

Research On The Pricing Of Rainbow Option Based On The Geometric Brownian Motion Model: Case Of Pfizer & Walmart

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Abstract. Trading financial derivatives become more and more popular in the modern life. The investor wants to purchase the financial derivative to receive more benefits. However, Option is the important component in the financial derivatives. Compared with Futures, Option can let investor feel more convenience since it acts as the contract. This research analyzes the price of the Rainbow Option in the Stock market. The Stock market includes two companies which are Pfizer and Walmart. The results shows that the Rainbow Option should be priced between max of call options and the sum of the options. It can testify the theory of rainbow Options' price successfully. Based on this research, the investor can understand the background and process of the rainbow Options' working. Combine with the feature of the rainbow options, the investor can make the proper investment decision in the different situations.

Keywords: Rainbow Options; Simulation; Geometric Brownian Motion Model; Stock price; Case of Walmart and Pfizer keywords covered.

1. Introduction.

The types of literature research on rainbow options can be regard as comprehensive and diverse. It covers about the mathematical model, such as the studies in Padovani & Vanini (2007), Ouwehand & West (2006), Rakhmatov & Rasulov (2016), Knox & Ouwehand (2015) [1-4]. The analysis of risk management of rainbow option is also included, which is presented is the research of Dockendorf (2010) and Topper (2001) [5,6]. Milanesi (2021), D.Ahmadian & L.V.Ballestra (2020), Wang & Zhang (2018) analyzes the application of rainbow option in the Economics [7-9].

This research focuses on the application of rainbow option in the stock market of Walmart and Pfizer. Since the rainbow option is customizable, it can provide any combinations for the investor and help them to hedge some risk. However, rainbow option is hard to regard as cost efficiency. The aim of rainbow option is using more capitals to reduce the risk in the investment. Generally, the price of rainbow call/put option should be smaller than the maximum in basket put/call option and smaller than the sum of basket put/call option. This research aims to testify it by simulating the Walmart's stock price and Pfizer's stock price.

This main contribution of this research is to help investor to understand the working of the rainbow option. Particularly, the pricing of rainbow option should be mentioned more. Since the world also experience the pandemic, the financial system become more sensitive and breakable. The company and investor also should pay more attention to reduce the risk in the investment. In that time, the rainbow option can show its functions to the public.

The rest of this paper is structured as follows. Section 2 covers the data and method, Section 3 presents the results and discussion, and the last section is conclusion to analyze the application of the rainbow option in Walmart and Pfizer.

2. Data and Method

2.1 Data

2.1.1 Data collection

This research selects the stock price of Walmart and Pfizer to price the Rainbow Options. In that case, the data sample is from Aug 6th, 2021, to Aug 6th, 2022(About 252 working days adjusted and

close price). The data are collected from Yahoo Finance. The reason to choose the adjusted close price is that this type of price can indicate the stock's value after company's behavior, like deduct the dividend. Based on above information, the risk-free rate will be one-year risk-free rate which should be 3.05%. The yearly risk-free rate is gained from U.S. Department of the Treasury.

2.1.2 Data Screening

After collecting the enough data, part of these data needs to take further calculation to get more details. Firstly, the returns should be calculated by using equation which is next time stock price minus original time stock price and then divide the original time stock price. In order to know the relationship between Walmart and Pfizer, the data also should include the correlation. In this situation, the returns of these two companies can help to find the correlation by taking the correlation function. Secondly, the research must consider the sigma value (standard deviation) of Walmart's returns and Pfizer's returns. Since the maturity is one year, the yearly sigma value of these two companies also needs to be showed in the data performance. The stock price of two companies in the original maturity should be remarked in these data since it is the necessary part in the calculation of the strike price and simulation.

2.2 Method

2.2.1 Monte Carlo Simulation

The z value is obtained by using the normal distribution. Combine with the z value for Walmart and Pfizer, this research uses Monte Carlo simulation [2] to simulating the stock price's return of Walmart and Pfizer 1000 times. In that case, this way can get overview of the simulation of the stock market. In this overview, Walmart and Pfizer also have 1000 strike price returns which can be used in the analyzing of European put and call option part.

2.2.2 Finding Optimization (return and loss)

This research let the 1000 results of strike price's return of Pfizer and Walmart as the example to calculate the maximum return and minimum loss in this procession. During the procession, the return will be gained by maximizing zero and the different returns. The loss will be gotten by minimizing zero with the different returns. However, the loss and return include the performance of the Walmart and Pfizer, it is hard to analyze each company. Based on the European option theory, this research also provides the situation of each company's European put and call option for investor. After that, the thousand values of these four situations will be taken the average to get the Euro call option and Euro put option of Walmart and Pfizer. The return and loss will also experience the same calculation to gain the basket call and basket put.

2.2.3 Validation of Assumption

Recall the assumption that the research points out in the introduction part, the research aims to testify the application of rainbow option in the Walmart and Pfizer. Since the rainbow options should be priced between max of call options and the sum of the options. In that case, the euro call option, euro put option, and the sum of option need to be compared with basket call and basket put. During this process, the application of rainbow option can be testified.

2.2.4 Sensitivity Analysis

In order to evaluate the equality of this simulation, sensitivity analysis helps creator to check the effect of the working procession. The sensitivity is divided into four parts:

First part is the relationship between correlation and rainbow call price, the second part refers the relationship between volatility and rainbow call price, third part indicate the relationship between time period and rainbow call price, and the last part represents the relationship between risk free rate and rainbow call price

These four different types of sensitivity analysis Keep the dependent (y-axis) always be the rainbow call price, the independent refer to correlation, risk rate, time, and volatility. It can help

investors to discover the influence of price (positively or negatively) by relying on the changing of risk, correlation, and time. Sensitivity analysis provide the opportunity for creator to make sure the efficiency of the simulation.

3. Result

3.1 Simulation

References are Based on the data in Table 1 and z-value which is get by normal distribution, the asset price can be simulated by using the geometric Brownian motion model [10]:

$$S_t = S_0 * e^{(\alpha - 0.5\sigma^2) * T + z\sigma\sqrt{T}} \quad (1)$$

Where S_t represents the future value of asset price; S_0 denotes the given value of asset (49.27 and 126.580002 in this research), T should be time period.

The equation of z-value:

$$Z_1 = \text{normsinv}(\text{rand}()) \quad (2)$$

$$Z_2 = \rho Z_1 + \text{normsinv}(\text{rand}()) * \sqrt{(1 - \rho)^2} \quad (3)$$

Table 1. The basic information of simulation

correlation	0.1257		
risk-free rate	0.0305	yearly risk-free rate	0.0305
sigma_pfz	0.0198	yearly sigma_pfz	0.3145
sigma_WMT	0.0154	yearly sigma_WMT	0.2446
time period	1		
S0_pfz	49.27		
S0_WMT	126.5800		
cash	1000000		

The Table 2 indicate the 1000 results of simulated stock price of the Pfizer and Walmart. The S_1 refer to the result of the Pfizer's stock price. The S_2 belongs to the result of Walmart's stock price.

Table 2. The procession of simulation

	Z_1	S_1	Z_2	S_2
1	0.8416	62.9934	0.0589	128.4901
2	0.3600	54.1407	0.8340	155.3189
3	-1.0751	34.4770	-1.1722	95.0768
4	0.0628	49.3102	0.1753	132.2030
5	-0.0286	47.9124	2.2174	217.8744
		...		
996	-1.2858	32.2669	0.4716	142.1396
997	0.3643	54.2130	0.9234	158.7522
998	0.4200	55.1721	1.3465	176.0640
999	-0.9846	35.4718	-0.1481	122.1465
1000	0.3366	53.7433	0.0683	128.7855

(The value of ρ in this research should be the correlation of Pfizer and Walmart's stock price return)

3.2 Return

The simulated data will help to get the value of maximum return, minimum loss, European call / put option for Pfizer and the European call/put option for Walmart. Since the rainbow option need creator to find the optimized return and minimum loss. In that case, the process of calculating need to use the MAX and MIN function which shows below.

$$\text{Max return} = \text{MAX} ((S_1 - S_{01})/S_{01}, (S_2 - S_{02})/S_{02}, 0) \quad (4)$$

$$\text{Min loss} = \text{MIN} ((S_1 - S_{01})/S_{01}, (S_2 - S_{02})/S_{02}, 0) \quad (5)$$

$$\text{European call option for Pfizer} = \text{Max} ((S_1 - S_{01})/S_{01}, 0) \quad (6)$$

$$\text{European call option for Walmart} = \text{Max} (S_2 - S_{02})/S_{02}, 0) \quad (7)$$

$$\text{European put option for Pfizer} = -\text{MIN} ((S_1 - S_{01})/S_{01}, 0) \quad (8)$$

$$\text{European put option for Walmart} = -\text{MIN} (S_2 - S_{02})/S_{02}, 0) \quad (9)$$

Where, S_{01} represents the sigma of Pfizer, and S_{02} denotes the sigma of Walmart (Since the data should shows the positive value, the min equation need contain “-” sign.)

Table 3. The optimization part of Walmart and Pfizer’s stock price

Max return	Max loss	European call option for pfz	European call option for WMT	European put option for pfz	European put option for WMT
return 1	loss 1	0.2785	0.0151	0	0
0.2785	0	0.0989	0.2270	0	0
0.2270	0	0	0	0.3002	0.2489
0	0.3002	0.0008	0.0444	0	0
0.0444	0	0	0.7212	0.0276	0
0.7212	0.0276	0.3070	0.4496	0	0
0.4499	0	0	0	0.0416	0.1316
0	0.1316	0	0	0.2233	0.1995
0	0.2234	0.0149	0.3924	0	0
0.3924	0	0.4614	0.1303	0	0
0.4614	0	0.1893	0.3061	0	0

3.3 Check the Application of Rainbow Option

References Based on the theory of rainbow option, the price of the rainbow option should between the Max of call options and sum of the out/call options. During this process, the Pfizer’s call / put option, Walmart’s call/put option, whole basket call, and basket put all need to be mentioned in the comparison.

There are equations to calculate above data:

Basket call = average of the value of returns in 1000 times simulation

Basket put = average of the value of loss in 1000 times simulation

European call option for Pfizer/Walmart = average of Euro call option in 1000 times simulation for each company

European put option for Pfizer/Walmart = average of Euro put option in 1000 times simulation for each company

Compared basket call/put with European put/call option of these two companies, then the result can show it fit the theory. It means that the rainbow option fit the range that the theory points out.

After that, using the assumed cash value one million can get the basket call discount and basket put discount (price). The formula is:

$$\text{Basket call discount} = \text{cash value} \times e^{(-\text{yearly risk-free rate} \times \text{time period} \times \text{maturity})} \times \text{basket call} \quad (10)$$

$$\text{Basket put discount} = \text{cash value} \times e^{(-\text{yearly risk-free rate} \times \text{time period} \times \text{maturity})} \times \text{basket} \quad (11)$$

Table 4. The final result of rainbow option price in simulation and optimization

Basket call	0.2113			
Basket put	0.1583			
price (The rainbow options should be priced between max of call options and the sum of the options.)		Euro call option of Pfz	0.1372	Euro put option of Pfz 0.1073
Basket call discount	Euro call option of WMT	0.1115	Euro put option of WMT	0.0860
Basket put discount	rainbow call option	TRUE	rainbow put option	TRUE
Euro call option of Pfz	0.1372	Euro put option of Pfz	0.1073	
Euro call option of WMT	0.1115	Euro put option of WMT	0.0860	
rainbow call option	TRUE	rainbow put option	TRUE	

3.4 Sensitivity Analysis

In figure 1, it indicates that the relationship between correlation and rainbow call price are negatively. It means that the correlation increased, the difference will go down. In the other view, the risk avoidance will become more rigid.

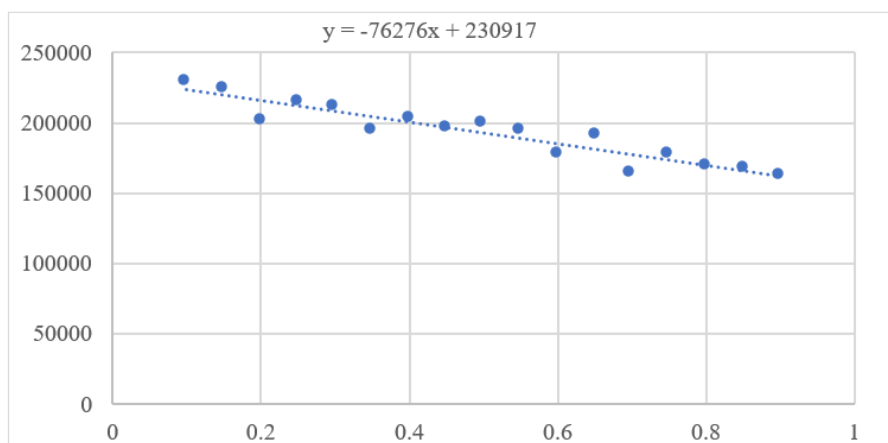


Figure 1. correlation and rainbow option price

The figure 2 and figure 3 both reveal the relationship are positively. Since the volatility denotes the risk of the rainbow option. It means that if the risk higher, the price will become higher. This kind of relationship express that the option can hedge they risk in the investment.

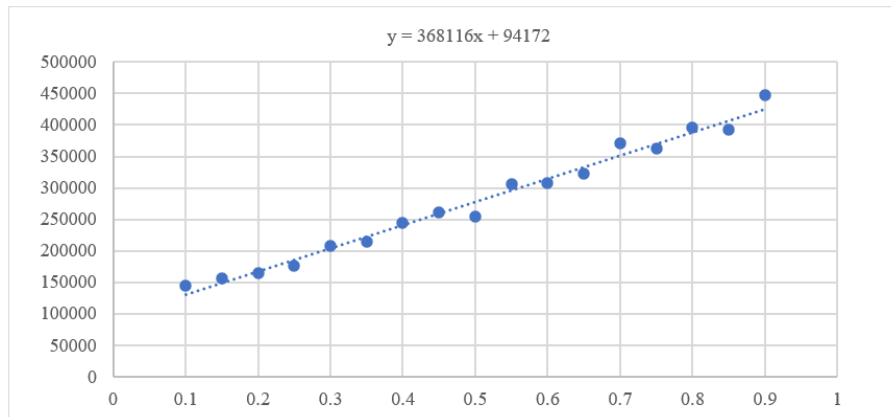


Figure 2. the relationship between rainbow call price and the volatility of Pfizer

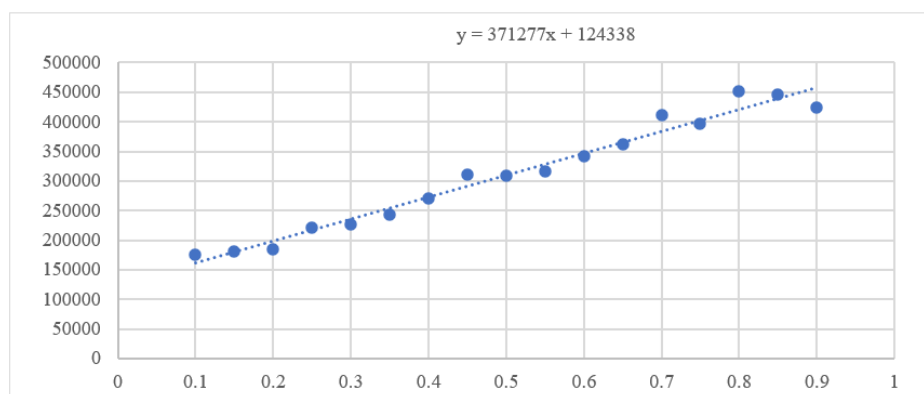


Figure 3. The Link Between Rainbow Call Price and The Volatility Of Walmart

Figure 4 shows that the increasing of the risk-free rate will influence the growth of the option price. It points out that if the call options bid fixed, the investor need not pay another fee to lead the price goes up.

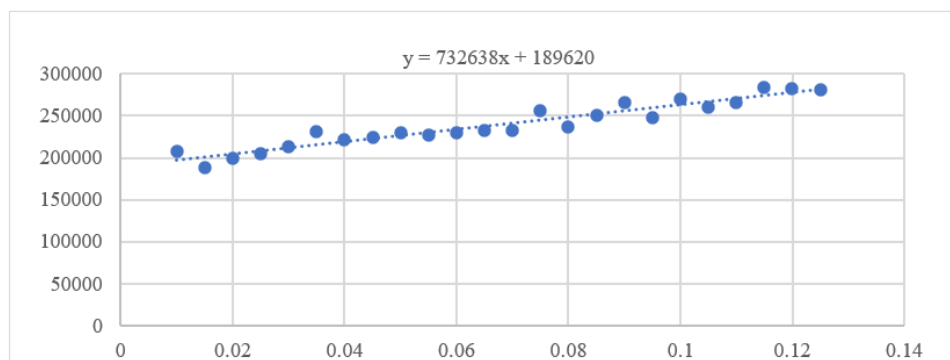


Figure 4. Interest Rate and Rainbow Option Price

4. Discussion

Based on the study of the applying the Rainbow Options of Pfizer and Walmart stock price, it proves that the rainbow options should be priced between max of call options and the sum of the

options. Rakhmatov & Rasulov (2016) also points out the same result about the Rainbow Options [3]. But it focuses on the mathematical method, like using some functions and derivatives to test the value of strike price. Based on these results and thinking, it helps Rainbow Options to show the features to the public. For the investor, the more capitals the investor investing, the less risk will be hedged. It means that the Rainbow Options can help investor to hedge the risk and provide any combinations for investor. On the other Hand, the Rainbow Options also expresses the disadvantage or bias in the investment. The Rainbow Options reduced the risk of the premise is to raise the investment of investors. In that case, Rainbow Options cannot be regards as the cost efficiency. If the investor plans to use the Rainbow Options, the cost part should This research uses the simulation and normal distribution to show the whole working process of the rainbow options' work in the stock market. Combine the process and the result, it indicates that the application of the rainbow option in the stock market fits the theory of rainbow options' pricing which is the price should be set between max of call options and the sum of the options. During this research, it provides the features of the rainbow options for the public. The investor can understand the advantages and drawbacks in the whole procession. Although the rainbow options can offer many combinations for people, the key point of the rainbow option is that investing more to hedge the risk. In that case, how to balance the investment and risk is the unavoidable question for every investor. If the investors want to use the rainbow option, considering comprehensively is necessary thing. On the other hand, the link between risk and investment in rainbow option also need more further investigation to get more details about these two elements. It is possible that the more effective combination in rainbow options will appear in the future.be considered more

5. Conclusion

This research uses the simulation and normal distribution to show the whole working process of the rainbow options' work in the stock market. Combine the process and the result, it indicates that the application of the rainbow option in the stock market fits the theory of rainbow options' pricing which is the price should be set between max of call options and the sum of the options. During this research, it provides the features of the rainbow options for the public. The investor can understand the advantages and drawbacks in the whole procession. Although the Rainbow Options can offer many combinations for investor, the key point of the rainbow option is that investing more to hedge the risk. In that case, how to balance the investment and risk is the unavoidable question for every investor. If the investors want to use the rainbow option, considering comprehensively is necessary thing. On the other hand, the link between risk and investment in rainbow option also need more further investigation to get more details about these two elements. It is possible that the more effective combination in rainbow options will appear in the future.

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