Comparing the Payoff Differences Between the Barrier and European Options Based on the Black-sholes Model

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Abstract. As more people want to invest in the options market, there are basic traditional options in the futures market, such as European options. Still, there are also some exotic options like the barrier option that is conditional on the stock price before expiration and is called a barrier (1). This paper analyzes the payoffs of European and barrier options based on the Black Scholes model by calculating them and comparing them. This study analysis of the results shows that the price of the barrier option is lower than that of the European option. The barrier option is essentially close to the European option under conditions. This article helps newcomers to the options and those unfamiliar with exotic options to understand the differences between barrier options and European options in terms of payoff.

Keywords: European options, Barrier options, Black-sholes Model

1. Introduction

As the investment environment and financial markets mature, more investment options are known to everyone. There are the most common European and American options, and securities firms have also introduced many exotic option products, such as barrier options. Barrier options are exotic, path-dependent, and in many ways identical to regular options. However, they are only activated or expire if the underlying stocks rise above a barrier. In-options are initially worthless and only become active if a predetermined knock-in barrier price is breached. Out-options begin their operational life and expire if a specific knock-out barrier limit is exceeded [1-3]. Barrier options can be more complicated for people new to financial products when they face multiple conditions.

Regarding the literature, Hui (2007) discussed their delta, gamma, and vega risks by analyzing barrier options with front-end barrier options and rear-end barrier options. And compare them with the risk factors of ordinary options [4]. Paul and Turtle (2003) proposed a framework for valuing corporate securities based on a path-dependent, barrier option model and confirmed that barriers remain essential in estimating a wide range of input variables and are, in most cases, z-scores [5]. Carr (1995) Two extensions are proposed for barrier options, the first allowing an initial protection period during which the option cannot be eliminated. The second considers an option that will only be eliminated if the second asset hits the cap [6]. Carl Chiarella, Boda Kang, and Gunter H. Meyer used the line method to evaluate prices and the delta and gamma of options in 2012. The process can effectively handle continuously monitored, discrete barrier options with early strike characteristics and calculate early strike bounds for American-style barrier options [7]. Paul Glasserman and Jeremy Staum (2001) use variance reduction techniques for the structure of barrier options, applicable to general simulation problems with similar structures, and also introduce algorithms that simultaneously combine measurement variation and conditional probability estimation [8]. Don R. Rich (2002) presents the mathematical foundations needed to evaluate barrier options in an intuitive and unified framework. Closed-form solutions are derived, and comparative static data are provided for European barrier options with fixed rebates and barrier levels [9]. Kai and Qin (2020) studied the barrier option and designed numerical algorithms to calculate the prices of barrier options based on these formulas by deriving pricing formulas based on the structure of the solutions of uncertain differential equations [10]. This study aims to compare traditional European options with exotic options such as barrier options to make it easier for readers to understand them. When looking for a barrier option instead of a European option, it is essential to understand the two options separately. A

European option is a type of option contract that restricts the holder's right to exercise the contract only on the expiration date. Still, the holder can buy or sell the option by trading in the market as one of the most widely used options. The call options in European options are used as the standard in this paper (2). This study compares the similarities and differences between European and barrier options to gain a deeper understanding of barrier options.

2. Data and Method

2.1 Data

Both options in this paper are linked to shares of Tesla Inc. The historical closing price of Tesla stock from January 2, 2020, to August 15, 2022, is selected. Furthermore, use the historical closing price to calculate the increase or decrease for each day.

$$\%change = \frac{current\ price - last\ price}{last\ price} \times 100\% \quad (1)$$

Table 1. Tesla Stock Historical Closing Price

Date	Adj Close	% Change
1/2/20	86.052002	
1/3/20	88.601997	2.88%
1/6/20	90.307999	1.89%
1/7/20	93.811996	3.74%
1/8/20	98.428001	4.69%
1/9/20	96.267998	-2.24%
1/10/20	95.629997	-0.67%
1/13/20	104.972	8.90%
1/14/20	107.584	2.43%
1/15/20	103.699997	-3.75%
1/16/20	102.697998	-0.98%
1/17/20	102.099998	-0.59%
1/21/20	109.440002	6.71%
…	…	…
8/8/22	871.27002	0.78%
8/9/22	850	-2.50%
8/10/22	883.070007	3.74%
8/11/22	859.890015	-2.70%
8/12/22	900.090027	4.47%
8/15/22	927.960022	3.00%

According to the return for each day, this study calculates the volatility needed in black-Scholes model based on the return for each day. This paper uses the standard deviation formula to calculate daily volatility based on each day's return in excel.

$$Excel\ function = stdv(\dots) \quad (2)$$

To convert to annual volatility, multiply by the square root of 252 trading days per year. After calculation, this paper obtains daily volatility of 4.65% and annual volatility of 74%.

The stock price is taken from the closing price of Tesla stock on yahoo finance on August 16, 2022. The risk-free rate is taken from the U.S. DEPARTMENT OF THE TREASURY website. which sets its strike price and up and down barrier prices.

Table 2. Basic information about Tesla stock

Stock Price on2022/8/16	919.69
Volatility	4.65%
Trading Data	252
Annualized Volatility	74%
Time	0.07936508
Strike	900
Risk free rate	0.0226
Div	0
Up barrier	1000
Down barrier	630

2.2 Method

Fischer Black and Myron Scholes published the BSM model in 1973, with significant assistance from Robert Merton (often called "Black-Scholes"). The basic principle of Fischer Black and Myron Scholes is that "if options are correctly priced in the market, it should not be possible to ensure profits by creating portfolios of long and short positions in options and their underlying stock."

Black -sholes Model:

$$C = S \cdot N(d1) - Xe^{-rt} \cdot N(d2) \tag{3}$$

$$d1 = \frac{ln(\frac{S}{X})+(r+\sigma^2/2)\tau}{\sigma\sqrt{\tau}};\ d2 = d1 - \sigma\sqrt{\tau} \tag{4}$$

They assumed that future stock prices were lognormally distributed, which meant that they could be represented as a function of "z." (2).

$$S_T = S_0 e^{(\alpha-\frac{1}{2}\sigma^2)T+z\sigma\sqrt{T}} \tag{5}$$

$$\alpha = r - \delta \quad for\ stock \tag{6}$$

3. Results and discussions

3.1 Simulation of stock prices

In this paper, the data in table 1 and table 2 are calculated using the BSM model in an Excel table, and a total of 1000 simulations are performed to estimate the stock prices for the next 30 trading days from August 16, 2022. The following table 3 was obtained.

Table 3. Simulation of stock prices

sim	day1	day2	day3	day4	day5	…	day29	day30
1	908.08	921.56	877.10	902.60	784.22	…	878.37	946.86
2	920.20	919.88	922.46	927.34	907.49	…	926.31	889.06
3	946.16	927.58	841.42	834.72	910.56	…	930.79	881.79
4	887.83	982.10	829.15	985.49	910.54	…	950.56	892.67
5	907.93	963.20	893.07	882.25	941.58	…	887.63	917.97
6	873.19	888.60	973.83	835.36	858.08	…	1001.20	969.89
7	946.16	886.63	951.00	922.43	878.22	…	886.51	972.28
8	878.85	922.27	955.21	867.38	878.03	…	908.55	914.49
9	874.57	884.47	837.93	895.34	968.51	…	921.14	1045.40
10	965.15	903.66	901.17	1014.88	868.26	…	954.51	991.55

…	…	…	…	…	…	…	…	…
995	1035.67	906.93	808.85	889.41	929.74	…	964.60	956.80
996	865.63	998.56	935.24	974.50	884.34	…	900.04	957.02
997	1004.84	896.37	932.75	966.32	937.82	…	979.51	897.37
998	934.13	946.95	930.90	905.11	857.99	…	972.16	878.19
999	936.73	925.60	863.00	966.37	920.36	…	901.11	1007.04
1000	962.50	896.66	935.40	946.16	937.75	…	915.86	898.12

3.2 pricing of European option

This paper calculates the European call option and barrier knock-in call options separately based on the prices calculated by BSM. The European call option gives the holder the right to purchase the security. Its payoff is $(S_t - K)^+$at the time of exercise at maturity. After 1000 more simulations of the return, we obtain the following data in table 4.

Table 4. simulation of European call

Sim	European call
1	46.86
2	0.00
3	0.00
4	0.00
5	17.97
6	69.89
7	72.28
8	14.49
9	145.40
10	91.55
11	11.51
12	0.00
13	0.00
14	0.00
15	0.00
16	0.00
17	22.97
18	24.85
19	13.15
…	…
996	57.02
997	0.00
998	0.00
999	107.04
1000	0.00

3.3 Pricing of up -and -out option

The payoff of an up-and-out barrier call option with exercise date T, strike price K, and barrier level (or call level) B on the underlying asset price St is,

$$C = (S_t - k)^+\{max_{0<t<T}(S_t < B)\} \quad (7)$$

After 1000 simulations of the payoff, we obtain the following data in table 5.

Table 5. simulation of Barrier (up and out) call

Sim	Barrier (up and out)
1	0.00
2	0.00
3	0.00
4	0.00
5	0.00
6	69.89
7	0.00
8	14.49
9	145.40
10	91.55
11	0.00
12	0.00
13	0.00
14	0.00
…	…
996	57.02
997	0.00
998	0.00
999	107.04

It is clear from the calculation that the barrier option is also based on the European option by adding some additional conditions and thus controlling the return. If we combine up-and-out and up-and-in calls, we get the same results as the European option. This paper does the same simulation, calculating the average payoff of European option, up- and- outcall, and up-and- in call, respectively, where the average of up and out call plus the average of up and in call equals the average of European option payoff, which further illustrates the close connection between barrier options and European options.

Table 6. Pricing of European and barrier options

	European call	up and out	up and in
Average	27.77860047	19.6348708	8.14372966
DISCOUNT	27.72882002	19.59968427	8.12913575

3.4 Sensitivity analysis

The sensitivity test in this article is done using the "what if" function in excel. By controlling the volatility of the stock separately and changing the value of the different barriers.

In this paper, the following simulation is done to compare the payoff of up and outcalls in barrier options and European options by controlling the volatility of the stock. The results of the sensitivity analysis are shown in figure 1.

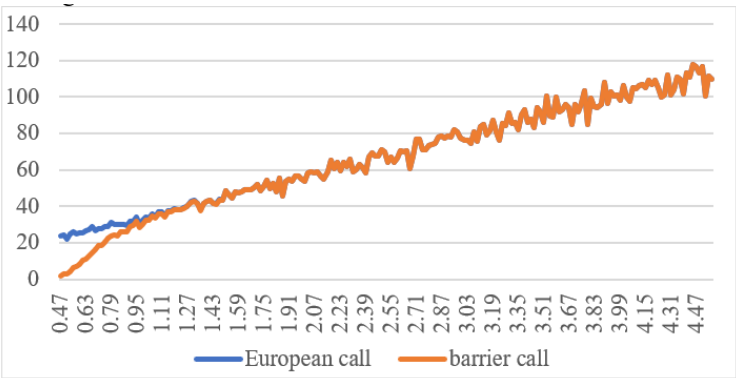

Fig 1. sensitivity test by change volatility

Moreover, comparing the payoff of up and outcalls and the European call option by changing the barrier price. The results of the sensitivity analysis shown in figure 2.

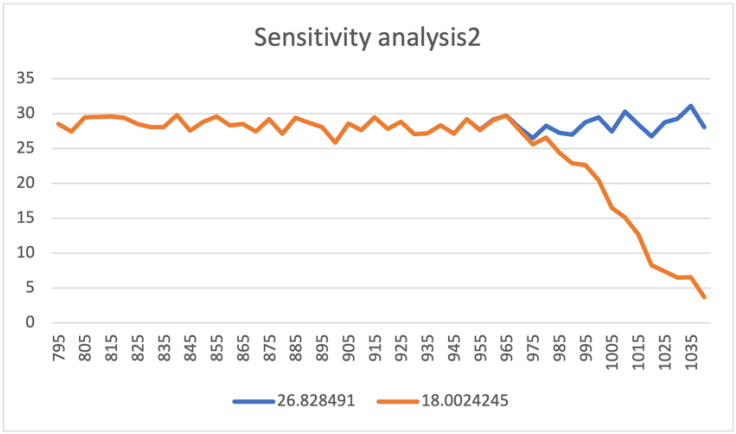

Fig 2. sensitivity test by change barrier price

4. Conclusion

By calculating the payoff of European options and barrier options separately, it is found that barrier options and European options are inextricably linked. Although in most cases barrier options are priced lower than European options, there are some special cases where the barrier option will be close to the European option. It is difficult for the novice to determine whether a barrier value will result in a greater profit, and barrier options have different forms of call-and-put options and different barrier values. Nevertheless, the final payoff shows us that the barrier option is just a variation of the European option. The barrier price is simply a threshold set to control the risk in the stock market. The sensitivity test shows that when stock becomes more volatile, the returns from the barrier option and the European option almost overlap in the test to control the barrier value. When the barrier value exceeds a specific price, the return values obtained by the two options gradually separate. Again, the close relationship between the barrier and European options is demonstrated. This article shows how barrier options work for the novice investor and how he differs from European options in terms of payoff.

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