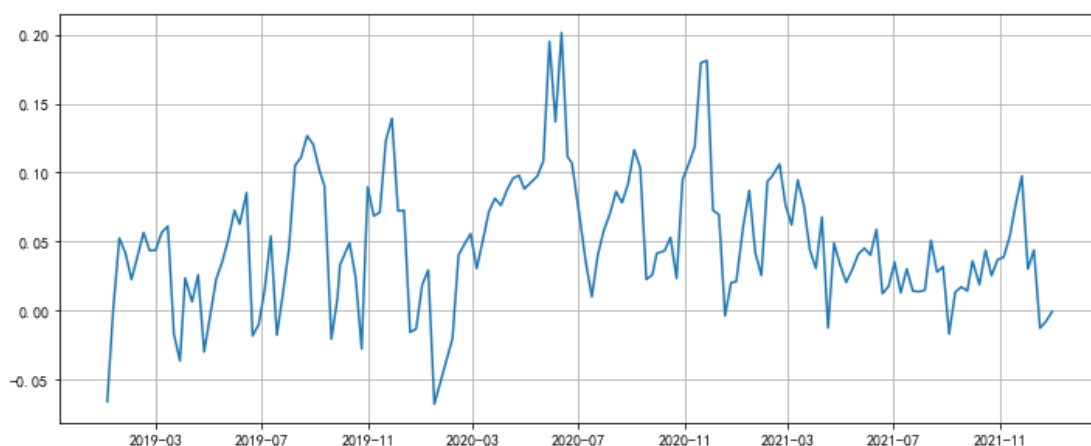


**Fig. 2** The smiling pattern of the volatility of SSE 50 ETF Options

### 3.1.3 The smiling slope of the implied volatility

This paper uses two methods to measure the smiling slope of the implied volatility.

For the first method, the indicator of the slope is the result of implied volatility of the OTM put option minus the implied volatility of ATM call option. The larger the value of the result, the more pronounced the smiling pattern of the volatility.[8] Fig.3 shows the time series trend of the results of this method.



**Fig. 3** The time series trend of the smiling slope of implied volatility (Method 1)

For the second method, this paper uses  $\log(\frac{K}{S})$  to represent the value state of the option, and regress with OLS:

$$\sigma^{imp} = \alpha + \beta \log(\frac{K}{S}) \tag{6}$$

The regression coefficient  $\beta$  is used as an indicator, the smaller the value of the  $\beta$ , the more pronounced the smiling pattern of implied volatility. Figure 4 shows the time series trend of the results of this method.



**Fig. 4** The time series trend of the smiling slope of the implied volatility (Method 2)

Further, this paper calculates the correlation coefficients between the slope K of the implied volatility obtained from the first method and the regression coefficients obtained from the second method (Table.1). The correlation coefficient between K and beta shows that those two methods are inversely correlated for measuring the smiling slope of the implied volatility.

**Table 1.** Correlation coefficients

	Alpha	Beta	K
Alpha	1.000000	0.088578	-0.166353
Beta	0.088578	1.000000	-0.937595
K	-0.166353	-0.937595	1.000000

### 3.2 The predictive function of the smiling slope of implied volatility

Further, this paper tests the correlation coefficients and significance levels of the slope of implied volatility on the future stock returns.

Based on the weekly trading price data of SSE 50 ETF from January 4, 2019 to December 31, 2021, this paper calculates the future stock return rates at each time point (4, 8, 12, 16, 20, and 24 weeks). Then, conduct the regression analysis with the return rates as the explained variables, and the implied volatility slopes measured by OMA and OLS as the explanatory variables, respectively. Table 2 and Table 3 show the regression coefficient and p-value for OMA and OLS respectively.

**Table 2.** Regression coefficient and p-value (OMA)

Week	Regression coefficient	p-value
4	0.194167	0.0271198
8	0.408678	0.000795186
12	0.511881	0.000556996
16	0.388403	0.0179437
20	0.456868	0.0130836
24	0.240428	0.2479

**Table 3.** Regression coefficient and p-value (OLS)

Week	Regression coefficient	p-value
4	-0.0318601	0.0337488
8	-0.0637903	0.00218646
12	-0.0710725	0.00527423
16	-0.0627994	0.0247453
20	-0.0893531	0.00452028
24	-0.0540569	0.133024

Table 2 shows that the explanatory power of the implied volatility slope on future return rates rises between the next 4 and 12 weeks, with a significant effect at 8 and 12 weeks. However, the coefficient decreases between the next 12 and 24 weeks, indicating that the explanatory power of the implied volatility slope on future return rates decreases at that time.

Table 3 shows that the coefficient of the slope decreases between 20 and 24 weeks in the future. It can be seen that using beta as an indicator of implied volatility slope has a stronger predictive power for future stock returns than the OMA method. Some scholars has pointed out that the slope of implied volatility implicates the tail risk of the underlying asset. Therefore, the increasing of smiling slope of implied volatility indicates that the traders' expectation of market downside risk rises, which also helps predict the downward movement of stocks.

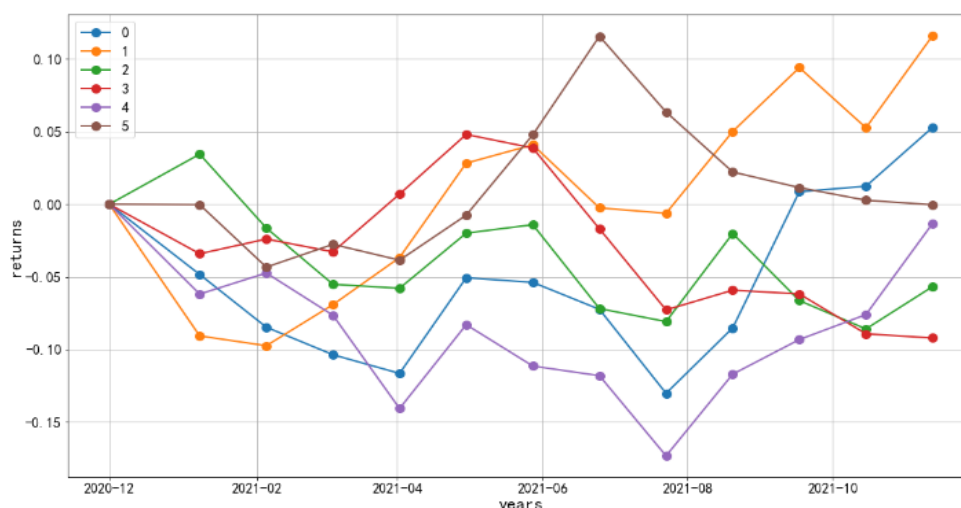
### 3.3 Construct the portfolio

#### 3.3.1 Divide the portfolios with beta

The beta of future returns of each stocks are ranked, grouped, and calculated based on the implied volatility of each share of the options. (1) First, calculate the future 4-week returns each stock at each historical point in time (with a sample of the past 103 weeks starting from the end of the first week of 2021). Then, conduct a regression to obtain the beta of future returns of each stock based on the slope of implied volatility of options (starting from the end of the first week of 2021), and rank the stocks by beta weekly. (2) Next, divide the pool of stocks into 5 tiers according to the value of beta, and calculate the average return rates of each 4-week for each tier. Finally, refresh the regression every four weeks and rebalance portfolio accordingly.

#### 3.3.2 Compare the portfolio returns

Finally, this paper divides the stocks into 6 groups according to the sensitive degree of each stock to the implied volatility of options, and calculates the average return of four weeks for each group of stocks, and then calculates the cumulative return for each group.



**Fig. 5** Cumulative return of each portfolio

Figure 5 shows that the returns of the implied volatility slope is distributed in U-shape, which means the returns of the stock pool are higher when beta is at a smaller or larger value. At the 11th month of the backtest period, the cumulative return of stocks with beta=1.0 reached the highest value of 11.6%, far exceeding the return of SSE 50 index, which equals to 2.7%, and the cumulative return of the stock with beta=0.0 also reached about 5.3%.

#### 4. Conclusion

This paper collects the trading data and stock price of SSE 50 ETF options from Choice financial data terminal, uses the B-S Model to calculate the implied volatility, portrays the morphological characteristics of the implied volatility, and verifies the existence of the option volatility smile in Chinese capital market. Furthermore, this paper demonstrates that the smile slope has significant predictive function on the market trend. Finally, this paper constructs investment portfolio using the tilt degree of the implied volatility smile as an indicator.

The significance of this paper is that it theoretically enriches the literature research, and argues that option implied volatility will show a smile pattern and that the tilt degree of the implied volatility smile will have an impact on future stock returns. In practice, this paper conducts stock selection and portfolio construction based on option implied volatility, which helps investors to obtain higher return.

However, this study still has shortcomings. First, in terms of data selection, the stock index used in this paper is SSE50, but other representative stock indexes such as CSI300 can also be selected for comparison using the same validation.[9] Second, in terms of model selection, this paper uses the B-S Model, which is limited by a set of strict assumptions, and thus the theoretical price does not match the actual price. Therefore, the use of nonparametric pricing models can be considered in future research to further investigate implied volatility.[10]

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